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Abstract

In this article, we use as case study the Spanish economy in the Early Modern period. We use recent time series data for the period 1492 - 1810. We consider the possibility that a linear cointegrated regression model with multiple structural changes would provide a good empirical description of the classical model of inflation for Spain over this long period. The principle testable implication is that money growth and inflation are cointegrated, ruling out speculative bubbles in the Spanish inflation rates.

Keywords: Classical model of inflation; Money demand; Money growth; Inflation; Explosiveness; Time-varying volatility; Cointegration: multiple structural changes

JEL classification: C22, E31, E51, N13

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Prices and Money in the Early Modern Period in Spain: Fresh Evidence from New Data, 1492 - 1810

1. Introduction

The discovery of massive deposits of precious metals in the Americas during the early modern period caused a large exogenous monetary injection into the money supply of Spain and other European countries (Palma, 2022).

Spain's money supply was strongly influenced by the inflow of precious metals from America (Desaulty et al., 2011). While precious metals (bullion) were not the same as money, silver and gold were key inputs for producing commodity money in the form of metallic coins. Money in early modern Spain consisted mainly of coins made of precious metals – above all silver (Nightingale, 1990). While other varieties of money existed, precious metal coins were more widely accepted than their surrogates, such as banknotes or bills of exchange. In terms of functionality, early modern precious metal money is comparable to narrow money aggregates today. There were no central banks in the modern sense, but there was a form of monetary policy since the monetary authority controlled the rate at which private agents could transform precious metals into currency.

How money affects the economy is a long-standing debate in economics. The arrival of massive amounts of precious metals from Spain's American colonies, together with a rising price level (and the rate of inflation), served as precursors to the quantity theory of money proposed at the Spanish School of Salamanca. In 16th- and 17th-century Spain, theologians at the school expressed the idea that an increase in money is absorbed by an equivalent increase in prices (de Azpilcueta, 1556; de Molina, 1597). Only a few decades later, in a context of currency shortages, English mercantilists theorized the hypothesis of the non-neutrality of money. They argued that an increase in money not only increases prices, but stimulates real economic activity (Misselden, 1622; de Malynes, 1623).

The link between the levels of prices and the production of gold and silver mines was also reported by the School of Classical Economics (Ricardo, 1810 - 1811; Tooke, 1838). These various observations were later analysed and named as the 'price revolution', which describes the economic effects in Europe from the specie flow stemming from the Spanish colonies (Clark and Hamilton, 1934; Hamilton, 1934). Empirical research on the effects of the inflow of American precious metal in Europe on prices and real output remains of great interest among economists and economic historians (see Brzezinski et al., 2024; Chen et al., 2021; Palma, 2020; Palma, 2022; Palma and Silva, 2024, among others).

The hypothesis that the inflow of precious metals from the Americas had a significant influence on the evolution of price inflation in Spain can be viewed through changes in the quantity of money (our study) or in its velocity, as tested by Chen et al. (2021). The goal of this article is to contribute to the debate about whether the increase in the Spanish money supply (measured in silver) raised the level of prices (and the rate of inflation) in the Early Modern period. In short, the present paper contributes to this controversial debate, in three important dimensions.

First, we test a classical model of inflation with rational expectations for the case of Spain during the period 1492 - 1810. The principle testable implication is that money growth and inflation are cointegrated, ruling out speculative bubbles, i.e., we test the proposition of long-run monetary neutrality, analysing the long-run relationship between monetary growth and inflation.

Second, we use recent times series data for the period 1492 - 1810, reconstructed by Álvarez-Nogal and Prados de la Escosura (2013) and Chen et al. (2021)

Third, we develop a cliometric analysis of the money supply and rate of inflation nexus using novel time series methods. On the one hand, to detect episodes of potential explosive behaviour in the Spanish inflation rate, we use the recursive unit root tests for explosiveness proposed by Phillips, Wu, and Yu (2011), and Phillips, Shi, and Yu (2015a,b); and we also use recent procedures to test for explosive bubbles under the presence of time-varying volatility (Harvey et al., 2016; Harvey et al., 2019; Harvey et al., 2020; Kurozumi et al., 2023). On the other hand, to control for structural breaks, we make use of recent developments in cointegrated regression models with multiple structural changes (Arai and Kurozumi, 2007; Kejriwal, 2008; Kejriwal and Perron, 2008, 2010). This approach could be classified within the subset of studies that look for structural breaks, where break dates and regimes are determined by the data. We use historical time series statistics for Spain during a 317-year span in which different debt crises episodes and institutional changes ran in parallel with debt restructuring and the use of the inflation tax.

The rest of the paper is organized as follows. A brief description of the underlying theoretical framework is provided in section 2, empirical results are presented in section 3, and the main conclusions are summarized in section 4.

2. A classical model of inflation with rational expectations

One of the central aspects of monetary theory deals with monetary models of inflation with forward-looking rational expectations. Such models impose structural restrictions that are easily evaluated with cointegration models. The solution for the inflation rate resembles the general form of the present value models, as proposed by Campbell and Shiller (1987, 1988a, 1988b). The principle testable implication is that money growth and inflation are cointegrated, ruling out speculative bubbles[1]. Specifically, we test for long-run money neutrality, which implies that there is an equilibrium relationship between the inflation rate and money growth with a known cointegrating vector $(1, -1)'$.

We use a classical model of inflation with rational expectations, as suggested by Feliz and Welch (1997). The model supposes rational expectations, i.e., that individuals use

[1] The presence of bubbles has several implications. For more details, see Diba and Grossman (1988a, 1988b).

all information available to them to form expectations about inflation rates. The model starts with a version of the Cagan (1956) money demand specification:

$$m_z - p_t = g_t - \alpha i_t + u_t \tag{1}$$

where m_{t} is the logarithm of the money stock at time t, p_t is the logarithm of the price level at time t, g_t is the logarithm of real output at time t, i_t is the nominal interest rate at time t, and u_t is a zero mean random error term describing a random walk.

Taking the first differences for equation (1), taking expectations, and solving forward n periods into the future, we can obtain the solution to the inflation rate:

$$\begin{aligned}\pi_t = \mu_t - \tilde{g} + \frac{\alpha}{(1+\alpha)} \sum_{i=0}^{\infty} \left(\frac{\alpha}{1+\alpha}\right)^i (E[\mu_{t+i+1} \mid \Phi_{t-k+1}] \\ -E[\mu_{t+i} \mid \Phi_{t-k}]) + \lim_{n\to\infty} \left(\frac{\alpha}{1+\alpha}\right)^{\mathrm{n}} E[\pi_{t+n} \mid \Phi_{t-k+1}] - \varphi_t\end{aligned} \tag{2}$$

where $\pi_{t+1} = p_{t+1} - p_t$ is the logarithmic inflation rate, μ_t is the logarithmic growth of money, and Φ_{t-k+1} is the information set at time $t-k+1$.

For a stable evolution of inflation expectations (and thus the inflation rate), the model imposes the following transversality condition (the 'no bubble' condition):

$$\lim_{n\to\infty} \left(\frac{a}{1+a}\right)^n E[\pi_{t+n} | \Phi_{t-k+1}| = 0 \tag{3}$$

From equations (2) and (3) we can obtain a long-run relationship between the inflation rate and money growth with a known cointegrating vector $(1,-1)'$[2 3]. In addition, if the inflation rate and money growth are cointegrated, no bubbles exist. In the empirical section, we test the classical model of inflation with rational expectations in the context of cointegration theory, using a linear model such as:

$$\pi_{\mathrm{t}} = c_1 + c_2 t + \gamma \mu_{\mathrm{t}} + \varepsilon_{\mathrm{t}} \tag{4}$$

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[2] We suppose both the inflation rate and money growth are stationary after first differencing [or $I(1)$], and the growth of real output held constant.

[3] For more details, see Feliz and Welch (1997).

3. Empirical results

In this section we re-examine the issue of a classical model of inflation with rational expectations by using instability tests to account for potential breaks in the long-run relationship between money growth and inflation, as well as by using the cointegration tests with multiple breaks. We use as case study the Spanish economy in the Early Modern period with the sample 1492 - 1810.

