

You can work on paper and scan your solutions or type out your answers onto the computer. Use of mathematical typing software is not required. You should **not use a calculator** or any other calculation device.

Paper D Total = 100 marks

1)

Let $f(x) = a^2x^2 - 2ax$ and $g(x) = x^2 - 2ax$

where a is a constant such that $a \in \mathbb{R}, a \neq 0$.

Any answers for this question can be left in terms of a where appropriate.

a) Write the following expressions as the product of two linear factors:

i) $f(x) + 1$ ii) $f(x) - 3$.

4 marks

b) Find the minimum value of $g(x)$.

3 marks

c) Find $\frac{ax}{f(x)} - \frac{x}{g(x)}$ in the form $\frac{Ax+B}{h(x)}$ where A, B are constants to be determined and $h(x)$ is a function of x .

6 marks

d) Find all values of x or a such that $f(x) = ag(x)$

4 marks

Total 17 marks

2)

Each of the following equations is written in terms of a where a is a real number.

i) Solve each equation exactly giving your answers in terms of a .

ii) State the set of values of a that ensure that x is a real number.

a) $2^{2x} - a2^{x+2} + 3a^2 = 0$

5 marks

b) $\cos^2(ax) + 3\sin(ax) = 3$

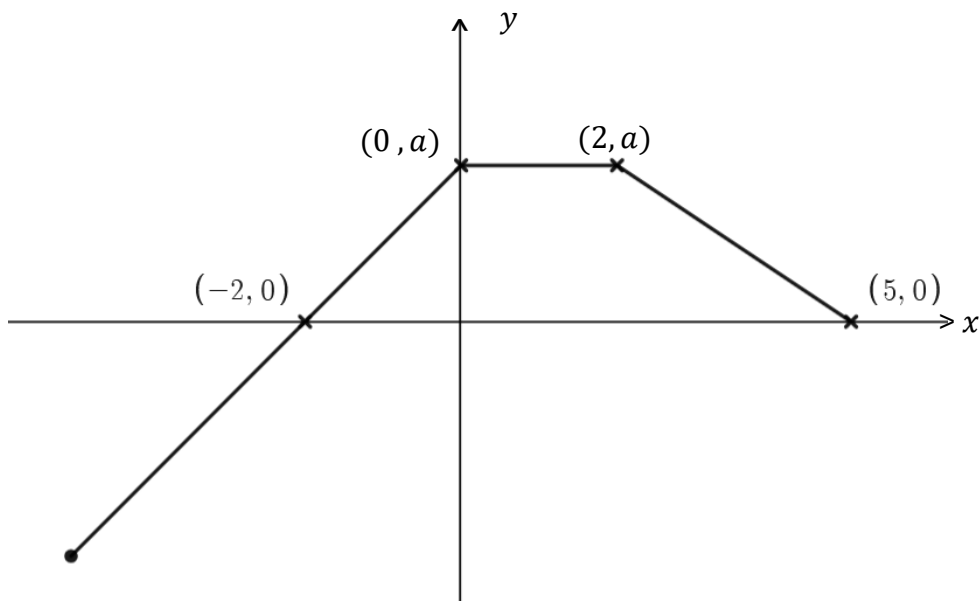
7 marks

c) $\log_{10}(10x) + 2\log_{10}x = 4 + \log_{10}a$

5 marks

Total 17 marks

3)



The function $f(x)$ for $-5 < x < 5$ is shown in the diagram above. where the constant $a > 1$.

The function $g(x) = -bx^2 + 6a, x \in \mathbb{R}$

a and b are constants such that $a > 1, b > 1$.

Any answers for this question can be left in terms of a and/or b where appropriate.

a)

Find :

i) $f(-2)$

ii) $ff(5)$

iii) $f'(-2)$

iv) $f'(3)$

v) $g'(3)$

vi) $gf(0)$

9 marks

b)

Solve the equation $fg(x) = 0$.

