2024 Colloquia in Combinatorics

UCL day (8 May 2024)

Schedule

10:00 Tea/Coffee

10:30 Paul Balister
11:20 Candy Bowtell

Lunch break

13:40 Marcelo Campos
14:30 Jinyoung Park

Coffee break

15:50 Gal Kronenberg
16:40 László Végh

All lectures will be in Chemistry Lecture Theatre XLG1 in the Christopher Ingold Building (see this map). Tea/Coffee breaks will be in the North Cloisters (see this map).

Abstracts

10:30 Paul Balister (University of Oxford)

Counting graphic sequences via integrated random walks

Given an integer n, let $G(n)$ be the number of integer sequences $n-1 \geq d_1 \geq d_2 \geq \cdots \geq d_n \geq 0$ that are the degree sequence of some graph. We show that $G(n) = (c + o(1))4^n/n^{3/4}$ for some constant $c > 0$, improving both the previously best upper and lower bounds by a factor of $n^{1/4+o(1)}$. The proof relies on a translation of the problem into one concerning integrated random walks.

Joint work with Serte Donderwinkel, Carla Groenland, Tom Johnston and Alex Scott.

11:20 Candy Bowtell (University of Warwick)

Matchings in multipartite hypergraphs

A well-known elementary exercise on matchings in graphs is to prove that if G is a bipartite graph whose vertex classes A and B each have size n, with $\deg(u) \geq a$ for every $u \in A$ and $\deg(v) \geq b$ for every $v \in B$, then G admits a matching of size $\min\{n, a+b\}$. We prove the natural analogue for large k-partite k-uniform hypergraphs.

This is joint work with Richard Mycroft.

13:40 Marcelo Campos (University of Cambridge)

New Lower Bounds for Sphere Packing

In this talk I'll show the existence of a packing of identical spheres in $\mathbb{R}^d$ with density

$$(1 - o(1))\frac{d \log d}{2^{d+1}},$$

as $d \to \infty$. This improves the best known asymptotic lower bounds for sphere packing density. The proof uses a connection with a new result about independence number of graphs which is proved probabilistically.

This is joint work with Matthew Jenssen, Marcus Michelen and Julian Sahasrabudhe.

14:30 Jinyoung Park (Courant Institute)

Lipschitz functions on expanders

We will discuss the typical behavior of M-Lipschitz functions on d-regular expander graphs, where an M-Lipschitz function means any two adjacent vertices admit integer values differ by at most M. While it is easy to see that the maximum possible height of an M-Lipschitz function on an n-vertex expander graph is about $C(M, d) * \log n$, it was shown by Peled, Samotij, and Yehudayoff (2012) that a uniformly chosen random M-Lipschitz function has height at most $C'(M, d) * \log \log n$ with high probability, showing that the typical height of an M-Lipschitz function is much smaller than the extreme case. Peled-Samotij-Yehudayoff's result holds under the condition that, roughly, subsets of the expander graph expand by the rate of about $M * \log(dM)$. We will show that the same result holds under a much weaker condition assuming that d is large enough.

This is joint work with Robert Krueger and Lina Li.

15:50 Gal Kronenberg (University of Oxford)

Balanced edge- and vertex-partitions of cubic graphs

When can we partition a graph into few isomorphic simple pieces? In this talk we will consider both edge and vertex decompositions of cubic graphs into isomorphic linear forests. We will present proofs of conjectures by Wormald (1987) and Abreu-Goedgebeur-Labbate-Mazzuoccolo (2018), and discuss the similarities and differences in the vertex and edge decompositions, as well as connections to more general settings.

This talk is based on joint works with Shoham Letzter, Alexey Pokrovskiy and Liana Yepremyan.

16:40 László Végh (LSE)

A strongly polynomial algorithm for the minimum-cost generalized flow problem

We give a strongly polynomial algorithm for minimum cost generalized flow and, as a consequence, for all linear programs with at most two nonzero entries per row, or at most two nonzero entries per column. Our result can be viewed as progress towards understanding whether all linear programs can be solved in strongly polynomial time, also referred to as Smale's 9th problem.