First, in order to detect episodes of potential explosive behaviour in the Spanish inflation rate, we use the recursive unit root tests for explosiveness proposed by Phillips, Wu, and Yu (2011, PWY henceforth), and Phillips, Shi, and Yu (2015a,b, PSY henceforth), and recent procedures to test for explosive bubbles in the presence of time-varying volatility (Harvey et al., 2016; Harvey et al., 2019, 2020; Kurozumi et al., 2023). Second, we use unit root tests to verify that the inflation rate and the money growth are integrated of order one. Third, we test the stability of the relationship between money growth and inflation (and select the number of breaks), using the test proposed in Kejriwal and Perron (2008, 2010). Next, we verify that the variables are cointegrated with tests for the presence/absence of cointegration, allowing for a single or multiple structural changes in the coefficients, as proposed by Arai and Kurozumi (2007) and Kejriwal (2008), respectively. Finally, we estimate the model incorporating the breaks to study how the relationship between the inflation rate and money growth (the slope parameter γ) has changed over time.

In our empirical analysis, we use new data in the Spanish economy for the period 1492 - 1810, with 319 annual observations. The data and their sources, which we shall expand on below, are: a) P_t, the price level (the consumer price index, $1790/99 = 100$) from Álvarez-Nogal and Prados de la Escosura (2013), Online Data Appendix, CPI; b) M_t, the money supply from Chen et al. (2021) (the money supply series as an 11-year moving average of the resulting baseline estimate, tonnes of silver, Appendix D, Table D.1); c) π_t, the growth of the price level; d) μ_t, the growth of money. Some descriptive statistics for both series are shown in Table 1. The estimate values of the standard deviation and the variance suggest high volatility[4].

Figure 1 presents the evolution of the price level, P_t, and the money supply, M_t, which shows a certain degree of co-movement in some periods. However, it should be noted that the P_t series could potentially suffer structural breaks in its trend, most likely in the second half of the 17th century. Moreover, the evolution of the inflation rate, π_t, and money growth, μ_t, appears in Figure 2, showing a co-movement between the two series but with possible structural changes in the long-run relationship. However, the plot also suggests that both variables show high volatility.

3.1 Explosive dynamics in the inflation rate

The classical model of inflation with rational expectations of equations (2) and (4) imposes the 'no bubble' condition from equation (3).

First, to detect episodes of potential explosive behaviour in the Spanish inflation rate, we use the recursive unit root tests for explosiveness proposed by PWY and PSY to examine whether the Spanish inflation rate series exhibits bubble behaviour at any time in the time series[5]. (This methodology was originally proposed to test for recurrent explosive behaviour for the US stock market).

For our empirical application, the lag order K is selected by using the Bayesian information criterion (BIC) with a maximum lag order of 6, as suggested by Campbell and Perron (1991). We set the smallest windows size according to the rule $r_0 = 0.01 + 1.8/\sqrt{T}$ recommended by PSY, giving the minimum length of a sub-sample as 35 years. The origination of an explosive episode is defined as the first chronological observation for which the test statistic exceeds its corresponding critical value. The termination of an explosive episode is instead defined as the first chronological observation for which the test statistic falls below its corresponding critical value.

Table 2 reports the $SADF$ and $GSADF$ tests for the null hypothesis of a unit root against

[4] Data is available on request from the authors.
[5] For more details on these tests, see Appendix A.

the alternative of an explosive root in the Spanish inflation rate. The various critical values for each of the two tests are also reported. We conduct a Monte Carlo simulation with 2,000 replications to generate the $SADF$ and $GSADF$ statistics sequences and the corresponding critical values at the 10.5% and1 % levels. As seen in Table 2, we cannot reject the unit root null hypothesis in favour of the explosive alternative at the 1% significance level for the $SADF$ and $GSADF$ tests. Neither test exceeds its respective 10%, 5%, and 1% right-tail critical values, giving no evidence that Spanish inflation rate had explosive subperiods. Consequently, we conclude from both summary tests that there is no evidence of bubbles.

Next, we conduct a real-time bubble monitoring exercise for the Spanish inflation rate using the PSY strategy. The PSY procedure also has the capability to identify downturns and adjustments in the inflation rate.

To locate the origin and conclusion of the explosive behaviour and the adjustments episodes, Figure 3 plots the profile of the $GSADF$ statistic for the Spanish inflation rate series. We compare the $GSADF$ statistic with the 99% and 95% $GSADF$ critical value for each observation of interest. The initial start-up sample for the recursive regression covers 14% of the full 1493 - 1810 period. Figure 3 identifies episodes of explosive inflation rate behaviour, allowing us to date-stamp their origination and termination, as well as their potential adjustments. Next, we also conduct a real-time bubble monitoring exercise for the Spanish inflation rates using the PWY strategy. Figure 4 plots the $SADF$ test statistics against the corresponding 99% and 95% critical value sequence.

According to Figures 3 and 4, there is no speculative bubble behaviour in the Spanish inflation rate series over the period 1493 - 1810.

However, the recursive unit root tests for explosiveness proposed by PWY and PSY assume constant unconditional volatility in the underlying error process, and recently Harvey, Leybourne, Sollis, and Taylor (2016) and Harvey, Leybourne, and Zu (2019) have demonstrated that the asymptotic null distribution of the PWY and PSY tests depends on the nature of the volatility through the variance profile under the existence of heteroskedasticity, so if the test is compared to critical values derived under a homoscedastic error assumption, its size is not controlled under time-varying volatility. This lack of size control typically leads to serious over-sizing, and consequently frequent spurious identification of a bubble.

Additionally, because the inflation rate exhibits high volatility, we also use recent procedures to test for explosive bubbles under conditions of time-varying volatility (Harvey, Leybourne, Sollis, and Taylor, 2016; Harvey, Leybourne, and Zu, 2019, 2020; Kurozumi, Skorobotov, and Tsarev, 2023)[6].

Figure 5 contains a plot of the first differences of π_t. A simple visual analysis of this plot suggests that the assumption of stationarity unconditional volatility could be unrealistic for this time series, since volatility appears over the sample period. Figure 6 shows the estimated variance profile of π_t, which is defined as $\hat{\eta}(s)$. We construct the estimator of the variance profile using the approach suggested by Cavaliere and Taylor (2007a, 2007b); Harvey, Leybourne and Zu (2023); and Kurozumi, Skorobotov, and Tsarev (2023). They use the kernel-type local least squares method to estimate the time varying parameter δ_t in (9) and (10)[7]. The inspection of the combination of the variance profile and the bubble tests will contribute to a more trustworthy result.

From Figure 6, note that the variance profile of this time series shows that there are two regimes in which volatility moves from high (1493 - 1691) to low (1692 - 1810). However, it is important to highlight that the regime in which volatility is high and of long duration represents 63% of the total sample.

Table 3 presents the results of the tests for explosive bubbles under stationarity volatility and the teats for explosive bubbles under time-varying volatility presented in the previous section: the standard $SADF$ and $GSADF$ tests, the wild bootstrap $SADF$ and $GSADF$ tests ($SADF_{\mathrm{b}}$ and $GSADF_{\mathrm{b}}$), a union of rejections of the $SADF_{\mathrm{b}}$ and SBZ tests and $GSADF_{\mathrm{b}}$ and SBZ tests (SBZ_{u}), a union of rejections of the $SADF_{\mathrm{b}}$ and sign-based tests $sSADF$, and of the $GSADF_{\mathrm{b}}$ and sign-based tests $sGSADF$ ($sSADF_{\mathrm{u}}$ and $sGSADF_u$)

[6] For more details of these tests, see Appendix A.
[7] For more details, see the Appendix.

and the $STADF$ and $GSTADF$ tests. We show the bootstrap p-values associated with the different tests: $SADF$ tests (Panel A) and $GSADF$ tests (Panel B)[8].

First, we observe that all tests for explosive bubbles under stationarity volatility do not reject again the null in favour of explosive behaviour at conventional significance levels. Second, this pattern of results is also obtained when considering the tests for explosive bubbles under time-varying volatility. Overall, the results in Table 3 suggest that for all tests there is no significant evidence of a bubble of the Spanish inflation rate during the period 1493 - 1810.

