Multiple Randomization Designs

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1. Introduction

Suppose we are thinking of offering customers more information about products. We could present them with reviews from other customers.

How do we estimate the causal effect of such a change?

Conventional experimentation: two possibilities:

Product Experiment
Randomize Products

$$\mathbf{W} = \begin{pmatrix} products & \rightarrow & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ & & \text{C} & \text{T} & \text{C} & \text{C} & \text{T} & \text{T} & \text{T} & \text{C} \end{pmatrix}$$

or:

Customer Experiment
Randomize Customers

$$\mathbf{W} = \begin{pmatrix} customers & \\ \downarrow & \\ 1 & \text{T} \\ 2 & \text{C} \\ 3 & \text{T} \\ 4 & \text{T} \\ 5 & \text{C} \end{pmatrix}$$

Conventional Analysis (going back to Fisher, Neyman): N units, $i = 1, \ldots, N$, randomly assigned to one of two treatments:

$W_i \in \{\mathsf{C}, \mathsf{T}\}$ is treatment indicator.

Two potential outcomes, $Y_i(\mathsf{C})$ and $Y_i(\mathsf{T})$, with unit-level causal effect $\tau_i = Y_i(\mathsf{T}) - Y_i(\mathsf{C})$.

Realized/observed outcome is

$$Y_i = Y_i(W_i) = \begin{cases} Y_i(\mathsf{C}) & \text{if} \quad W_i = \mathsf{C}, \\ Y_i(\mathsf{T}) & \text{if} \quad W_i = \mathsf{T}. \end{cases}$$

Interest is in average causal effect:

$$\tau = \frac{1}{N}\sum_{i=1}^{N}\Big(Y_i(\mathsf{T}) - Y_i(\mathsf{C})\Big)$$

Average causal effect is estimated as difference in average outcomes by treatment status.

$$\hat{\tau} = \overline{Y}_{\mathsf{T}} - \overline{Y}_{\mathsf{C}} = \frac{1}{N_{\mathsf{T}}}\sum_{i:W_i=\mathsf{T}} Y_i - \frac{1}{N_{\mathsf{C}}}\sum_{i:W_i=\mathsf{C}} Y_i$$

Estimated (conservative) variance:

$$\hat{\mathbb{V}} = \frac{S_{\mathsf{T}}^2}{N_{\mathsf{T}}} + \frac{S_{\mathsf{C}}^2}{N_{\mathsf{C}}}$$

where

$$S_{\mathsf{C}}^2 = \frac{1}{N_{\mathsf{C}} - 1}\sum_{i:W_i=\mathsf{C}}\left(Y_i - \overline{Y}_{\mathsf{C}}\right)^2, \quad S_{\mathsf{T}}^2 = \frac{1}{N_{\mathsf{T}} - 1}\sum_{i:W_i=\mathsf{T}}\left(Y_i - \overline{Y}_{\mathsf{T}}\right)^2$$

What is the problem with conventional A/B testing in market place settings?

Main problem is units interact in complex, intentional ways, leading interference / spillovers at some level.

- Treating customer i may have an effect on outcome for control customer i'.

- Treating product j may have an effect on outcome for control product j'.

A/B experiments (i) ignore this, (ii) do not not allow us to detect the problem, (iii) do not allow us to address the resulting bias.

Spillovers/Interactions are intrinsic to market places: Market places bring together different parties.

Recent Literature on "Complex" Experiments

- Munro, Evan, Stefan Wager, and Kuang Xu. "Treatment effects in market equilibrium." arXiv preprint arXiv:2109.11647 (2021).

- Papadogeorgou, Georgia, Fabrizia Mealli, and Corwin M. Zigler. "Causal inference with interfering units for cluster and population level treatment allocation programs." Biometrics 75, no. 3 (2019): 778-787.

- Zigler, Corwin M., and Georgia Papadogeorgou. "Bipartite causal inference with interference." Statistical science: a review journal of the Institute of Mathematical Statistics 36, no. 1 (2021): 109.

