

Optimal Credit Market Policy*

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Abstract

We study optimal credit market policy in a stochastic, quantitative, general equilibrium, infinite-horizon economy with collateral constraints tied to housing prices. Collateral constraints yield a competitive equilibrium that is Pareto inefficient. Taxing housing in good states and subsidizing it in recessions leads to a Pareto-improving allocation for borrowers and savers. Quantitatively, the welfare gains afforded by the optimal tax are significant. The optimal tax reduces the covariance of collateral prices with consumption, and, by doing so, it increases asset prices on average, thus providing welfare gains both in steady state and around it. We also show that the welfare gains stem from mopping up after the crash rather than a pure ex-ante macroprudential aspect, aligning with prior research that emphasizes the importance of ex-post measures compared to preventative policies alone.

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The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of anyone else associated with the Federal Reserve System.

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1 Introduction

In the aftermath of the global financial crisis, policymakers and researchers have studied tools to address financial instability through two distinct approaches: macroprudential policies that preemptively restrict lending, and ex-post interventions that mitigate crisis effects once they occur.¹ While both approaches have received significant attention separately, their relative effectiveness and quantitative merits remain open questions. In this paper we develop a quantitative dynamic stochastic general equilibrium (DSGE) model with housing and occasional financial crises and study optimal credit market intervention. We focus on simple tax rules on housing and debt, allowing taxes to respond to the level of productivity, in order to capture the cyclical nature of policy interventions. Our key finding is that a procyclical tax can achieve significant welfare gains. The optimal tax is positive in expansions and becomes a subsidy in recessions, thus capturing both the macroprudential and the ex-post intervention components of policy intervention. Quantitatively, we find that the welfare gains from the implementation of the tax come entirely from the subsidy part, thus pointing to an important role for ex-post intervention in our model economy.

The model features two types of agents, borrowers and savers. Borrowers are impatient and rely on housing as collateral for borrowing, while savers are patient, accumulate capital, and lend to borrowers. The framework is a closed economy setting with an endogenous interest rate. The economy features financial frictions, where borrowing is constrained by a collateral requirement linking debt capacity to the value of a borrower’s housing. Financial crises in the model happen when negative productivity shocks cause a decline in house prices and lead borrowing constraints to become binding. With limited borrowing capacity, the decline in borrower’s consumption and housing investment is amplified through a financial accelerator channel.

We use the model to investigate whether taxes on borrower’s housing investment or debt holdings can improve welfare. With collateral constraints that depend on the value of housing, the market equilibrium is inefficient because of the presence of pecuniary externalities. Specifically, atomistic agents fail to internalize how their allocation choices affect the price of housing and hence the tightness of the borrowing constraint.² We study whether a simple tax policy can address this pecuniary externality and deliver Pareto improvements relative to the decentralized equilibrium allocation. We choose this approach—relative to setting up the constrained planner problem—

¹ See [Bianchi \(2011\)](#), [Bianchi and Mendoza \(2018\)](#), [Davila and Korinek \(2017\)](#), [Gertler, Kiyotaki, and Prestipino \(2020\)](#), [Jeanne and Korinek \(2020\)](#), and [Benigno, Chen, Otrok, Rebucci, and Young \(2013\)](#) for examples of work on macroprudential policies. [Gertler and Kiyotaki \(2010\)](#), [Gertler and Karadi \(2011\)](#), [Bornstein and Lorenzoni \(2018\)](#) are papers that study ex-post intervention in the form of monetary policy accommodation, unconventional monetary policy, and credit market policies.

² Papers studying optimal macroprudential regulation focus on this externality to argue for macroprudential policies that limit borrowing ex-ante to avoid large asset price declines when the constraint binds ex-post. These arguments are typically developed in economies with a single agent, in which the only way to address the large asset price declines due to a binding constraint is to reduce borrowing ex-ante. Our economy has different channels through which this pecuniary externality could be addressed, in particular through trade among the two agents.

for two reasons. First, our environment includes two agents with different discount factors, which complicates the welfare analysis. Second, the optimal allocation choices associated with the solution of a constrained planner problem are usually implemented with state-contingent policies that would require, in practice, a very detailed amount of information in the hands of the planner. Our policies, on the other hand, are simple enough to be easily implemented, communicated, and interpreted. In particular, we study taxes on borrower’s housing investment or borrower’s debt which vary with the level of productivity. We also impose that the tax intervention is revenue-neutral and all revenues or subsidies are rebated back in a lump-sum fashion to the borrower. This ensures that the welfare gains are not achieved because the planner’s intervention facilitates borrowing and lending on behalf of private agents. By contrast, we focus on how a distortionary, yet welfare-improving, tax policy can shift borrowers’ demand schedules for housing and debt.