4 marks

c)

Find the following integrals:

i) $\int_{-2}^2 f(x)dx$

ii) $\int_{-4}^{-2} g(x) - f(x)dx$

iii) $\int g(x)e^{\frac{x}{2}}dx$

11 marks

Total 24 marks

4)

A curve is defined by the following equation:

$$5x + A^2x^2y - 27B^2y^3 = B^2$$

where A and B are constants and $A > 0, B > 0$.

Any answers for this question can be left in terms of A and/or B where appropriate.

a)

Find $\frac{dy}{dx}$ giving your answer in the form $\frac{f(x,y)}{g(x,y)}$.

6 marks

b)

Find an equation for the normal to the curve at the point where $x = 0$.

4 marks

Total 10 marks

5)

In each of the following expressions k is a constant and $k > 3$.

Answers may be left in terms of k where appropriate.

For each series write the sum as an algebraic expression $f(x)$.

Do not use Σ in your answers.

a)

$$x + (x + 1) + (x + 2) + \dots + (x + k)$$

3 marks

b)

$$2^x + 2^{x+1} + 2^{x+2} + \dots + 2^{x+k}$$

3 marks

c)

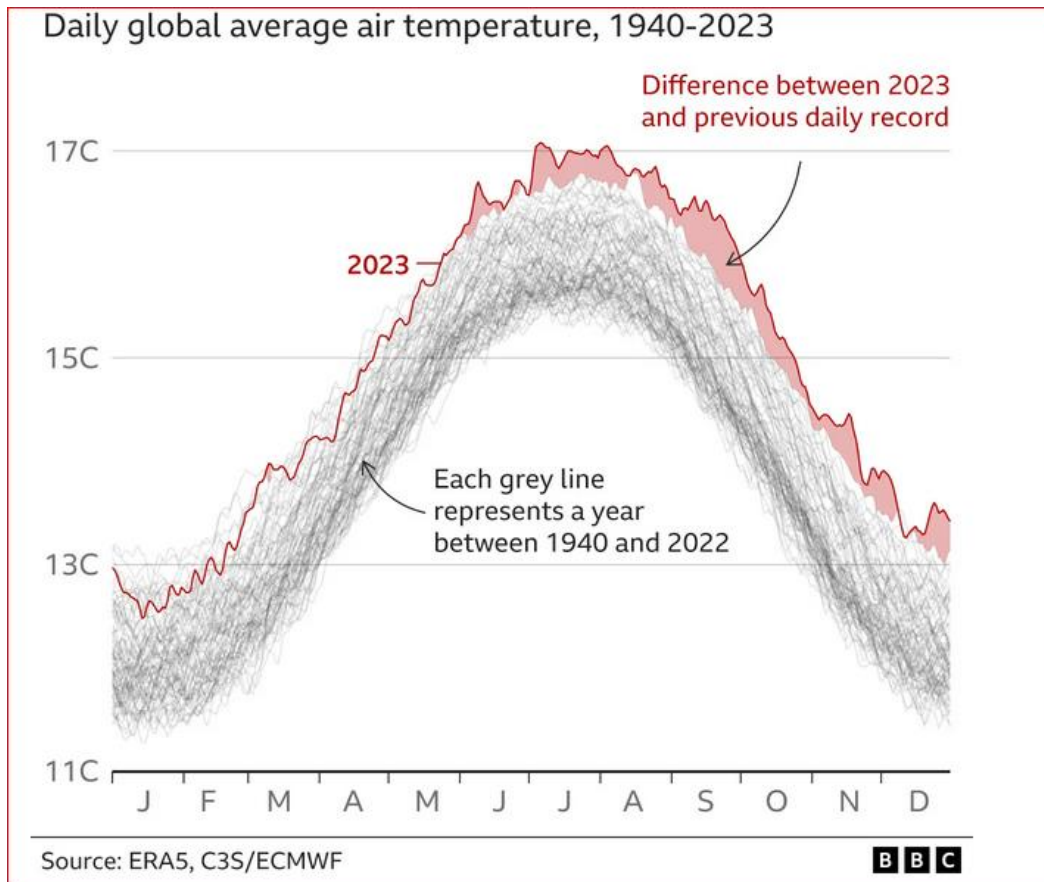
i) $x + x^2 + x^3 + \dots + x^k$

ii) $1 + 2x + 3x^2 + \dots + kx^{k-1}$

5 marks

Total 11 marks

6)