- Johari, Ramesh, Hannah Li, Inessa Liskovich, and Gabriel Y. Weintraub. "Experimental design in two-sided platforms: An analysis of bias." Management Science (2022).

Main Idea of current paper:

Can think of (binary) treatment assignment as **matrix** instead of **vector**

$$\mathbf{W} = \begin{pmatrix} products & \rightarrow & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ customers & & & & & & & & & \\ \downarrow & & & & & & & & & \\ 1 & & ? & ? & ? & ? & ? & ? & ? & ? \\ 2 & & ? & ? & ? & ? & ? & ? & ? & ? \\ 3 & & ? & ? & ? & ? & ? & ? & ? & ? \\ 4 & & ? & ? & ? & ? & ? & ? & ? & ? \\ 5 & & ? & ? & ? & ? & ? & ? & ? & ? \end{pmatrix}$$

In this set up, standard experiments are special case:

Customer Experiment
Randomize Customers

$$\mathbf{W} = \begin{pmatrix} products & \rightarrow & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ customers & & & & & & & & & \\ \downarrow & & & & & & & & & \\ 1 & & C & C & C & C & C & C & C & C \\ 2 & & C & C & C & C & C & C & C & C \\ 3 & & T & T & T & T & T & T & T & T \\ 4 & & C & C & C & C & C & C & C & C \\ 5 & & T & T & T & T & T & T & T & T \end{pmatrix}$$

or

Product Experiment
Randomize Products

$$\mathbf{W} = \begin{pmatrix} products & \rightarrow & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ customers & & & & & & & & & \\ \downarrow & & & & & & & & & \\ 1 & & C & T & C & C & T & T & T & C \\ 2 & & C & T & C & C & T & T & T & C \\ 3 & & C & T & C & C & T & T & T & C \\ 4 & & C & T & C & C & T & T & T & C \\ 5 & & C & T & C & C & T & T & T & C \end{pmatrix}$$

But we can do more interesting things than customer or product experiments:

Simple Multiple Randomization Design

$$\mathbf{W} = \begin{pmatrix} products \rightarrow & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ customers & & & & & & & & \\ \downarrow & & & & & & & & \\ 1 & \color{red}{\text{C}} & \color{red}{\text{C}} & \color{red}{\text{C}} & \color{red}{\text{C}} & \color{blue}{\text{C}} & \color{blue}{\text{C}} & \color{blue}{\text{C}} & \color{blue}{\text{C}} \\ 2 & \color{red}{\text{C}} & \color{red}{\text{C}} & \color{red}{\text{C}} & \color{red}{\text{C}} & \color{blue}{\text{C}} & \color{blue}{\text{C}} & \color{blue}{\text{C}} & \color{blue}{\text{C}} \\ 3 & \color{green}{\text{C}} & \color{green}{\text{C}} & \color{green}{\text{C}} & \color{green}{\text{C}} & \text{T} & \text{T} & \text{T} & \text{T} \\ 4 & \color{green}{\text{C}} & \color{green}{\text{C}} & \color{green}{\text{C}} & \color{green}{\text{C}} & \text{T} & \text{T} & \text{T} & \text{T} \\ 5 & \color{green}{\text{C}} & \color{green}{\text{C}} & \color{green}{\text{C}} & \color{green}{\text{C}} & \text{T} & \text{T} & \text{T} & \text{T} \end{pmatrix}$$

Three control groups that are *ex ante* comparable, but *ex post* possibly different: $\color{green}{\text{C}}$, $\color{blue}{\text{C}}$, $\color{red}{\text{C}}$

2. Multiple Randomization Designs

General: Two or more populations, with outcomes indexed by both (customers/products, drivers/riders, products/days, drivers/riders/days).

Could choose to randomize units from one of the two populations and use conventional A/B experiments.

But: Could assign treatment to pair customer/product.

Three benefits:

Benefit 1: Multiple randomization designs can be more powerful in estimating average treatment effects

Benefit 2: Multiple randomization designs can detect presence of spillovers.