We show that a procyclical tax on housing or debt improves welfare for both borrowers and savers. The key mechanism driving this result is that the tax stabilizes “inefficient” house price fluctuations. In particular, by mitigating sharp declines in house prices during downturns, the policy prevents the collateral constraint from tightening as severely, thereby reducing the frequency and magnitude of fire sales that depress the price of housing. As a result, house prices also rise in normal times, as expected future house prices are higher because fire-sales are less likely to occur and less severe if they do. This mechanism is closely related to standard asset pricing models in which expectations of higher future prices raise current valuations. With higher and more stable house prices, collateral constraints are less tight and borrowers’ precautionary motives decrease, causing them to increase debt and consumption.³ The larger borrowing by borrowers is financed by savers, who lower their consumption to save more on impact, but end up consuming more in the long run thanks to a larger stock of savings. As a result, both agents are better off with the introduction of the tax policy.

We decompose welfare changes into three components: (i) short-run utility effects due to changes in average consumption, (ii) long-run utility effects due to changes in average consumption, and (iii) utility effects arising from changes in consumption volatility. We find that borrowers’ welfare increases due to the first component, short-run transition path, and the third component, lower volatility. The second component is detrimental as higher borrowing leads to lower consumption in the long-run stochastic steady state. Our analysis quantifies these trade-offs and finds that the borrower is still better off. Savers are also better off, mainly because the tax system allows them to save more. As a result, they consume less on impact but more in the long run and still achieve welfare gains when the tax system is introduced.

While our quantitative analysis studies housing taxes, we show that taxes on borrowing and

³There may be other benefits of less volatile house prices. For instance, [Dhamija, Nunes, and Tara \(2023\)](#) empirically show that households tend to overweight house price inflation when forming inflation expectations. Hence, more stable house prices could also lead to more stable and anchored inflation expectations.

taxes on housing investment are close substitutes and yield very similar welfare effects. This result occurs because the collateral constraint is related to the borrowers' portfolio choices between housing investment and borrowing. We show that the two sets of taxes affect the optimality conditions of the borrower in a similar way. Finally, we study the effects of asymmetric taxes on housing. In particular, we look separately at the effects of introducing only positive taxes when productivity is above average, and only subsidies (negative taxes) when productivity is below average. We interpret the positive-tax-only policy as a macroprudential policy that limits borrowing during booms, and the subsidy-only policy as an ex-post intervention type policy that dampens the macroeconomic effects of financial crises. We find that the policy with only subsidies achieves welfare gains very similar to the optimal symmetric policy, while the positive-tax-only policy reduces welfare for both agents. This result illustrates that, in our setup, ex-post policy intervention is more effective than macroprudential policy.

Literature Review: Our model is based on a variant of the models [Kiyotaki and Moore \(1997\)](#) and [Iacoviello \(2005\)](#).⁴ Our contribution is to extend this model to allow for the possibility of occasionally binding constraints—in the spirit of [Guerrieri and Iacoviello \(2017\)](#)—and to examine optimal policy in this context. A common finding in the literature is that economies with borrowing constraints and multiple goods feature inefficiencies (e.g. [Kehoe and Levine, 1993](#)), an observation that in turn connects to the broader principle of inefficiency in economies with incomplete markets (e.g. [Geanakoplos and Polemarchakis, 1986](#); [Stiglitz, 1982](#)). We apply the insights and tools of this literature to a heterogeneous agents model with natural borrowers and savers.