3.2 Stationarity of the time series

The first step in our analysis is to examine the properties of the time series by testing for a unit root over the full sample. Trend breaks appear to be prevalent in macroeconomic time series, and therefore unit root tests need to make allowances for these breaks if they are to avoid the serious effects that unmodelled trend breaks have on power[9]. In a seminal paper, Perron (1989) shows that failure to account for trend breaks present in the data results in unit root tests with zero power, even asymptotically. Consequently, when testing for a unit root, allowing for this kind of deterministic structural change should become a matter of regular practice. To avoid this pitfall, we run tests to assess whether structural breaks are present in π_t and μ_t series.

We have used the GLS-based unit root tests with multiple structural breaks under both the null and the alternative hypotheses proposed in Carrion-i-Silvestre et al. (2009). The commonly used tests for unit root with a structural change in the case of an unknown break date (Zivot and Andrews, 1992; Perron, 1997; Vogelsang and Perron, 1998; Perron and Vogelsang, 1992a, 1992b) assume that if a break occurs, it does so only under the alternative hypothesis of stationarity. The methodology developed by

[8] Following Kurozumi, Skorobotov, and Tsarev (2023) for the wild bootstrap p-values, $B = 999$ bootstrap replications were used. For the standard $SADF$ and $GSADF$ tests and the time-transformed tests $STADF$ and $GSTADF$, the p-values are obtained by simulations of the asymptotic distributions of the test statistics under homoskedasticity. We use $r_0 = \lfloor 0.01 + 1.8/\sqrt{T} \rfloor$ for calculations of the p-values.
[9] See, inter alia, Stock and Watson (1996, 1999, 2005) and Perron and Zhu (2005).

Carrion-i-Silvestre et al. (2009) solves many of the topical problems in standard unit root tests with a structural change in the case of an unknown break date[10]. Carrion-i-Silvestre et al. (2009) consider the modified unit root tests (M-class tests) analyzed by Perron and Ng (1996) and Ng and Perron (2001), and the P_T^{GLS}, MP_T^{GLS}, MZ_a^{GLS}, MSB^{GLS}, and MZ_t^{GLS} tests.

Carrion-i-Silvestre et al. (2009) consider three models: Model 0 ('level shift' or 'crash'), Model I ('slope change' or 'changing growth'), and Model II ('mixed change'). We consider that both the inflation rate and money growth series are characterized by stationary fluctuations without a clear trend profile. Moreover, these series are possibly affected by structural changes in their mean (in level). Consequently, for our empirical application we have used Model 0.

The results of applying the Carrion-i-Silvestre-Kim-Perron tests to Model 0 are shown in Table 4, allowing for either one or two breaks[11]. As Table 2 shows, the null hypothesis of a unit root with one or two structural breaks affecting the level (intercept) of the times series cannot be rejected by any of the tests at the 5% level of significance[12]. Consequently, we can conclude that the variables π_t and μ_t could be $I(1)$ with one or two different structural breaks.

3.3 Structural changes in the variance of the time series

The second step in our analysis is to examine the properties of the time series by testing structural changes in the variance over the full sample. These testing problems are important for practical applications in macroeconomics and finance to detect structural changes in the variability of shocks in time series. In empirical applications based on linear regression models, structural changes often occur in both the error variance and regression coefficients, possibly at different dates.

[10] See Carrión-i-Silvestre et al. (2009) for more details.
[11] We have used the tests allowing for up three, four, or five breaks. The results do not change. These results are available from the authors upon request.
[12] The critical values were obtained from simulations using 1,000 steps to approximate the Wiener process and 10,000 replications.

From an applied perspective the existence of breaks in variance has also attracted considerable interest following the work of McConnell and Perez-Quiros (2000), who documented the existence of a break in US output volatility occurring in the early mid-1980s. Building on this line of research, van Dijk and Sensier (2001) also explored the existence of a break in the volatility of a large database of U.S. macroeconomic series and found that the vast majority of the real series were also characterized by a variance shift that occurred during the early mid-1980s; see also Gadea et al. (2018), Perron and Yamamoto (2022), and Stock and Watson (2002, 2003a, 2003b).

We have used the test statistics to test jointly for structural changes in both mean and variance as proposed by Perron et al. (2020). More specifically, these authors provided a comprehensive treatment of the problem of testing jointly for structural changes in both the regression coefficients and the variance of the errors in a single equation regression model involving stationary regressors, allowing the break dates for the two components to be different or overlap.

Perron et al. (2020) consider several types of test statistics for testing structural changes in mean and/or variance: 1) the sup LR_T test statistic for m coefficient changes given no variance changes; 2) the sup $LR_{1,T}$ test statistic for n variance changes given no coefficient changes; 3) the sup $LR_{2,T}$ test statistic for n variance changes given m coefficient changes; 4) the sup $LR_{3,T}$ test statistic for m coefficient changes given n variance changes; 5) the sup $LR_{4,T}$ test statistic for m coefficient changes and n variance changes; 6) The UD max tests for each version, which can be computed by taking a maximum over a range of $1 \leq n \leq N$ for sup $LR_{1,T}$ and sup $LR_{2,T}$, a range of $1 \leq n \leq M$ for supLR_T and sup $LR_{3,T}$, and ranges of $1 \leq n \leq N$ and $1 \leq m \leq N$ for the sup$LR_{4,T}$; 7) the seq $LR_{9,T}$ test statistic for m coefficient changes versus $m+1$ coefficient changes given n variance changes; 8) the seq $LR_{10,T}$ test statistic for n variance changes versus $n+1$ variance changes given m coefficient changes. M and N denote the maximum number of breaks for the coefficients and the variance, respectively.

First, we investigate structural changes in the conditional mean and in the error variance of the Spanish inflation rate (see Figure 2). We use $M = 3$ and $N = 2$ and consider any potential serial correlations in the error term via a HAC variance estimator following Bai and Perron (1998, 2003). Table 5 reports the results of the test statistics for testing structural changes in mean and/or variance. All tests including the sequential procedure using the sup $LR_{9,T}$ test and the sequential test sup $LR_{10,T}$ suggest no presence of breaks in the conditional mean coefficients or in the error variance. Hence, we conclude for no structural change in the conditional mean or in the error variance.

Second, we investigate structural changes in the conditional mean and in the error variance of Spanish money growth (see Figure 2). We also use $M = 3$ and $N = 2$ and consider any potential serial correlations in the error term via a HAC variance estimator. Table 6(a) presents the results for the sup $LR_{4,T}$ and the $UD\ max\ LR_{4,T}$ tests, which suggest clear rejections of the null hypothesis of no breaks.

Table 6(b) presents the results when testing for mean breaks and accounting for possible variance breaks using the sup $LR_{3,T}$ and the $UD\ max\ LR_{3,T}$ tests and also the seq $LR_{9,T}$ test to determine the number of breaks. We obtain evidence for a mean break in 1584, regardless of how many variance breaks are accounted for. The change is such that the mean went from 1.60 in the 1493 - 1583 period to 0.58 in 1584 - 1810 period.

Table 6(c) presents the results of the tests for variance breaks accounting for mean breaks. If we account for one ($m_a = 1$) or two breaks in the mean ($m_a = 2$), two variance breaks are found in 1584 and 1728. The changes are such that the variance went from 0.14 in 1493 - 1583 period to 0.75 in 1584 - 1727, and then to 0.05 in 1728 - 1810. Moreover, with three breaks in the mean ($m_a = 1$) we find again two breaks in the variance: one in 1584 and the other in 1714. In this case, the variance changed from 0.14 in 1493 - 1583 to 0.81 in 1584 - 1713, and then to 0.07 in 1714 - 1810. The periods of greater variance correspond to the periods of greater maritime disasters for the ships that transported the silver of the Spanish American Empire (1567 - 1733), and therefore, represent a slowdown in the monetary supply[13]. Hence, we conclude for one structural change in the conditional mean and two changes in the error variance.

[13] For more details, see Brzezinski et al. (2024).

3.4 Long-run relationship

Once the order of integration of the series has been analysed, we estimate the long-run or cointegration relationship between π_t and μ_t.

If there is cointegration in the demeaned specification given in (4), such cointegration would occur when $c_2 = 0$, which corresponds to deterministic cointegration and implies that the same cointegrating vector eliminates both the deterministic and stochastic trends. However, if the linear stationary combinations of $I(1)$ variables have nonzero linear trends (which occurs when $\Phi \neq 0$), as given in (4), this would correspond to a stochastic cointegration. In both cases, the parameter γ is the estimated long-run cointegrating coefficient between π_t and μ_t.