Benefit 3: Multiple randomization designs can adjust for richer patterns of spillovers.

Generic Double Randomization Example:

Population 1: Customers

Population 2: Products

Treatment: provide more information about product to customer (e.g., pictures instead of written description, or ratings of previous customers) or shipping discount.

Decision: should we implement the treatment for all customers and products or for no one?

Statistical Question to Inform Decision: By how much would exposing all customers/products to the treatment improve average customer satisfaction/purchases?

Key: In experiment we can vary the treatment and measure the outcome at the level of the customer/product pair (not possible in many traditional settings, and even here sometimes fraught with issues)

Simple Multiple Randomization Design

$$\mathbf{W} = \begin{pmatrix} \textit{products} \rightarrow & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \textit{customers} & & & & & & & & \\ \downarrow & & & & & & & & \\ 1 & \color{red}{C} & \color{red}{C} & \color{red}{C} & \color{red}{C} & \color{blue}{C} & \color{blue}{C} & \color{blue}{C} & \color{blue}{C} \\ 2 & \color{red}{C} & \color{red}{C} & \color{red}{C} & \color{red}{C} & \color{blue}{C} & \color{blue}{C} & \color{blue}{C} & \color{blue}{C} \\ 3 & \color{green}{C} & \color{green}{C} & \color{green}{C} & \color{green}{C} & T & T & T & T \\ 4 & \color{green}{C} & \color{green}{C} & \color{green}{C} & \color{green}{C} & T & T & T & T \\ 5 & \color{green}{C} & \color{green}{C} & \color{green}{C} & \color{green}{C} & T & T & T & T \end{pmatrix}$$

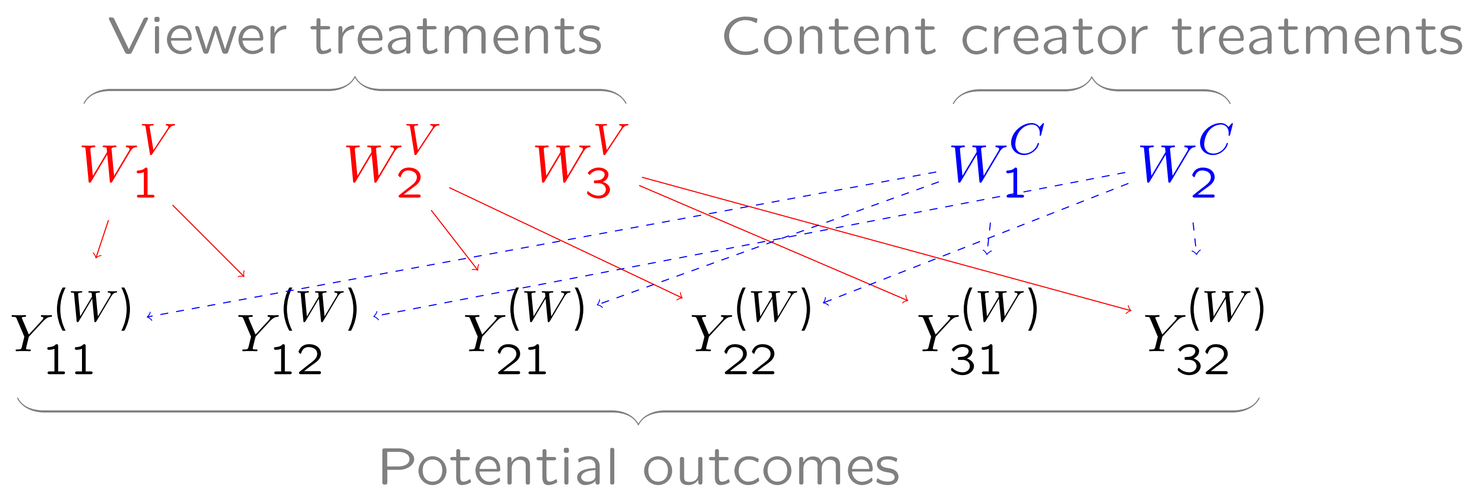
Questions:

1. (Estimand) What are we interested in?