In the aftermath of the global financial crisis, there was a strong push to develop macroprudential policies to prevent the buildup of risks associated with high debt levels, as highlighted by [Bernanke \(2009\)](#). Papers by [Davila and Korinek \(2017\)](#), [Lorenzoni \(2008\)](#), [Bianchi \(2011\)](#) and [Bianchi and Mendoza \(2018\)](#) made groundbreaking theoretical contributions to this literature. [Davila and Korinek \(2017\)](#) and [Lorenzoni \(2008\)](#) conducted theoretical studies of efficient borrowing choices within three-period models, while [Bianchi \(2011\)](#) and [Bianchi and Mendoza \(2018\)](#) examined small open economy models with exogenous interest rates.⁵

Our model differs in several important respects. First, our interest rate is endogenously determined in a full-blown infinite-horizon model, rather than exogenous like in many small-open economy models. Second, our framework incorporates two distinct types of agents, requiring that any Pareto-improving tax policy enhance welfare for both agents. Finally, our model specifically addresses housing markets, whereas [Bianchi \(2011\)](#) focused on foreign borrowing and collateral constraints involving prices of tradable and non-tradable goods, and [Bianchi and Mendoza \(2018\)](#)

⁴ See also [Aoki, Proudman, and Vlieghe \(2004\)](#) for a benchmark model in this literature and more recently [Garriga, Kydland, and Sustek \(2017\)](#).

⁵ See also [Benigno et al. \(2013\)](#) and [Schmitt-Grohé and Uribe \(2020\)](#).

concentrated on capital investment.⁶

Finally, our paper is also related to the literature on housing and collateral constraints. [Guerrieri and Iacoviello \(2017\)](#) estimated model indicates that during the housing boom from 2001 to 2006, the loosening of collateral constraints allowed increased housing wealth to contribute to consumption growth. Conversely, the subsequent housing collapse tightened these constraints, significantly worsening the recession from 2007 to 2009. The empirical relevance of housing and collateral constraints highlights the importance of the policy analysis in our paper.

The remainder of the paper is organized as follows. Section 2 introduces the model, detailing the behavior of natural savers and natural borrowers and the market equilibrium conditions. Section 3 provides several model experiments. Section 4 examines the tax policy intervention and the decomposition of welfare. Section 5 presents extensions. Section 6 concludes. The Appendix provides additional technical details.

2 The Model

We consider a discrete-time infinite-horizon model, with two private agents — natural borrowers and natural savers — in the spirit of [Kiyotaki and Moore \(1997\)](#). Natural borrowers represent a fraction n of the population, and natural savers make up the remaining fraction $1 - n$. Borrowers operate a production technology that uses housing as the only input, while savers have access to a more flexible production function that also includes capital. We assume that borrowers are more impatient than savers so that, in equilibrium, they borrow to finance consumption and housing purchases. The amount they can borrow is limited by a collateral constraint that depends on the value of housing. Finally, a tax authority sets taxes, as will be explained later.

We proceed to describe the environment in detail. We start with the description of the savers' problem.

2.1 Natural Savers

Natural savers, denoted with a prime, produce a final good using capital and housing as inputs. Their production function is Cobb-Douglas:

$$y'_t = A_t h'^{\gamma}_{t-1} k'^{\alpha}_{t-1}, \tag{1}$$

where y'_t is the final good produced, h'_{t-1} is the housing input, k'_{t-1} is the capital input, and A_t is a technology parameter common to the production technologies of savers and borrowers. Throughout

⁶ Another branch of the literature has examined whether monetary policy should lean against the wind. Among others, [Goodhart and Hofmann \(2010\)](#), [Gali \(2014\)](#), [Walentin \(2014\)](#), [Svensson \(2017\)](#), [Schularick, ter Steege, and Ward \(2021\)](#), and [Bijlsma and van Ewijk \(2021\)](#) examine this question. Our paper instead focuses on fiscal rather than monetary policy. Our tax prescription has elements of leaning against the wind.

the analysis, A_t is stochastic and fluctuates around its long-run mean of unity. Labor is fixed and normalized to unity.

Savers can spend their resources on consumption (c'_t), investment in physical capital (k'_t) and housing (h'_t), or to save in a risk free bond (b'_t). Their budget constraint is:

$$c'_t + k'_t + q_t h'_t + b'_t = y'_t + (1 - \delta) k'_{t-1} + q_t h'_{t-1} + R_{t-1} b'_{t-1}, \quad (2)$$

where q_t is the price of housing, R_{t-1} is the risk-free interest rate on bonds, and δ is the rate of capital depreciation.

Savers maximize expected utility, which is assumed to be time-separable with period utility $u(c_t)$ of the logarithmic form. Expected lifetime utility is, therefore:

$$E_0 \sum_{t=0}^{\infty} \beta'^t \log c'_t, \quad (3)$$

where β' denotes the discount rate.