We start by estimating and testing the coefficients of the cointegration equation by means of the dynamic ordinary least squares (DOLS) method of Saikkonen (1991) and Stock and Watson (1993) and following the methodology proposed by Shin (1994). This estimation method provides a robust correction to the possible presence of endogeneity in the explanatory variables, as well as serial correlation in the error terms of the OLS estimation. Additionally, to overcome the problem of the low power in classical cointegration tests in the presence of persistent roots in the residuals from the cointegration regression, Shin (1994) suggests a new test in which the null hypothesis is that of cointegration. Therefore, in the first place, we estimate a long-run dynamic equation that includes the leads and lags of all the explanatory variables, i.e., the so-called DOLS regression:

$$\pi_t = c + \Phi t + \gamma \mu_t + \sum_{j=-q}^{q} \gamma_j \Delta \mu_{t-j} + v_t \tag{5}$$

Then, we use the Shin test, based on the calculation of two LM statistics from the DOLS residuals, C_μ and C_τ, to test for stochastic and deterministic cointegration respectively. If there is cointegration in the demeaned specification given in (5), which occurs when $\Phi = 0$, this corresponds to a deterministic cointegration, which would imply the same cointegrating vector eliminates both deterministic and stochastic trends. But, as already stated, if the linear stationary combinations of $I(1)$ variables have nonzero linear trends as given in (5), this corresponds to a stochastic cointegration.

In both cases, the parameter γ is the long-run cointegrating coefficient estimated between π_t, and μ_t. The coefficient from the DOLS regression and the results of the Shin test are reported in Table 7. The null of deterministic cointegration between π_t and μ_t is not rejected at the 1% level, with an estimated value for γ of 0.41. The results obtained are consistent with the existence of linear cointegration between the inflation rate, π_t, and money growth, μ_t, with a vector $(1, -0.41)$. Thus, the cointegration vector is not $(1, -1)$, as predicted by the theory.

Overall, the results of the estimated value for γ using the DOLS method imply that a 10 percentage-point increase in money growth is associated with a 4.1 percentage-point higher inflation rate in the full sample. This suggests the presence of a partial effect in the long run, in the sense that the inflation rate was not adjusted to fully compensate for higher money growth. Recently, Chen et al. (2021) use the same data from 1492 to 1810 in the equation of exchange ($\mathrm{MV} = \mathrm{PY}$) to account for Spain's price level rise in terms of money growth, velocity changes, and real output growth. Their results show that the money supply increase accounts for most of Spain's Early Modern price level rise. More specifically, money supply increase accounts for 70% of Spain's price level increase over the whole sample. This effect is clearly higher than the 41% estimated in our work.

Accounting for parameter shifts is crucial in cointegration analysis since this type of analysis normally involves long spans of data, which are more likely to be affected by structural breaks. Our data cover three hundred and eighteen years in the history of the series, and during that period of time, the long-run relationship between the inflation rate and money growth has probably changed due to alterations in monetary and fiscal policy, as well as reforms in the financial market. Thus, the information content of the linear classical model of inflation with rational expectations is subject to change over time, and all the empirical modelling studies that have not taken the possible changes and instabilities into account have likely failed to explain the variations in the relationship between the inflation rate and money growth. Therefore, as we argued before, it is very important to allow for structural breaks in our cointegration relationship.

We now consider the tests for structural changes proposed in Kejriwal and Perron $(2008, 2010)$[14]. Since we have used a 20% trimming, the maximum numbers of breaks we may have under the alternative hypothesis is three. Moreover, the intercept and the slope in equation (5) are permitted to change. Table 8 presents the results of the stability tests as well as the number of breaks selected by the sequential procedure (SP) and the BIC and LWZ proposed by Bai and Perron (2003). The sup $F_T^*(3)$ test statistic results and the $UD\ max\ F_T^*(M)$ do suggest instability at the 1% level of significance. Further, the SP, BIC, and LWZ results suggest instability and select three breaks, which provides evidence against the instability of the long-run relationship. Overall, the results of the Kejriwal-Perron tests suggest a cointegrated model with three breaks estimated at 1582, 1629, and 1676.

Since the above reported stability tests also reject the null coefficient of stability when the regression is spurious, we still need to confirm the presence of cointegration among the variables. With that end in mind, we use the residual based test of the null of cointegration against the alternative of cointegration with unknown multiple breaks proposed in Kejriwal (2008), $\tilde{V}_k(\hat{\lambda})$.

Arai and Kurozumi (2007) show that the limit distribution of the test statistic, $\tilde{V}_k(\hat{\lambda})$, depends only on the timing of the estimated break fraction $\hat{\lambda}$ and the number of $\mathrm{I}(1)$ regressors m[15]. To account for serial correlation, we use the Newey and West (1987) method to compute a weighted variance with Bartlett's kernel applied to the autocovariances. The maximum number of allowed breaks is five, and the optimal number of breaks is selected based on the model that minimizes the Bayesian Information Criterion (BIC). Since we are interested in the stability of the inflation rate-

[14] For more details of these tests, see Appendix B.
[15] In our case, the Monte Carlo simulation generates critical values for the cointegration tests under the null hypothesis with structural breaks. The simulation implements up to five structural breaks and includes changes in the intercept and trend. It iterates over a large number of trials (e.g., 10,000).

money growth coefficient, γ, we only consider Model 1, which permits a level shift. Table 9 presents the results of the Arai-Kurozumi-Kejriwal cointegration tests allowing for three breaks[16]. As before, the level of trimming used is 10%. The optimal number of breaks selected is two, and we find that test $\tilde{V}_k(\hat{\lambda})$ cannot reject the null of cointegration with two structural breaks at the 1% level of significance[17]. Therefore, we conclude that π_t and μ_t are cointegrated with two structural changes estimated at 1576 and 1765.

Overall, the dates of the estimated structural changes, as identified by the tests for structural changes of Kejriwal and Perron (2008, 2010) and the residual-based tests for the null hypothesis of cointegration against the alternative of cointegration by Arai and Kurozumi (2007) and Kejriwal (2008), align with periods of major maritime disasters involving ships transporting silver from the Spanish American Empire (1567 - 1733). These events corresponded to a slowdown in the monetary supply and triggered money supply shocks[18].

4. Conclusions

The discovery of extensive deposits of precious metals in the Americas during the early modern period led to a significant exogenous increase in the money supply in Spain and other European nations. This article aims to contribute to the ongoing debate about whether the surge in the Spanish money supply, measured in silver, drove up price levels and inflation rates during this era. Specifically, the paper addresses this somewhat contentious issue through three key dimensions.

First, we evaluate a classical inflation model with rational expectations, focusing on Spain between 1492 and 1810. The central hypothesis tested is the cointegration of

[16] The long-run variance is estimated by the method proposed by Newey and West (1987).
[17] The critical values used in this test were calculated via Monte Carlo simulation. The method considers up to five structural breaks in models with a constant or with both a constant and a trend and uses a simulated sample of 318 observations with 10,000 replications for each configuration. Critical values are derived from the quantiles of the empirical distribution of the maximum cumulative deviation statistic adjusted for autocorrelation.
[18] For more details, see Brzezinski et al. (2024).

money growth and inflation, which excludes speculative bubbles and supports the concept of long-run monetary neutrality by examining the relationship between monetary growth and inflation over time. Second, we utilize newly reconstructed time series data for the period 1492 - 1810, as provided by Álvarez-Nogal and Prados de la Escosura (2013) and Chen et al. (2021). Third, we conduct a cliometric analysis of the link between money supply and inflation rates, employing innovative time series methods.

On the one hand, we find that there is not speculative bubble behaviour in the Spanish inflation rate series. On the other hand, the results obtained in our study are consistent with the existence of linear cointegration between the inflation rates and money growth series, with a vector $(1, -0.41)$. Thus, the cointegration vector is not $(1, -1)$, as predicted by the theory. Nevertheless, we provide evidence against the instability of the long-run relationship using several tests for structural changes in cointegrated regression models, as well as the residual-based test for the null hypothesis of cointegration against the alternative hypothesis of cointegration.

The estimated dates of structural changes, as determined by tests for structural shifts in cointegrated regression models and residual-based tests evaluating the null hypothesis of no cointegration versus the alternative of cointegration, coincide with significant maritime disasters involving ships carrying silver from the Spanish American Empire (1567 - 1733). These incidents led to a reduced monetary supply, causing money supply shocks.