2. (Design) How do we choose distribution $p(\mathbf{w})$?

3. (Estimation) How do we estimate things?

4. (inference) How we do inference?



Bipartite graph representation ($I = 3, J = 2$) of a simple double randomization design. Viewers $i \in \{1, ..., I\}$ have treatment indicators $W_i^V \in \{0, 1\}$. Content creators $j \in \{1, ..., J\}$ have treatment indicators $W_j^C \in \{0, 1\}$. Treatment assignment for each (viewer, creator) pair (i, j) is $W_{ij} = W_i^V W_j^C$ so that it is treated iff both treatment indicators are 1. Potential outcome for pair (i, j) is $Y_{ij}^{(W)} = Y_{ij}(S_{ij}^{\mathtt{dbr}}) = Y_{ij}(\text{type}(W_i^V, W_j^C))$, where 'type' is given by equation **??**.

Double Randomization Design

- I Customers, $i = 1, \ldots, I$.

- J Products, $j = 1, \ldots, J$

Outcomes and treatments are measured for pair customer/product:

- Y_{ij} is outcome for customer i and product j

- $W_{ij} \in \{\mathsf{C}, \mathsf{T}\}$ is binary treatment for customer i and product j (information / no information)

$\mathbf{Y}$ and $\mathbf{W}$ are $I{\times}J$ matrices of outcomes and treatments.

General Question:
Design of Experiment, what is good/optimal choice of distribution $p(\mathbf{w})$.

In standard experimental setting often the optimal design is simple: randomly select half the population and assign those to treatment and the others to control.

Here: What should the correlation be within rows and columns of treatment matrix $\mathbf{W}$?

Depends on
(i) question of interest
(ii) assumptions about potential spillovers and correlation of outcomes:
Not assumption-free

Completely Randomized Design

(attractive in absence of spillovers, and in that case easy efficiency gain over customer or product experiment)

$$\mathbf{W} = \begin{pmatrix} products & \rightarrow & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ customers & & & & & & & & & \\ \downarrow & & & & & & & & & \\ 1 & & \text{C} & \text{T} & \text{C} & \text{T} & \text{T} & \text{T} & \text{C} & \text{C} \\ 2 & & \text{C} & \text{C} & \text{T} & \text{T} & \text{C} & \text{T} & \text{C} & \text{T} \\ 3 & & \text{T} & \text{T} & \text{T} & \text{C} & \text{C} & \text{C} & \text{T} & \text{C} \\ 4 & & \text{T} & \text{C} & \text{C} & \text{C} & \text{T} & \text{C} & \text{T} & \text{T} \end{pmatrix}$$

- More efficient than Customer or Product experiment.

- Optimal to balance treatment for Customers and Products.

- Estimate average treatment effect as $\hat{\tau} = \overline{Y}_{\text{T}} - \overline{Y}_{\text{C}}$

Suppose

$$Y_{ij}(\mathsf{C}) = \mu + \alpha_i + \beta_j + \varepsilon_{ij}, \qquad Y_{ij}(\mathsf{T}) = Y_{ij}(\mathsf{C}) + \tau,$$

$$E[\alpha_i] = 0, \quad V(\alpha_i) = \sigma_\alpha^2, \quad E[\beta_j] = 0, \quad V(\beta_j) = \sigma_\beta^2,$$

$$E[\varepsilon_{ij}] = 0, \quad V(\varepsilon_{ij}) = \sigma_\varepsilon^2, \quad \alpha_i, \beta_j, \varepsilon_{ij} \text{ independent.}$$

$$V(\hat{\tau}|\text{customer} - \text{experiment})) = 4\frac{\sigma_\alpha^2}{I} + 4\frac{\sigma_\varepsilon^2}{IJ}$$

$$V(\hat{\tau}|\text{product} - \text{experiment})) = 4\frac{\sigma_\beta^2}{J} + 4\frac{\sigma_\varepsilon^2}{IJ}$$

$$V(\hat{\tau}|\text{completely} - \text{randomized})) = 4\frac{\sigma_\varepsilon^2}{IJ}$$ much smaller!