The optimization problem of savers is to maximize expected lifetime utility, equation (3), subject to their production function, equation (1), and their budget constraint, equation (2). The optimality conditions of the saver yield the following equilibrium conditions:

$$\frac{1}{c'_t} = \beta' R_t E_t \frac{1}{c'_{t+1}}, \quad (4)$$

$$\frac{1}{c'_t} = \beta' E_t \frac{1}{c'_{t+1}} \left(\alpha \frac{y'_{t+1}}{k'_t} + 1 - \delta \right), \quad (5)$$

$$\frac{q_t}{c'_t} = \beta' E_t \frac{1}{c'_{t+1}} \left(\gamma \frac{y'_{t+1}}{h'_t} + q_{t+1} \right). \quad (6)$$

Equation (4) is the first-order condition for bond holdings that equalizes the marginal utility of consuming one unit today versus saving one unit in a bond for future consumption. Equation (5) is the first-order condition for capital. It equates marginal costs and marginal benefits of investing in capital, where the marginal benefits depend on capital's marginal productivity next period, $\alpha \frac{y'_{t+1}}{k'_t}$, and its depreciated amount $1 - \delta$. Finally, equation (6) is the optimality condition for housing and reflects how the marginal costs and benefits of housing investments also depend on fluctuations in the price of housing q_t and q_{t+1} .

2.2 Natural Borrowers

The production technology of the borrower uses housing as its only input:

$$y_t = A_t h_{t-1}^\gamma, \quad (7)$$

where the technology parameter A_t is common with that of the saver in equation (1).⁷ Letting b_t denote the amount borrowers borrow in the bond market, the budget constraint of these agents is given by:

$$c_t + q_t(1 + \tau_t^h)h_t - b_t = y_t + q_t h_{t-1} - R_{t-1}(1 + \tau_{t-1}^b)b_{t-1} + T_t, \quad (8)$$

where τ_t^h and τ_t^b are taxes on housing and borrowing, respectively, that are levied by the tax authority as discussed in section 2.4 below, and T_t are lump sum transfers.

Borrowers do not hold capital and must finance their consumption and housing investment by borrowing from savers. The amount they can borrow is subject to a collateral constraint that limits borrowing to a fraction m of the value of their housing,

$$b_t \leq m q_t h_t, \quad (9)$$

where $0 \leq m \leq 1$. Similarly to Kiyotaki and Moore (1997), this condition imposes a limit on the ability of borrowers to take on debt, which arises from an agency problem between savers and borrowers. If borrowers renege on their debt obligations, then lenders can repossess the borrowers' assets by paying a proportional transaction cost $(1 - m)q_t h_t$. As a result, the maximum amount b_t that a borrower can obtain is bound by $m q_t h_t$.

Similar to savers, borrowers have logarithmic preferences over consumption. Borrowers are more impatient and discount the future at the rate $\beta < \beta'$. Their expected lifetime utility is:

$$E_0 \sum_{t=0}^{\infty} \beta^t \log c_t. \quad (10)$$

Borrowers maximize equation (10) subject to their production function (7), budget constraint (8), and borrowing collateral constraint (9). The inequality constraint (9) needs to be dealt with the usual Karush-Kuhn-Tucker formulation and the associated complementary-slackness conditions. The optimality conditions of the borrower are

$$\frac{1}{c_t} = \beta R_t (1 + \tau_t^b) E_t \left(\frac{1}{c_{t+1}} \right) + \lambda_t, \quad (11)$$

$$\frac{q_t(1 + \tau_t^h)}{c_t} = \beta E_t \left(\frac{1}{c_{t+1}} \left(\gamma \frac{y_{t+1}}{h_t} + q_{t+1} \right) \right) + \lambda_t m q_t, \quad (12)$$

$$\lambda_t \geq 0, \quad b_t \leq m q_t h_t, \quad \lambda_t (b_t - m q_t h_t) = 0, \quad (13)$$

where λ_t is the Lagrange multiplier on the collateral constraint.

Equation (11) is the first-order condition for consumption. When the collateral constraint binds, borrowing becomes restricted and the marginal utilities of consuming and saving are no

⁷ It is qualitatively inconsequential to include capital in the production function of borrowers and we exclude it for simplicity. An alternative would be to assume that borrowers provide labor and housing enters the utility function.

longer equalized, reflected by $\lambda_t > 0$. In this case, borrowers would like to borrow more, as interest rates are relatively low, as $\beta R_t(1 + \tau_t^b)E_t \frac{c_t}{c_{t+1}} < 1$, but cannot do so because the value of their housing restricts their borrowing capacity, i.e. $b_t = mq_t h_t$. Equation (12) is the optimality related to housing. The last term, $\lambda_t mq_t$, reflects the extra benefit to the borrower from investing in housing associated with its effect on relaxing the borrowing constraint. Finally, equation (13) is the usual complementarity-slackness condition.