Our approach is based on the recent 'subspace layered least squares' interior point method, an earlier joint work with Allamigeon, Dadush, Loho and Natura. They show that the number of iterations needed by the IPM can be bounded in terms of the 'straight line complexity' of the central path. Roughly speaking, this is the minimum number of pieces of any piecewise linear curve that multiplicatively approximates the central path. Our main contribution is a combinatorial analysis showing that the straight line complexity of any minimum cost generalized flow instance is polynomial in the number of arcs and vertices.

This is joint work with Daniel Dadush, Zhuan Khye Koh, Bento Natura, and Neil Olver.

British Combinatorial Committee
Heilbronn Institute for Mathematical Research
UKRI Engineering and Physical Sciences Research Council
ICA The Institute of Combinatorics and its Applications

2024 Colloquia in Combinatorics

LSE day (9 May 2024)

Schedule

10:00 Tea/Coffee

10:30 Yoshiharu Kohayakawa
11:20 Anurag Bishnoi

Lunch break

13:40 Jo Ellis-Monaghan
14:30 Nemanja Draganić

Coffee break

15:50 Kristina Vušković
16:40 Alan Sokal

Reception

All lectures will be in the Sheikh Zayed Theatre located on the lower ground floor of the Cheng Kin Ku Building (CKK building on this map).

Abstracts

10:30 Yoshiharu Kohayakawa (University of São Paulo)

Arithmetic progressions in sumsets and in subsetsums of sparse random sets

Given a set A, its sumset $A + A$ is defined as the set of all sums of pairs of elements of A. Given $p\colon \mathbb{N} \to [0, 1]$, we let $A_n = [n]_p$ be the p-random subset of $[n] = \{1, \dots, n\}$ and consider $m^* = m^*(n)$, the largest m for which $A_n + A_n$ contains an m-element arithmetic progression with high probability. We shall see that for certain choices of $p_- = p_-(n)$ and $p_+ = p_+(n)$ with $p_- \leq n^{-1/2} \leq p_+$ and $p_+/p_- = (\log n)^\omega$, where ω can be chosen to grow arbitrarily slowly, for $p \leq p_-$ we have $m^* \ll (\log n)/\log\log n$ and for $p \geq p_+$ we have $m^* = \Theta(n)$. Furthermore, if $p \geq n^{-1/2+\varepsilon}$ for any fixed $\varepsilon > 0$, then long progressions exist in the sumset of any positive density subset of A_n: with high probability, for any subset S of A_n with a fixed proportion of elements of A_n, the sumset $S+S$ contains arithmetic progressions with $2^{\Omega(\sqrt{\log n})}$ elements.

Based on joint with with Rafael K. Miyazaki and with Marcelo Campos and Gabriel Dahia.

11:20 Anurag Bishnoi (Delft University of Technology)

Linear trifferent codes

A trifferent code is a set of ternary strings with the property that for any three distinct strings in the set there must be a coordinate where they have distinct values. Finding the largest size $T(n)$ of a trifferent code as a function of its length n is a major open problem in information theory. In this talk, we will focus on a linear version of this problem, where the set of strings is constrained to be a vector subspace of $\mathbb{F}_3^n$, and the largest size of such a subspace with the trifference property is denoted by $T_L(n)$. Pohoata and Zakharov recently obtained upper bounds on $T_L(n)$ that are exponentially better than the best bounds on $T(n)$. We prove new upper, and lower, bounds on $T_L(n)$ using coding theoretic and probabilistic arguments, after translating it to a problem in finite geometry on blocking sets. We also obtain new explicit constructions. The methods also generalise to other problems on minimal codes.

Joint work with Jozefien D'haeseleer, Dion Gijswijt and Aditya Potukuchi.