Overall, the results suggest that ignoring structural changes in the long-run cointegration relationships may understate the extent of correlation between the inflation rate and money growth, since the response of the present value of inflation to a change in money growth changes over time. Our results support the existence of a partial effect in the long run, in the sense that the inflation rate was not fully adjusted to compensate for higher money growth.

5. Appendix A: Testing explosive bubbles

5.1 The heteroskedastic bubble model

Kurozumi, Skorobotov, and Tsarev (2023) consider the time series process $\{y_t\}$ generated according to the following DGP that allows one explosive regime with a subsequent collapsing regime,

$$y_t = \eta + u_t \tag{6}$$

$$u_t = \begin{cases} u_{t-1} + \varepsilon_t, & t = 1, \dots, \lfloor \tau_{1,0} T \rfloor, \\ (1+\delta_1) u_{t-1} + \varepsilon_t, & t = \lfloor \tau_{1,0} T \rfloor + 1, \dots, \lfloor \tau_{2,0} T \rfloor \\ (1-\delta_2) u_{t-1} + \varepsilon_t, & t = \lfloor \tau_{2,0} T \rfloor + 1, \dots, \lfloor \tau_{3,0} T \rfloor \\ u_{t-1} + \varepsilon_t, & t = \lfloor \tau_{3,0} T \rfloor + 1, \dots, T, \end{cases} \tag{7}$$

$$\varepsilon_t = \sigma_t e_t \tag{8}$$

where $\delta_1 \geqslant 0, \delta_2 \geqslant 0, 0 \leq \tau_{1,0} < \tau_{2,0} \leq \tau_{3,0} \leq 1$. The process $\{y_t\}$ evolves as a unit root process, but a bubble possibly emerges at $\lfloor \tau_{1,0} T \rfloor + 1$ with the explosive AR(1) coefficient given $1 + \delta_1$, followed by the collapsing regime from $\lfloor \tau_{2,0} T \rfloor + 1$ to $\lfloor \tau_{3,0} T \rfloor$ generated as a stationary process, which is interpreted as the return to the normal time series behavior. The magnitude of δ_2 specifies the extent of the collapse of the bubble with a duration between $\lfloor \tau_{2,0} T \rfloor + 1$ to $\lfloor \tau_{3,0} T \rfloor$. In the presence of heteroskedasticity the volatility of the innovations is given by σ_t in (8); and it can be non-stationary, whilst the conventional homoskedasticity assumption, as employed in PWY and PSY and other papers, implies that $\sigma_t = \sigma$ for all t.

On the other hand, the time series process $\{y_t\}$ can simply be rewritten as,

$$y_t = (1 + \delta_t) y_{t-1} + \varepsilon_t \tag{9}$$

or

$$\Delta y_t = \delta_t y_{t-1} + \varepsilon_t \tag{10}$$

The null hypothesis, H_0, is that no bubble is present in the series and y_t follows a unit root process throughout the sample period, i.e., $\delta_t = 0$ in expression (9)[19].

The alternative hypothesis H_1 is that a bubble is present in the series, which corresponds to the case where δ_t in (9) is not stable at 1 and the model is given by $(6) - (8)$ with $\delta_1 > 0$.

5.2 Test for explosive bubbles under stationarity volatility

Phillips, Wu, and Yu (2011) and Phillips, Shi, and Yu (2015a, 2015b) proposed a test for explosive bubbles based on recursive right-tailed Dickey-Fuller-type unit root tests which can detect evidence of the explosive behaviour of a time series $\{y_t\}$.

First, Phillips, Wu, and Yu (2011) suggested to employ the maximum of the ADF test statistics constructed using subsamples. The testing procedure developed from a regression model of the form:

$$\Delta y_t = \mu + \delta y_{t-1} + \varepsilon_t \tag{11}$$

for $t = \lfloor \tau_1 T \rfloor + 1$ to $\lfloor \tau_2 T \rfloor$. The key parameter of interest is δ. We want to test the null hypothesis of a unit root, $H_0\colon \delta = 1$, against the right-tailed alternative, $H_1\colon \delta > 1$, at least in some subsample. The model is estimated by Ordinary Least Squares (OLS), and the t-statistics associated with the estimated δ is referred to as ADF statistic.

The $SADF$ test is then a supremum test statistic based on the forward recursive regression and is simply defined as,

$$SADF(r_0) = \sup_{r_2 \in [r_0, 1]} ADF_0^{r_2} \tag{12}$$

where the right-tail is the rejection region. This test can be used for testing for a unit root against explosive behaviour in some subsample.

[19] The null hypothesis can be expressed using (7) in several ways such that $\tau_{1,0} = 1, \delta_1 = 0$, $\tau_{2,0} = 1$, or $\delta_1 = \delta_2 = 0$.

Second, Phillips, Shi, and Yu (2015a, 2015b) proposed a generalized version of the sup $ADF(SADF)$ test of Phillips, Wu, and Yu (2011). Their Generalized Supremum ADF $(GSADF)$ test is,

$$GSADF(r_0) = \sup_{r_2 \in [r_0, 1], r_1 \in [0, r_2 - r_0]} ADF_0^{r_2} \tag{13}$$

The statistic (13) is used to test the null of a unit root against the alternative of recurrent explosive behaviour, as in the statistic (12).

Note that the $SADF$ test previously proposed by Phillips, Wu, and Yu (2011) is a special case of $GSADF$ test, obtained by setting $r_1 = 0$ and $r_2 = r_\omega \in [r_0, 1]$[20].

The $SADF$ and $GSADF$ assume constant unconditional volatility in the underlying error process, and recently Harvey, Leybourne, Sollis, and Taylor (2016) and Harvey, Leybourne, and Zu (2019) demonstrated that the asymptotic null distribution of the Phillips, Wu, and Yu (2011) and Phillips, Shi, and Yu (2015a, 2015b) test depends on the nature of the volatility through the variance profile $\eta(s)$ under the existence of heteroskedasticity; so if the test is compared to critical values derived under a homoskedastic error assumption, its size is not controlled under time-varying volatility. This lack of size control typically leads to serious over-sizing, and consequently frequent spurious identification of a bubble[21].

5.3 Test for explosive bubbles under time-varying volatility

To account for this issue, several tests for explosive bubbles have recently been proposed under the assumption of time-varying volatility:

- Harvey, Leybourne, Sollis, and Taylor (2016); Harvey, Leybourne, and Zu (2019); and Kurozumi, Skorobotov, and Tsarev (2023) developed a wild bootstrap algorithm for the $SADF$ and the $GSADF$ tests. They propose to use

[20] Phillips and Shi (2018) showed that although the $GSADF$ procedure is designed to detect bubble behaviour, it can also detect crisis periods (see also Phillips and Shi, 2019; Phillips and Shi, 2020) which are often observed in empirical applications, for example Esteve and Prats (2023a, 2023b).

[21] Some classical unit root tests are severely oversized because their limiting distributions depend on a particular function, the so-called variance profile, of the underlying volatility process (see Cavaliere, 2004; Cavaliere and Taylor, 2007a, 2007b, 2008, 2009, and references therein).

this bootstrap scheme, applied to the first differences of the data, to replicate in the bootstrap data the pattern of non-stationarity volatility present in the original innovations. We called these tests $SADF_b$ and $GSADF_b$.

- Harvey, Leybourne, and Zu (2019) proposed two tests:
  - A weighted least squares (WLS) modification of the Phillips, Wu, and Yu (2011) test. Their supremum-based test is,

$$SBZ(r_0) = \sup_{r \in [r_0, 1]} BZ_r \tag{14}$$

  - A union U test (test of rejections/testing strategy) whenever none of the tests, SBZ and $SADF$, dominate each other across all volatility specifications. We called these tests SBZ_u.

- Harvey, Leybourne, and Zu (2020) proposed another method which controls size under time-varying volatility. They proposed two tests:
  - A sign-based variant of the Phillips, Shi, and Yu (2015a, 2015b) test for explosive behaviour, the supremum sign-based test, is,

$$sGSADF(r_0) = \sup_{r_2 \in [r_0, 1], r_1 \in [0, r_2 - r_0]} sADF_{r_1}^{r_2} \tag{15}$$

  - A union U test (test of rejections/testing strategy) with wild bootstrap implementation with $GSADF$ and $sGSADF$ tests with an approach similar to Harvey, Leybourne, and Zu (2019). We called this test the $sGSADF_u$ test (and $sSADF_u$).