2.3 Implications of Collateral Constraints

The collateral constraint prevents beneficial trade between borrowers and savers.⁸ To illustrate the inefficiencies associated with a binding collateral constraint, we consider the equilibrium with no taxes, i.e. $\tau_t^b = \tau_t^h = 0$. Combining the optimality conditions for borrowing for the saver and the borrower, equations (4) and (11), and using $\lambda_t > 0$, we get:

$$\beta' R_t E_t \frac{c'_t}{c'_{t+1}} = 1 > \beta R_t E_t \frac{c_t}{c_{t+1}}. \quad (14)$$

This inequality shows that when the constraint is binding, the borrower would find it optimal to borrow more at the current interest rate. The current consumption of borrowers c_t is ‘too low,’ and the borrower would like to increase borrowing and increase c_t , but the collateral constraint prevents this from happening.

Turning to housing investment, equations (11) and (12) with a binding constraint, i.e. $\lambda_t > 0$, and zero taxes yield :

$$\beta E_t \left(\frac{c_t}{c_{t+1}} \frac{(q_{t+1} + \gamma \frac{A_{t+1}}{h_t^{1-\gamma}})}{q_t} \right) > \beta E_t \left(\frac{c_t}{c_{t+1}} R_t \right). \quad (15)$$

This relationship shows that borrowers do not equate the marginal benefit of housing investment to the marginal cost of funds. The marginal return on housing is too high because the investment in housing is too low. Borrowers would like to obtain additional funds and invest in housing, which would reduce the left-hand side, but they cannot due to collateral constraints. Alternatively, one can see that the price of housing q_t is too low. The collateral constraint would be alleviated if q_t increases. This is one of the mechanisms that could lead to welfare gains.

The welfare analysis in the next section will explore different ways in which a planner, although forced to respect collateral and all constraints, i.e., to operate through the same markets as private agents, can reduce the extent of such unexploited trade opportunities by affecting borrowers’ demand for housing and bonds with tax instruments.

⁸ See Araujo, Fajardo, and Páscoa (2005) for a discussion on the existence of equilibrium and Fernandes and Páscoa (2024) for applications in the context of repo markets and macroprudential policy.

2.4 The Tax

As it is well known, economies with endogenous borrowing constraints and multiple goods can be inefficient, which relates to the generic inefficiency result in economies with incomplete markets.⁹ Various papers have studied optimal policy interventions in economies with collateral constraints of the type considered in our framework by analyzing the problem of a planner that faces the same borrowing constraints as private agents, but internalizes the effects of their allocation choices on asset prices.¹⁰

Here, we are going to consider a different approach. We will restrict the set of policies available to the planner to specific simple rules and quantitatively study the optimal policy rule for the planner. We do so for two rationales. First, a constrained planner’s welfare analysis is unclear due to the differing discount factors of the two agents. Second, the optimal allocation choices derived from solving a constrained planner’s problem typically involve state-contingent policies, which in practice would demand an extremely detailed level of information available to the planner. In contrast, the policies we examine here are simple rules that are straightforward to interpret and implement. In particular, we consider tax rules of the following form:

$$\tau_t^x = \varepsilon^x (A_t - 1), \tag{16}$$

for $x = h, b$.

The parameter ε^x determines the elasticity of the tax with respect to TFP. For instance, a positive value of ε^h implies that housing holdings are taxed in good times, when A_t is above one, and subsidized during recessions, when A_t is below one. A similar interpretation applies to taxes on borrowing. This allows us to capture the countercyclical nature of credit market intervention, restricting housing purchases and debt in good times, and subsidizing borrowers during crises. In our policy exercises, described in section 4 below, we will study how the competitive equilibrium and agents’ welfare vary as we vary ε^x .