13:40 Jo Ellis-Monaghan (University of Amsterdam)

Combinatorial, topological, and computational approaches to DNA self-assembly

Applications of immediate concern have driven some of the most interesting questions in the field of graph theory, for example graph drawing and computer chip layout problems, random graph theory and modeling the internet, graph connectivity measures and ecological systems, etc. Currently, scientists are engineering self-assembling DNA molecules to serve emergent applications in biomolecular computing, nanoelectronics, biosensors, drug delivery systems, and organic synthesis. Often, the self-assembled objects, e.g. lattices or polyhedral skeletons, may be modeled as graphs. Thus, these new technologies in self-assembly are now generating challenging new design problems for which graph theory is a natural tool. We will present some new applications in DNA self-assembly and describe some of the graph-theoretical design strategy problems arising from them. We will see how finding optimal design strategies leads to developing new algorithms for graphs, addressing new computational complexity questions, and finding new graph invariants corresponding to the minimum number of components necessary to build a target structure under various laboratory settings.

14:30 Nemanja Draganić (University of Oxford)

Hamiltonicity of expanders: optimal bounds and applications

An n-vertex graph G is a C-expander if $|N(X)| \geq C|X|$ for every $X \subseteq V(G)$ with $|X| < n/2C$ and there is an edge between every two disjoint sets of at least $n/2C$ vertices.

We show that there is some constant $C > 0$ for which every C-expander is Hamiltonian. In particular, this implies the well known conjecture of Krivelevich and Sudakov from 2003 on Hamilton cycles in (n, d, λ)-graphs. This completes a long line of research on the Hamiltonicity of sparse graphs, and has many applications.

Joint work with Richard Montgomery, David Munhá Correia, Alexey Pokrovskiy and Benny Sudakov.

15:50 Kristina Vušković (University of Leeds)

Structure and algorithms for even-hole-free graphs

The class of even-hole-free graphs (i.e. graphs that do not contain a chordless cycle of even length as an induced subgraph) has been studied since the 1990's, initially motivated by their structural similarity to perfect graphs. It is known for example that they can be decomposed by star cutsets and 2-joins into algorithmically well understood subclasses, which has led to, for example, their polynomial time recognition. Nevertheless, the complexity of a number of classical computational problems remains open for this class, such as the coloring and stable set problems.

In this talk we survey some of the algorithmic techniques developed in the study of this class.

16:40 Alan Sokal (UCL)

Coefficientwise Hankel-total positivity in enumerative combinatorics

A matrix M of real numbers is called *totally positive* if every minor of M is nonnegative. Gantmakher and Krein showed in 1937 that a Hankel matrix $H = (a_{i+j})_{i,j\geq 0}$ of real numbers is totally positive if and only if the underlying sequence $(a_n)_{n\geq 0}$ is a Stieltjes moment sequence, i.e. the moments of a positive measure on $[0,\infty)$. Moreover, this holds if and only if the ordinary generating function $\sum_{n=0}^{\infty} a_n t^n$ can be expanded as a Stieltjes-type continued fraction with nonnegative coefficients:

$$\sum_{n=0}^{\infty} a_n t^n \;=\; \cfrac{\alpha_0}{1 - \cfrac{\alpha_1 t}{1 - \cfrac{\alpha_2 t}{1 - \cfrac{\alpha_3 t}{1 - \cdots}}}}$$

(in the sense of formal power series) with all $\alpha_i \geq 0$. So totally positive Hankel matrices are closely connected with the Stieltjes moment problem and with continued fractions.

Here I will introduce a generalization: a matrix M of polynomials (in some set of indeterminates) will be called *coefficientwise totally positive* if every minor of M is a polynomial with nonnegative coefficients. And a sequence $(a_n)_{n\geq 0}$ of polynomials will be called *coefficientwise Hankel-totally positive* if the Hankel matrix $H = (a_{i+j})_{i,j\geq 0}$ associated to (a_n) is coefficientwise totally positive. It turns out that many sequences of polynomials arising naturally in enumerative combinatorics are (empirically) coefficientwise Hankel-totally positive. In some cases this can be proven using continued fractions, by either combinatorial or algebraic methods; I will sketch how this is done. There is also a more general algebraic method, called *production matrices*. In a vast number of cases, however, the conjectured coefficientwise Hankel-total positivity remains an open problem.