- Kurozumi, Skorobotov, and Tsarev (2023) proposed a test based on the sup-type t-statistics expanded under the null hypothesis, using the time transformed data based on the variance profile, $\eta(s)$. They consider the $SADF$ and $GSADF$ test statistics with a version of the GLS-type demeaning. Their test statistics based on the time-transformed ADF test statistics is,

$$SADF = \sup_{r_2 \in [r_0, 1]} TADF_{r_0}^{r_2} \tag{16}$$

and

$$GSTADF = \sup_{r_2 \in [r_0,1], r_1 \in [0, r_2 - r_0]} TADF_{r_1}^{r_2} \tag{17}$$

6. Appendix B: Structural break tests

Kejriwal and Perron (2008, 2010) present issues related to structural changes in cointegrated models that allow for both $I(1)$ and $I(0)$ regressors as well as multiple breaks. They propose a sequential procedure that permits consistent estimation of the number of breaks, as in Bai and Perron (1998).

Kejriwal and Perron (2010) consider three types of test statistics for testing multiple breaks. First, they propose a sup *Wald* test of the null hypothesis of no structural break ($m = 0$) versus the alternative hypothesis that there are a fixed (arbitrary) number of breaks ($m = k$):

$$\sup F_T^*(k) = \sup_{\lambda \in \Lambda \varepsilon} \frac{SSR_0 - SSR_k}{\hat{\sigma}^2} \tag{18}$$

where SSR_0 denotes the sum of squared residuals under the null hypothesis of no breaks; SSR_k denotes the sum of squared residuals under the alternative hypothesis of k breaks; $\lambda = \{\lambda_1, \dots, \lambda_m\}$ is the vector of break fractions defined by $\lambda_i = T_i/T$ for $i = 1, \dots, m, T_i$ and T_i are the break dates; and where $\hat{\sigma}^2$ is:

$$\hat{\sigma}^2 = T^{-1} \sum_{t=1}^{T} \tilde{u}_t^2 + 2T^{-1} \sum_{j=1}^{T-1} \varpi\left(\frac{j}{\hat{h}}\right) \sum_{t=j+1}^{T} \tilde{u}_t \tilde{u}_{t-j} \tag{19}$$

and $\hat{u}_t (t = 1, \dots, T)$ are the residuals from the model estimated under the null hypothesis of no structural change. Additionally, for some arbitrarily small positive numbers $\epsilon, \Lambda_\epsilon = \{\lambda: |\lambda_{i+1} - \lambda_i| \geq \epsilon, \lambda_1 \geq \epsilon, \lambda_k \leq 1 - \epsilon\}$. Second, they consider testing the null hypothesis of no structural break ($m = 0$) versus the alternative hypothesis that there is an unknown number of breaks, given some upper bound $M (1 \leq m \leq M)$:

$$UD \max F_T^*(M) = \max_{1 \leq k \leq m} F_T^*(k) \tag{20}$$

In addition to the tests above, Kejriwal and Perron (2010) consider a sequential test of the null hypothesis of k breaks versus the alternative hypothesis of $k+1$ breaks (sequential procedure, SP):

$$\mathrm{SEQ}_T(k+1 \mid k) = \max_{1 \le j \le k+1} \sup_{\tau \in \Lambda_{j,\varepsilon}} T\left\{SSR_T\left(\hat{T}_1, \dots, \hat{T}_k\right\} - \left\{\frac{SSR_T(\hat{T}_1, \dots \hat{T}_{j-1}, \tau, \hat{T}_j, \dots, \hat{T}_k)}{SSR_{k+1}}\right\} \qquad (21)$$

where $\Lambda_{j,\varepsilon} = \left\{\tau: \hat{T}_{j-1} + \left(\hat{T}_j - \hat{T}_{j-1}\right)\varepsilon \le \tau \le \hat{T}_j - \left(\hat{T}_j - \hat{T}_{j-1}\right)\varepsilon\right\}$. The model with k breaks is obtained by a global minimisation of the sum of squared residuals, as in Bai and Perron (1998).

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Tables

Table 1
Descriptive Statistics: 1493 - 1810

Statistics	π_t	μ_t
Mean	0.80	0.88
Minimum	-32.40	-1.26
Maximum	30.50	2.53
Standard deviation	7.28	0.78
Variance	53.13	0.60
Skewness	-0.04	-0.21
Kurtosis	2.14	-0.45

Table 2
Tests for explosive behaviour under stationarity volatility in the Spanish inflation rate, π_t

Unit root tests	Estimated value	Finite critical value		
		1%	5%	10%
SADF	-5.710	1.964	1.363	1.101
GSADF	-3.980	2.761	2.171	1.907

Note: Superscripts *,**,*** indicate significance at the $10\%, 5\%$ and 1% levels, respectively.

Table 3
Test for explosive bubbles under non-stationarity volatility in the Spanish inflation rate, π_t, p-values

Panel (a) $SADF$ tests				
$SADF$	$SADF_b$	SBZ_u	$sSADF_u$	$STADF$
0.999	0.999	0.998	0.995	0.994
Panel (b) $GSADF$ tests				
$GSADF$	$GSADF_b$	$GSBZ_u$	$sGSADF_u$	$GSTADF$
0.999	0.999	0.999	0.993	0.999

Note: Superscripts [1,2,3] indicate significance at the $1\%, 5\%$, and 10% levels, respectively.

Table 4
M unit root tests with multiple structural breaks from Carrion-i-Silvestre et al. (2009) [a,b,c]

Variable	Model	MP_T^{GLS}	MZ Z_a^{GLS}	MSB ^{GLS}	MZ_t^{GLS}
π_t	$0(\hat{T}_1)$	132.780	0.326	0.785	0.256
μ_t	$0(\hat{T}_1)$	118.180	0.395	0.708	0.280
π_t	$0(\hat{T}_1, \hat{T}_2)$	93.540	0.129	0.652	0.084
μ_t	$0(\hat{T}_1, \hat{T}_2)$	250.350	0.883	1.065	0.941

Notes:
[a] Superscript * denotes rejection of the null at the 5% level.
[b] Structural breaks affect the intercept (Model 0: level shift or 'crash'). $\hat{T}$ numbers of breaks.
[c] The critical values were obtained from simulations using 1,000 steps to approximate the Wiener process and 10,000 replications.

Table 5

Tests for structural changes in mean and variance from Perron et al. (2020): Spanish inflation rate, π_t

(a) Tests for structural changes in mean and/or variance

	$\sup LR_{4,T}$			$UD\max LR_{4,T}$
	$m_a = 1$	$m_a = 2$	$m_a = 3$	$M = 3, N = 2$
$n_a = 1$	3.19	2.39	2.15	6.55
$n_a = 2$	6.55^{1}	5.23	4.18	

(b) Tests for structural changes in mean

	$\sup LR_{3,T}$			$UD\max LR_{3,T}$	seq $LR_{9,T}$			
	$m_a = 1$	$m_a = 2$	$m_a = 3$	$M = 3$	$m_a = 1$	$m_a = 2$	$m_a = 3$	Break dates
$n_a = 0$	1.55	2.72	1.97	2.72	2.56	3.48	1.63	-
$n_a = 1$	0.61	0.84	0.98	0.98	2.56	2.39	1.56	-
$n_a = 2$	1.69	1.48	0.99	1.60	2.48	1.63	1.63	-

(c) Tests for structural changes in variance

	$\sup LR_{2,T}$		$UD\max LR_{2,T}$	seq $LR_{10,T}$		
	$n_a = 1$	$n_a = 2$	$N = 2$	$n_a = 1$	$n_a = 2$	Break dates
$m_a = 0$	5.81	9.23^{2}	9.23^{1}	15.62^{3}	4.27	-
$m_a = 1$	5.37	9.16^{2}	9.16^{1}	15.08^{3}	4.27	-
$m_a = 2$	4.30	8.82^{2}	9.82^{1}	16.04^{3}	4.08	-
$m_a = 3$	4.71	8.89^{2}	8.89^{1}	16.04^{3}	4.08	-

Note: Superscripts ^{1,2,3} indicate significance at the 10%, 5% and 1% levels, respectively. The critical values are taken from Bai and Perron (1998), Perron et al. (2020), and Perron and Yamamoto (2022).