We consider the tax intervention to be revenue-neutral as this allows us to focus on how to correct market failures rather than on the fiscal deficit policy of the government. Accordingly, the lump sum transfers to borrowers satisfy:

$$T_t = \tau_t^h q_t h_t + \tau_{t-1}^b R_{t-1} b_{t-1}. \tag{17}$$

This equation also represents the budget constraint of the fiscal authority. Individual borrowers do not influence the lump sum rebate. This is a common assumption, which follows from considering that there is a continuum of atomistic borrowers. Since all borrowers are identical ex-post, we

⁹ See, e.g., [Kehoe and Levine \(1993\)](#), [Geanakoplos and Polemarchakis \(1986\)](#), and [Stiglitz \(1982\)](#).

¹⁰ See, e.g., [Lorenzoni \(2008\)](#), [Bianchi \(2011\)](#), [Benigno et al. \(2013\)](#), [Bianchi and Mendoza \(2018\)](#), and [Benigno, Rebucci, and Zaretski \(2024\)](#).

can solve the borrower’s problem as a representative borrower who optimizes for individual-level variables.

2.5 Market Clearing and Equilibrium

Bonds are in zero net supply, and housing is in fixed supply, normalized to 1. Market clearing requires:

$$nb_t = (1 - n)b'_t, \tag{18}$$

$$nh_t + (1 - n)h'_t = 1. \tag{19}$$

The stochastic process governing total factor productivity follows:

$$A_t - 1 = \rho(A_{t-1} - 1) + \sigma\varepsilon_t, \quad \varepsilon \sim N(0, 1), \tag{20}$$

where $|\rho| < 1$.

The definition of equilibrium is standard. A competitive equilibrium consists of sequences of allocations for borrowers and savers, including consumption, housing, capital, and borrowing, along with sequences of housing prices and interest rates that satisfy the optimality conditions for each agent, the borrowing constraint, and the market-clearing conditions.

2.6 Calibration

The model is based on the framework of [Kiyotaki and Moore \(1997\)](#), with several key modifications. First, we adopt a parametrization that results in a borrowing constraint that binds only occasionally. This feature aligns with the dynamics studied by [Bianchi \(2011\)](#), [Mendoza \(2010\)](#), and more recently [Guerrieri and Iacoviello \(2017\)](#), where borrowing limits become relevant only in specific states of the economy. Second, we distinguish between two types of agents by endowing natural savers with access to a distinct production technology that allows them to accumulate variable capital, unlike borrowers who rely solely on a fixed factor of production. Finally, we introduce a fully stochastic environment with recurring productivity shocks and employ global solution methods to solve the model. These methods are crucial for accurately capturing the nonlinear effects of borrowing constraints and conducting welfare analysis.

We set the parameters as in [Table 1](#). These parameters result in a steady state where the wealth-to-annual GDP ratio is around 5.1, the debt-to-annual GDP ratio is around 2.3, the standard deviation of log aggregate consumption is 5.2 percent, and the standard deviation of log GDP is

around 5.7 percent, broadly in line with the data.¹¹ The standard deviation of consumption for borrowers is 8.6 percent, while for savers it is around 3.7 percent.¹² Finally, the standard deviation of house prices is about 3.7 percent.

We compute policy functions using the parametrized expectations algorithm. Conditional expectations are approximated as polynomial functions of state variables. The state variables are the previous period housing stock (h_{t-1}), capital stock (k_{t-1}), debt level (b_{t-1}), and the current productivity shocks (A_t). The solution of the model follows an iterative procedure:

1. Initialize polynomial coefficients η^0 using the solution from OccBin (Guerrieri and Iacoviello, 2015) as a candidate solution.
2. Solve and simulate the nonlinear equilibrium conditions for a large number of periods.
3. Generate a new set of time series conditional expectations.
4. Update polynomial coefficients via OLS.
5. Iterate until $\|\eta^j - \eta^{j-1}\| < \zeta$.

The Technical Appendix describes our approach in detail.¹³

3 Model Experiments

We start by illustrating the model’s behavior in response to productivity shocks. Figure 1 shows the response of the economy to a positive and a negative shock to productivity, starting from the risk-adjusted steady state of the economy. Due to the precautionary behavior of the borrower, the constraint is not binding in the risk-adjusted steady state. However, the negative productivity shock lowers house prices and pushes the economy into the region where the constraint is binding. The result is a larger drop in consumption, housing investment, and debt after the negative productivity shock than the corresponding increases in response to the positive shock.

The lower panels of Figure 1 show the responses of the economy with and without housing taxes, represented by dashed and solid lines, respectively. For this experiment we set the tax on borrowing to zero, $\epsilon^b = 0$, and the elasticity of the tax on housing to a value of 3 percentage points, $\epsilon^h = .03$.¹⁴ The responses of the economy with taxes are computed in deviation from its own risk-adjusted steady state, which, as we further discuss below, is different from the risk-adjusted steady state of the economy without taxes.