Cross-over Experiment

(familiar from old agricultural experimental design literature)

$$\mathbf{W} = \begin{pmatrix} time \;\rightarrow & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ customers & & & & & & & & \\ \downarrow & & & & & & & & \\ 1 & C & T & C & T & T & T & C & C \\ 2 & C & C & T & T & C & T & C & T \\ 3 & T & T & T & C & C & C & T & C \\ 4 & T & C & C & C & T & C & T & T \end{pmatrix}$$

⋄ More efficient than Customer or Time experiment.

⋄ Optimal to balance treatment for Customers and Time Periods.

3. Simple Double Randomization Designs (more complex)

Randomize customers into $N^{\mathsf{C}} \geq 2$ groups with indicator W_i^{C}

Randomize products into $N^{\mathsf{P}} \geq 2$ groups with indicator W_j^{P}

Assignment is $W_{ij} = f\left(W_i^{\mathsf{C}}, W_j^{\mathsf{P}}\right)$

For example $N^{\mathsf{C}} = N^{\mathsf{P}} = 2$:

$$\mathbf{W} = f\left(\mathbf{W}^{\mathsf{C}}, \mathbf{W}^{\mathsf{P}}\right) = \begin{pmatrix} \color{red}{\mathsf{C}} & \color{red}{\mathsf{C}} & \color{red}{\mathsf{C}} & \color{blue}{\mathsf{C}} & \color{blue}{\mathsf{C}} & \color{blue}{\mathsf{C}} \\ \color{red}{\mathsf{C}} & \color{red}{\mathsf{C}} & \color{red}{\mathsf{C}} & \color{blue}{\mathsf{C}} & \color{blue}{\mathsf{C}} & \color{blue}{\mathsf{C}} \\ \color{green}{\mathsf{C}} & \color{green}{\mathsf{C}} & \color{green}{\mathsf{C}} & \mathsf{T} & \mathsf{T} & \mathsf{T} \\ \color{green}{\mathsf{C}} & \color{green}{\mathsf{C}} & \color{green}{\mathsf{C}} & \mathsf{T} & \mathsf{T} & \mathsf{T} \end{pmatrix}.$$

• Creates $N^{\mathsf{C}} \times N^{\mathsf{P}}$ (= 4 here) *ex ante* comparable groups that have systematically different experiences *ex post*.

Four groups of pairs, T, C, C, and C.

Analyzing a Simple Double Randomization Design

The pair $(W_i^{\mathsf{C}}, W_j^{\mathsf{P}})$ defines four types of customer/product pairs:

$$T_{ij} = \begin{cases} \text{cc (consistent control)} & \text{if } W_i^{\mathsf{C}} = 0, W_j^{\mathsf{P}} = 0, \\ \text{ic (inconsistent customer control)} & \text{if } W_i^{\mathsf{C}} = 1, W_j^{\mathsf{P}} = 0, \\ \text{ia (inconsistent product control)} & \text{if } W_i^{\mathsf{C}} = 0, W_j^{\mathsf{P}} = 1, \\ \text{tr (treated)} & \text{if } W_i^{\mathsf{C}} = 1, W_j^{\mathsf{P}} = 1. \end{cases}$$