Table 6

Tests for structural changes in mean and variance from Perron et al. (2020): Spanish money growth, μ_t

(a) Tests for structural changes in mean and/or variance

	$\sup LR_{4,T}$			$UD\max LR_{4,T}$
$n_a = 1$	$m_a = 1$	$m_a = 2$	$m_a = 3$	$M = 3, N = 2$
$n_a = 2$	80.48^{3}	61.49^{3}	52.15^{3}	80.48^{3}

(b) Tests for structural changes in mean

	$\sup LR_{3,T}$			$UD\max LR_{3,T}$	seq $LR_{9,T}$			
	$m_a = 1$	$m_a = 2$	$m_a = 3$	$M = 3$	$m_a = 1$	$m_a = 2$	$m_a = 3$	Break dates
$n_a = 0$	136.50^{3}	75.95^{3}	63.87^{3}	136.50^{3}	2.49	2.49	2.99	1584
$n_a = 1$	123.30^{3}	64.28^{3}	51.54^{3}	123.30^{3}	2.49	2.49	2.99	1584
$n_a = 2$	183.90^{3}	93.67^{3}	68.49^{3}	183.90^{3}	2.49	2.49	2.99	1584

(c) Tests for structural changes in variance

	$\sup LR_{2,T}$		$UD\max LR_{2,T}$	seq $LR_{10,T}$		Break dates	
	$n_a = 1$	$n_a = 2$	N = 2	$n_a = 1$	$n_a = 2$		
$m_a = 0$	31.67^{3}	14.58^{3}	31.67^{3}	14.79^{3}	14.74^{2}	-	-
$m_a = 1$	11.86^{2}	11.69^{3}	11.86^{2}	9.85^{1}	7.46	1584	1728
$m_a = 2$	12.49^{3}	12.27^{3}	12.49^{2}	10.61^{2}	7.73	1584	1728
$m_a = 3$	12.72^{2}	11.95^{3}	12.72^{2}	12.65^{2}	7.91	1584	1714

Note: Superscripts ^{1,2,3} indicate significance at the 10%, 5%, and 1% levels, respectively. The critical values are taken from Bai and Perron (1998), Perron et al. (2020), and Perron and Yamamoto (2022).

Table 7
Estimation of long-run relationships: Stock-Watson-Shin cointegration tests: equation (5) [a,b,c]

Parameter estimates	
c	0.460
γ	0.410
Tests:	
p, q	6
l	11
C_{μ}	0.239^{3}

Notes:
[a] The lags and leads for DOLS regression are approximately equal to $p = q = (T)^{1/3}$, as proposed in Stock and Watson (1993).
[b] C_{μ} is LM statistics for cointegration using the DOLS residuals from deterministic cointegration, as proposed in Shin (1994). Superscripts [1,2,3] indicate significance at the $10\%, 5\%$, and 1% levels, respectively (the null cointegration is rejected). The critical values are taken from Shin (1994), table 1, from $m = 1$.
[c] The lag truncation parameter for C_{μ} is aproximately equal to $\ell = INT\left(\alpha(T/100)^{1/3}\right)$ as proposed in Andrews (1991).

Table 8
Kejriwal-Perron tests for testing multiple structural breaks in cointegrated regression models equation (5)[a,b,c]

Specifications [a]				
$y = \{\pi_t\}$	$z_t = \{1, \mu_t\}$ q = 2	$x_t = \{\emptyset\}$ $p = 0$	$M = 3$ $h = 63$	
Tests [b]				
	$\sup F_T$ (3)	$UD\max$		
	101.69^3	70.11^3		
	Number of breaks selected	Breaks $\widehat{T_1}$	$\widehat{T_2}$	$\widehat{T_3}$
SP	3	1582	1582	1582
LWZ	3	1629	1629	1629
BIC	3	1676	1676	1676

Notes:
[a] y_t, z_t, q, p, h, and M denote the dependent variable, the regressors, the number of $I(1)$ variables (and the intercept) allowed to change across regimes, the number of $I(0)$ variables, the minimum number of observations in each segment, and the maximum number of breaks, respectively.
[b] Superscripts [1,2,3] indicate significance at the 10%, 5%, and 1% levels, respectively.
[c] The critical values are taken from Kejriwal and Perron (2010), Table 1.10 (critical values are available on Pierre Perron's website), non-trending case with $q_b = 1$ in cointegrated regression models: equation (5).

Table 9
Arai-Kurozumi-Kejriwal cointegration tests with multiple structural breaks: equation (5) [a]

Two-breaks model				
Test $\tilde{V}_k(\hat{\lambda})$	$\hat{\lambda}_1$	$\hat{T}_1$	$\hat{\lambda}_2$	$\hat{T}_2$
0.619^3	0.840	1576	0.273	1765

Notes:
[a] Superscripts [1,2,3] indicate significance at the 10%, 5%, and 1% levels, respectively.

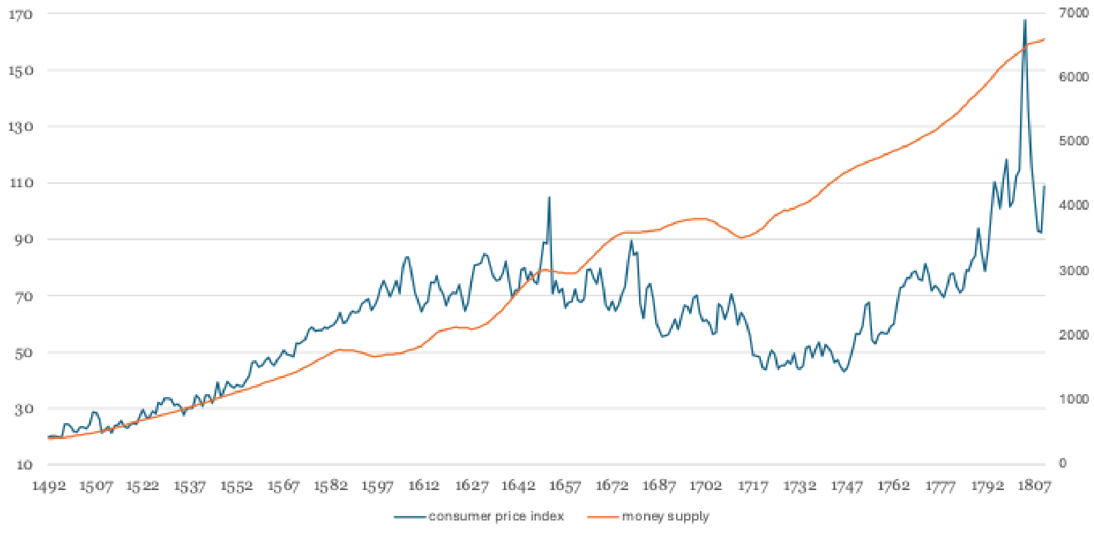


Figure 1. Spain: consumer price index (1790/99 = 100) and money supply (tonnes of silver), 1493-1810

Figure 2. Spain: inflation rate (change in price level,%) and money growth (change in money stock %), 1493-1810

Figure 3
Date-stamping bubble periods in the Spanish inflation rate:
The GSADF test