Following equation (16), the tax is positive after the increase in aggregate productivity and is a

¹¹ In U.S. data from 1947:Q1 through 2024:Q4, the standard deviation of linearly detrended log GDP is 7.5 percent. It drops to 6.7 percent if the sample runs from 1947:Q1 through 2019:Q4. If one applies the HP-filter to both data and model simulations (with a smoothing parameter of $\lambda = 1,600$), the standard deviations for filtered log GDP drops to around 2 percent both in the data and in the model.

¹² In U.S. data from 1947:Q1 through 2024:Q4, the standard deviation of linearly detrended log personal consumption is 6.9 percent.

¹³ See den Haan and Marcet (1990) for a description of the parametrized expectation algorithm.

¹⁴ As we discuss in Section 5.2, varying ϵ^b has very similar effects to varying ϵ^h . Therefore, we mostly focus on housing taxes in our analysis.

subsidy when productivity is below average. The subsidy on housing after a negative productivity shock dampens the drop in housing investment and house prices. With a smaller drop in the value of collateral, the constraint is less tight and the borrower can prevent borrowing from declining as much, thus softening the decline in consumption. The increase in taxes on housing after a positive technology shock causes housing investment by the borrower to actually drop and, hence, house prices to increase less. However, the response of borrowers' consumption to a positive technology shock is almost identical with and without the tax.

Figure 2 depicts the policy functions of the models with and without taxes. The panels show how policies depend on productivity while keeping all other state variables constant at the risk-adjusted steady state of the economy without taxes. With the housing tax, house prices become less sensitive to productivity fluctuations; in particular, the (negative) tax cushions sharp declines after negative shocks, resulting in higher prices, looser collateral constraints, and increased borrowing capacity. Conversely, housing is taxed during expansions, which lowers house prices after large technology increases. Besides a change in the slope, the policy function shifts upward, leading to slightly higher average house prices.¹⁵ The reason for this is that future house prices tend to be higher on average as the likelihood and severity of fire sales decrease, resulting in higher prices in the present. This mechanism is similar to Tobin's Q-theory or traditional asset pricing models, where expectations of higher future prices lead to increased current valuations. In our framework, this effect is reflected in equation (6), where, all else being equal, a higher q_{t+1} leads to a higher q_t .

All told, the tax policy raises consumption of the borrower both after negative productivity shocks, because it softens the decline in house prices and allows the borrower to borrow more, and after positive productivity shocks, because it lowers housing investment. As a result, borrowers' welfare is higher in the economy with the tax. Conversely, consumption of the saver decreases. These policy functions, however, keep the level of debt constant. As we discuss below, consumption patterns change after a transition period because the borrower and saver accumulate more debt and assets, respectively.

4 Taxes and Welfare

Does the tax intervention result in welfare Pareto improvements? In this economy, agents only trade one riskless non-contingent bond; hence, state-contingent taxes on particular assets may improve welfare because they substitute for the absence of a complete set of state-contingent securities, given the presence of aggregate risk. However, the welfare improvement afforded by a particular tax policy that operates through this channel turns out to be minuscule, analogous to the well-known result in open economy macro that, with highly correlated productivity shocks, one riskless

¹⁵Note, for instance, in Figure 2 that when productivity is at steady state, house prices are higher with the tax intervention.

asset comes very close to completing the markets. Here, instead, the welfare comparison is made interesting by the presence of the uninternalized pecuniary externality coming from the collateral constraint.

Figure 3 plots the welfare effects of varying the elasticity of housing taxes to technology, ϵ^h , while keeping taxes on debt fixed at zero, i.e. $\epsilon^b = 0$. Each point corresponds to a different elasticity of the tax rate to aggregate productivity and the origin corresponds to the allocations without taxes. All welfare changes are expressed as lifetime consumption equivalent compensation relative to the equilibrium without taxes. For each model, we evaluate welfare at a point of the state space corresponding to the stochastic steady state of the economy without taxes. Pareto improvements relative to the no-tax equilibrium correspond to all the allocations that lie north-east of the zero-tax equilibrium. It is evident from this figure that a procyclical housing tax with intermediate values ϵ^h leads to a Pareto improvement, whereas too low or too high values do not.