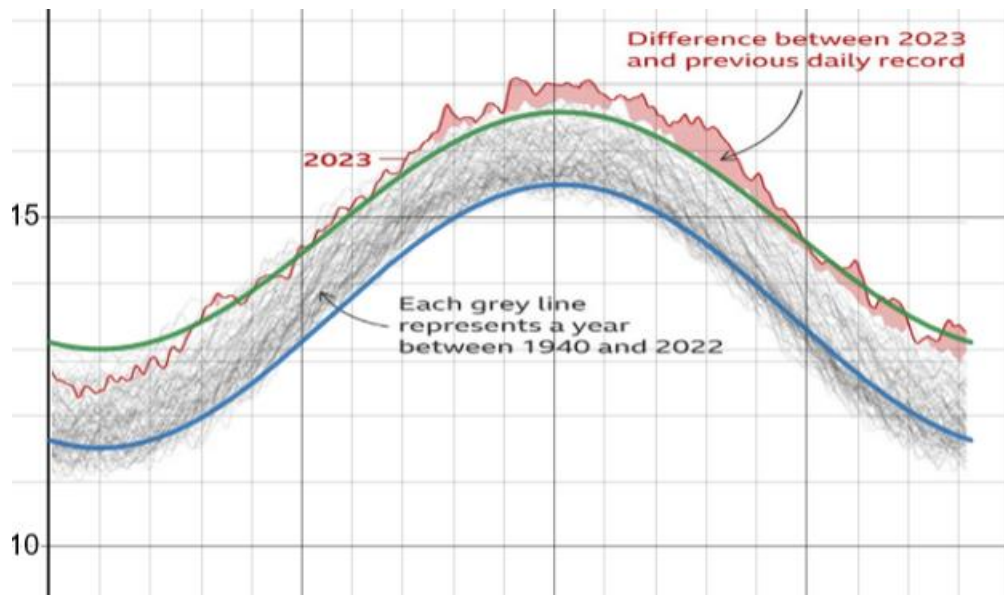


Andi finds the graph above concerning global temperatures for the 365 days in the year.

Andi decides to model the lowest and highest temperatures between 1940 and 2022 using a cosine function:

$y = -A\cos(B(x - C)) + D$ where $0 \leq x \leq 365$ and A, B, C, D are positive constants.

The two models are shown in the following graph.



a)

For the lowest temperatures (the bottom of the grey area) Andi uses the lowest temperature of 11.5 on the 20th day of the year and the highest temperature of 15.5 with period of 365 days.

Find the exact values of A, B, C, D .

6 marks

b)

For the highest temperatures Andi transforms the original equation using a vertical stretch of 0.9 and a translation of $\begin{pmatrix} 0 \\ 2.65 \end{pmatrix}$.

Find the new equation giving your answer in the form

$$y = -A\cos(B(x - C)) + D$$

5 marks

c)

Find the maximum difference between Andi's equations for the lowest and highest temperatures.

3 marks

d)

Do you think that the two models are realistic for the years between 1940 and 2022, giving a short reason?

2 marks

e)

The data for 2023 indicates a maximum temperature during the year of about 17 degrees.

Andi considers adjusting the model for the highest temperature to ensure its maximum value is 17.

Andi considers **either** a vertical translation **or** a vertical stretch of the equation for the highest temperature found in b) to ensure a maximum of 17.

- i) Describe the vertical translation Andi could use **and** write down the equation Andi would find.
- ii) Describe the vertical stretch Andi could use.

5 marks

Total 21 marks