The assignment and type matrices for a simple double randomization design are

$$\mathbf{W} = \begin{pmatrix} \mathsf{C} & \mathsf{C} & \mathsf{C} & \mathsf{C} & \mathsf{C} & \mathsf{C} \\ \mathsf{C} & \mathsf{C} & \mathsf{C} & \mathsf{C} & \mathsf{C} & \mathsf{C} \\ \mathsf{C} & \mathsf{C} & \mathsf{C} & \mathsf{T} & \mathsf{T} & \mathsf{T} \\ \mathsf{C} & \mathsf{C} & \mathsf{C} & \mathsf{T} & \mathsf{T} & \mathsf{T} \end{pmatrix} \qquad \mathbf{T} = \begin{pmatrix} \text{cc} & \text{cc} & \text{cc} & \text{cc} & \text{ia} & \text{ia} & \text{ia} & \text{ia} \\ \text{cc} & \text{cc} & \text{cc} & \text{cc} & \text{ia} & \text{ia} & \text{ia} & \text{ia} \\ \text{ic} & \text{ic} & \text{ic} & \text{ic} & \text{tr} & \text{tr} & \text{tr} & \text{tr} \\ \text{ic} & \text{ic} & \text{ic} & \text{ic} & \text{tr} & \text{tr} & \text{tr} & \text{tr} \end{pmatrix}$$

Possible Comparisons:

1. Treated versus **all** controls: $\overline{Y}_{\mathsf{T}} - \overline{Y}_{\mathsf{C}}$

2. Treated versus consistent controls: $\overline{Y}_{\mathsf{T}} - \overline{Y}_{\mathsf{C}}$ (informative about total effect of intervention)

3. Inconsistent Products versus Consistent Controls:
$\overline{Y}_{\mathsf{C}} - \overline{Y}_{\mathsf{C}}$ (informative about spillovers within products)

4. Inconsistent customers versus Consistent Controls:
$\overline{Y}_{\mathsf{C}} - \overline{Y}_{\mathsf{C}}$ (informative about spillovers within customers)

5. Difference In Differences Comparison:
$\overline{Y}_{\mathsf{T}} - \overline{Y}_{\mathsf{C}} - \left(\overline{Y}_{\mathsf{C}} - \overline{Y}_{\mathsf{C}}\right)$ (direct effect of treatment)

Local Interference

Potential outcomes satisfy the local interference assumption if, for any pair (i,j), and $\mathbf{w}, \mathbf{w}'$, such that (a) the assignments for the pair (i,j) coincide, $w_{ij} = w'_{ij}$, (b) the fraction of treated s for the same coincide, $\overline{w}_i^{\mathrm{P}} = \overline{w}_i'^{\mathrm{C}}$, and (c) the fraction of treated s for the same coincide, $\overline{w}_j^{\mathrm{C}} = \overline{w}_j'^{\mathrm{P}}$,

$$y_{ij}(\mathbf{w}) = y_{ij}(\mathbf{w}').$$

Inference

$$\alpha_\omega^{\mathsf{C}} := \frac{1}{I-1}\frac{I-I_\omega}{I_\omega} \quad \text{and} \quad \alpha_\omega^{\mathsf{P}} := \frac{1}{J-1}\frac{J-J_\omega}{J_\omega},$$

Let

$$\widehat{\Sigma}_\omega := \frac{I_\omega^{-1}\alpha_\omega^{\mathsf{C}}\omega\mathsf{C} + J_\omega^{-1}\alpha_\omega^{\mathsf{P}}\omega\mathsf{P} + (I_\omega J_\omega)^{-1}\alpha_\omega^{\mathsf{C}}\alpha_\omega^{\mathsf{P}}\omega\mathsf{CP}}{1-\alpha_\omega^{\mathsf{C}}-\alpha_\omega^{\mathsf{P}}+\alpha_\omega^{\mathsf{C}}\alpha_\omega^{\mathsf{P}}} - \left[\sum_{i\in_\omega, j\in_\omega} \frac{\left(y_{i,j}(\omega) - i\mathsf{C}(\omega)\right)^2}{\frac{(1-\alpha_\omega^{\mathsf{C}})(J_\omega-1)}{\alpha_\omega^{\mathsf{C}}}} + \frac{\left(y_{i,j}(\omega) - i\mathsf{P}(\omega)\right)^2}{\frac{(1-\alpha_\omega^{\mathsf{P}})(I_\omega-1)}{\alpha_\omega^{\mathsf{P}}}}\right].$$