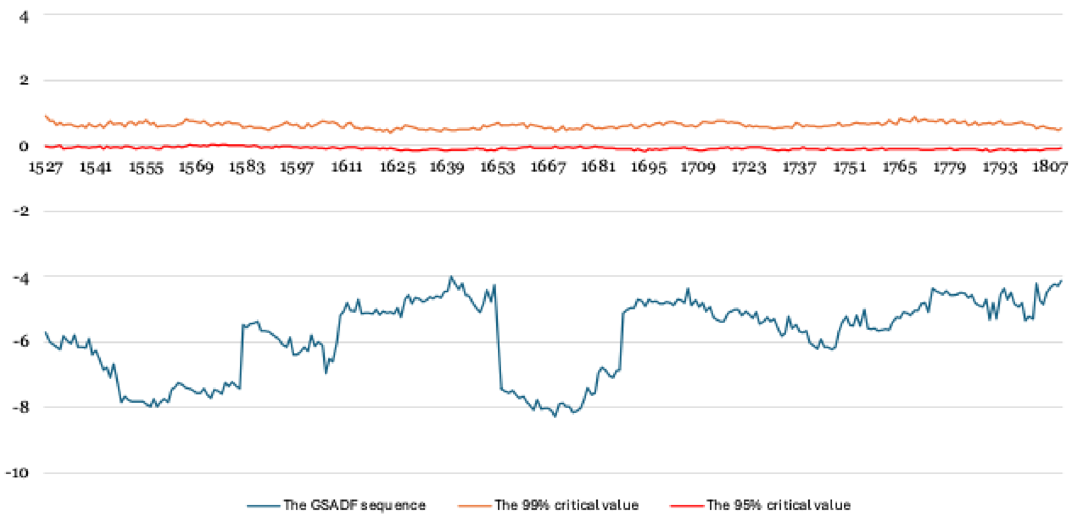
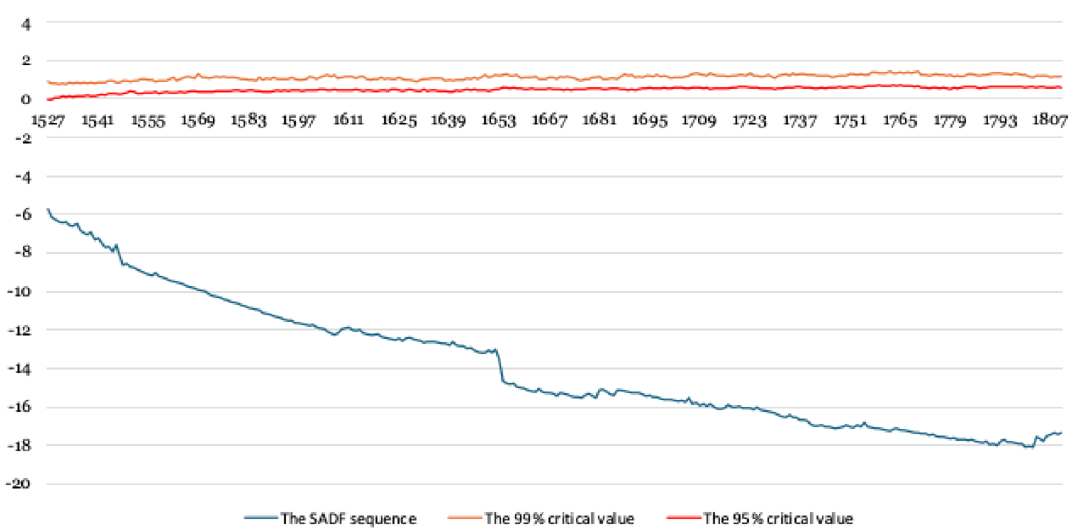


Figure 4
Date-stamping bubble periods in the Spanish inflation rate:
The SADF test



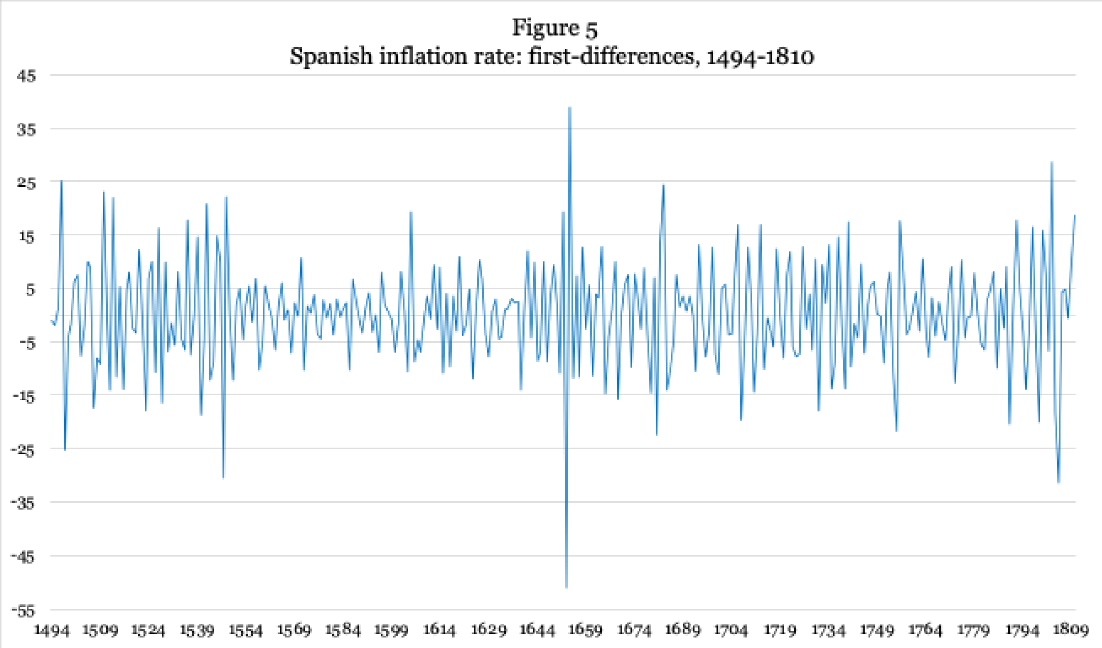


Figure 5
Spanish inflation rate: first-differences, 1494-1810
45
35
25
15
5
-5
-15
-25
-35
-45
-55
1494 1509 1524 1539 1554 1569 1584 1599 1614 1629 1644 1659 1674 1689 1704 1719 1734 1749 1764 1779 1794 1809

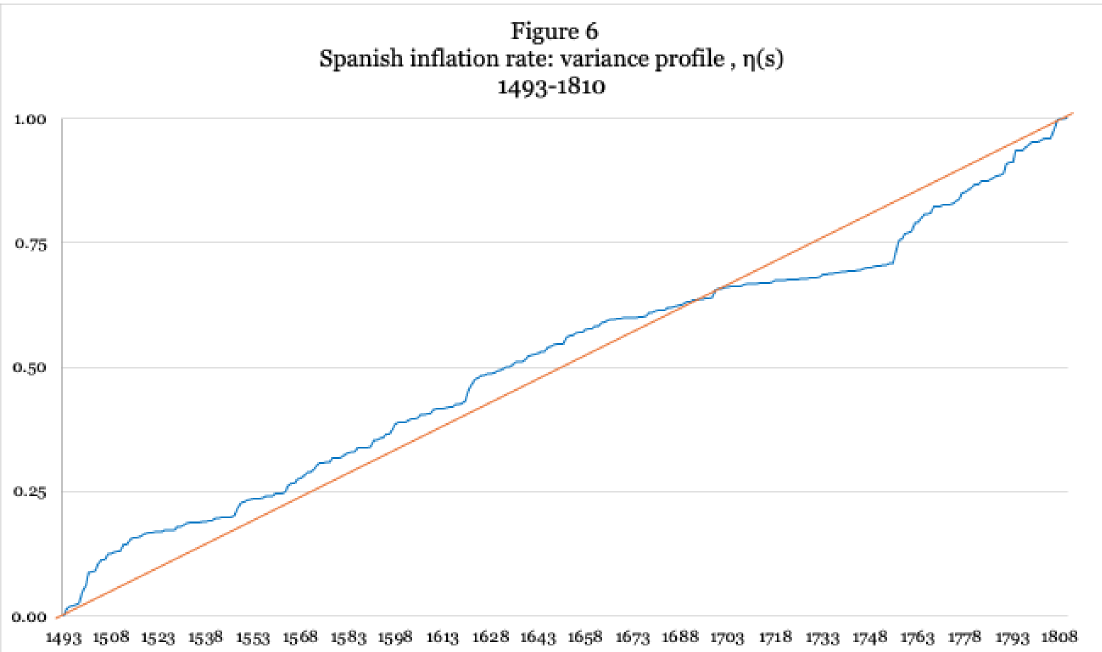


Figure 6
Spanish inflation rate: variance profile , η(s)
1493-1810
1.00
0.75
0.50
0.25
0.00
1493 1508 1523 1538 1553 1568 1583 1598 1613 1628 1643 1658 1673 1688 1703 1718 1733 1748 1763 1778 1793 1808