Improvements in the sequential change-point detection schemes for finance

Theoretical developments in the sequential testing and quickest change-point detection problems for diffusion processes were used for on-line detection of spontaneously occurring effects in financial data.

What was the problem?

Statistical analysis has recently shown that the dynamics of logarithms of certain market quotes can be efficiently described by continuous diffusion processes with random drift rates. More precisely, the shape of the drift rate can be modelled as either unknown but fixed from the beginning or changing between some unobservable random times called change points. The arising statistical problems when observing such data on-line are then to test the hypotheses about the value of the drift rate or to detect the appropriate change points as soon and as correctly as possible, keeping the costs for observations and errors in decisions as well as for delays in detections to a minimum. The problem of sequential testing of two simple hypotheses seeks to determine the time at which the observations should be stopped and one of the hypotheses about the shape of the drift rate of an observable diffusion process be accepted. The problem of quickest change-point detection is to search for a time at which a change is detected in the shape of the drift rate of an observable diffusion process.

What did we do?

Dr Pavel Gapeev (LSE Associate Professor in Financial Mathematics) and Prof Albert Shiryaev (Visiting Professor in the LSE Department of Statistics since 2011) studied the problems of sequential testing of two simple hypotheses and quickest change-point detection in the shape of the drift rate of observable general diffusion processes. They derived solutions of these problems in the Bayesian formulations in which the two simple hypotheses and the time of change are assumed to have a priori given distributions. More precisely, in Gapeev and Shiryaev (2011) [1], the authors studied the Bayesian formulation of the problem of sequential testing of two simple hypotheses about the drift rate of an observable diffusion process. This problem consisted of minimisation of a linear combination of expected observation time and probabilities of errors in acceptance of either of the hypotheses. The optimal decision rule contained the optimal detection time at which the observations should be stopped and the optimal decision function indicating which of the two hypotheses should be accepted. This time was characterised as the first time at which the conditional posterior probability of validity of one of the hypotheses under given observations or its equivalent likelihood ratio exits a region restricted by two stochastic boundaries depending on the running value of the observation process. The optimal decision function indicated the acceptance of the corresponding hypotheses when the posterior probability or the likelihood ratio crosses either of the boundaries. It was also shown that these boundaries can be characterised as unique solutions of a system of arithmetic equations, under certain nontrivial relations between the coefficients of the observable diffusion process. In Gapeev and Shiryaev (2013) [2], the authors studied the Bayesian formulation of the problem of quickest detection of a change in the shape of the drift rate of an observable diffusion process. This problem consisted of minimisation of a linear combination of the probability of false detection and the expected linear or exponential penalty costs for a detection delay. The optimal detection times were characterised as the first times at which the weighted likelihood ratios based on the given observations hit stochastic boundaries depending on the running value of the observation process. Arithmetic equations were derived for some upper bounds of these boundaries, under certain nontrivial relations between the coefficients of the observable diffusion process.

[1] Gapeev P. V., Shiryaev A. N. (2011). On the sequential testing problem for some diffusion processes. *Stochastics: An International Journal of Probability and Stochastic Processes* **83**(4–6) (519–535).

[2] Gapeev P. V., Shiryaev A. N. (2013). Bayesian quickest detection problems for some diffusion processes. *Advances in Applied Probability* **45**(1) (164–185).

What happened?

The implementation of the resulting detection procedures by certain analytic research groups in financial industry led to improvements in the existing methods for trading and hedging in financial markets.

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