Consider a SMRD, with $I \times J$ total units, $I > I_{\mathsf{T}} \geq 2, J > J_{\mathsf{T}} \geq 2$, and for which local interference holds. For all $\omega \in \{\mathsf{c}, \mathsf{im}, \mathsf{iv}, \mathsf{t}\}$,

$$\mathbb{E}\left[\widehat{\Sigma}_\omega\right] = \mathbb{V}\left(\widehat{\overline{\overline{Y}}}_\omega\right).$$

More complex **Multiple Randomization Design:**

$\mathbf{W} =$

	Customer Experiment								Product Experiment					
products →	1	2	3	4	5	6	7	8	9	10	11	12	13	14
customers ↓	*A*	*A*	*A*	*A*	*A*	*A*	*A*	*A*	*B*	*B*	*B*	*B*	*B*	*B*
1	C	C	C	C	C	C	C	C	T	C	T	C	C	T
2	C	C	C	C	C	C	C	C	T	C	T	C	C	T
3	T	T	T	T	T	T	T	T	T	C	T	C	C	T
4	C	C	C	C	C	C	C	C	T	C	T	C	C	T
5	T	T	T	T	T	T	T	T	T	C	T	C	C	T
6	T	T	T	T	T	T	T	T	T	C	T	C	C	T
7	C	C	C	C	C	C	C	C	T	C	T	C	C	T
8	T	T	T	T	T	T	T	T	T	C	T	C	C	T
9	T	T	T	T	T	T	T	T	T	C	T	C	C	T
10	C	C	C	C	C	C	C	C	T	C	T	C	C	T

Questions:

What are we interested in?

How do we choose assignment distribution $p(\mathbf{w})$?

How do we estimate things?

How we do inference?

Multiple Randomization for Clustering Problems: We could combine a cluster-randomized product experiments for one sets of customers with an product experiment for a second set of customers.

Suppose the treatment is a shipping discount, and as a result of the treatment customers switch their purchases from control products to treated products within the same cluster.

Customers assigned to group A are part of a cluster randomized experiment, customers assigned to group B are assigned to completely randomized experiment.

$$
\mathbf{w} = \begin{pmatrix}
\multicolumn{2}{c}{\text{customers} \rightarrow} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
 & & A & A & A & A & B & B & B & B \\
\text{product} & \text{cluster} & & & & & & & & \\
\downarrow & \downarrow & & & & & & & & \\
1 & 1 & \text{C} & \text{C} & \text{C} & \text{C} & \text{T} & \text{T} & \text{T} & \text{T} \\
2 & 1 & \text{C} & \text{C} & \text{C} & \text{C} & \text{C} & \text{C} & \text{C} & \text{C} \\
3 & 2 & \text{T} & \text{T} & \text{T} & \text{T} & \text{T} & \text{T} & \text{T} & \text{T} \\
4 & 2 & \text{T} & \text{T} & \text{T} & \text{T} & \text{T} & \text{T} & \text{T} & \text{T} \\
5 & 3 & \text{C} & \text{C} & \text{C} & \text{C} & \text{T} & \text{T} & \text{T} & \text{T} \\
6 & 3 & \text{C} & \text{C} & \text{C} & \text{C} & \text{C} & \text{C} & \text{C} & \text{C} \\
7 & 4 & \text{T} & \text{T} & \text{T} & \text{T} & \text{C} & \text{C} & \text{C} & \text{C} \\
8 & 4 & \text{T} & \text{T} & \text{T} & \text{T} & \text{C} & \text{C} & \text{C} & \text{C}
\end{pmatrix}
$$

• This design is be informative re within-cluster spillovers by creating multiple *ex ante* comparable control groups.

4. Conclusion

• In settings with multiple populations more complex experimental designs are possible.

• Such designs (*e.g.* **multiple randomization designs**) can answer more questions about interference/spillovers than conventional designs by creating multiple comparison groups.

• Opens up lots of design questions.

• Opens up lots of inference questions.

• Important role for (economic/substantive) modeling (limits on) spillovers.