For an elasticity around $\epsilon^h = 0.03$, the tax improves borrower’s welfare as much as a permanent change in consumption of about 0.2 percent, a value that is as large as the welfare gains of eliminating business cycles altogether in representative-agent business cycle models. In the latter class of models, the welfare gains can be shown to be proportional to half the squared variance of log consumption, as first shown by Lucas (1987).¹⁶ In the next subsection we provide a detailed decomposition of the origin of the welfare gains in our model.

Figure 4 shows the welfare effects of introducing housing taxes not only in the steady state, but also in a boom and during a crisis. The welfare effects are similar to the ones in steady state, with larger benefits for the borrower when taxes are introduced in a crisis, and larger benefits for the saver when taxes are introduced during an expansion. Of note, a tax rule with an elasticity between 2.5 and 3, which roughly optimizes borrower’s welfare subject to the saver’s welfare being the same as in the no-tax equilibrium, risk-adjusted steady state, delivers Pareto improvements also when introduced either in an expansion or in a recession.

To understand the workings of the policy intervention, consider a tax that improves welfare of the borrower by the maximum amount while holding the saver’s welfare unchanged. Such tax has an elasticity of 2.8 percentage points to aggregate productivity. To get some quantitative flavor, remember that the fiscal instrument is a holding tax — similar to a property tax — and is levied as a percentage of the value of borrowers’ housing holdings. From the borrower’s budget constraint, we know that $c_t + (1 + \tau_t) q_t h_t = resources$, so that a tax τ_t raises the holding cost of housing by $100 \times \tau_t$ percent. An x percent negative productivity shock calls for a reduction in the holding cost of housing of $100 \times \epsilon^h \times x$ percent. With $\epsilon^h = .028$ and assuming a negative productivity shock of 3 percent, $x = -0.03$, the reduction in the housing holding cost is $100 \cdot 0.028 \cdot 0.03 = .084$ percent.

¹⁶ Lucas (1987) shows that the welfare cost of business cycles can be approximated as $(\gamma/2)\sigma^2$, where γ is the coefficient of relative risk aversion and σ^2 is the variance of log consumption. A welfare gain of 0.2 percent can be obtained by eliminating business cycles if the initial standard deviation of consumption is around 6.3 percent and the coefficient of relative risk aversion is unity.

For a house of 420,000 US dollars, this corresponds to a subsidy of about 350 dollars per year.¹⁷

While the tax magnitudes seem small, the tax/subsidy is very effective because τ_t^h is a holding tax, not a purchase tax. With forward-looking rational agents, housing demand is very sensitive to changes in the tax, as the housing price depends on current and future expected taxes. This can be seen by solving equation (12) forward:

$$q_t = E_t \sum_{j=0}^{\infty} \beta^{1+j} \frac{c_t}{c_{t+1+j}} \left(\prod_{i=0}^j \frac{1}{(1 + \tau_{t+i}^h - \lambda_{t+i} m c_{t+i})} \right) \left(\gamma \frac{y_{t+1+j}}{h_{t+j}} \right). \quad (21)$$

As the equation shows, under persistent productivity shocks, small changes in the tax can exert large effects on today's asset prices.

Figure 5 plots the transition of the economy from the risk-adjusted steady state of the case in which taxes are zero to the new risk-adjusted steady state of the economy with a procyclical housing tax. The transition happens unexpectedly after 20 quarters. Throughout the simulation, we shut down aggregate shocks. The economy starts from the risky adjusted steady state without taxes, where aggregates are constant, but equilibrium prices and quantities fully reflect the possibility that shocks might happen in the future.¹⁸ In period 20, we artificially and unexpectedly move the no-tax economy to a new regime, in which agents still expect future shocks but at the same time anticipate that the tax rule will be of the form described above.

The introduction of the tax policy raises the price of housing as it lowers the sensitivity of house prices to technology shocks, and in particular, the subsidy to housing investment dampens house price drops after negative productivity shocks, thus reducing the covariance of consumption with house prices. As house prices increase and become less variable, the borrower increases its borrowing and consumption. As a result, the borrower's welfare increases by about 0.2 percentage points of permanent consumption equivalent gains. The increase in borrower's debt is financed by larger savings by the saver, who, on impact, decreases consumption and lends more to the borrower. However, the saver's welfare also increases on impact as their consumption is permanently higher in the new risk-adjusted steady state.

4.1 Decomposing Welfare Effects of Taxes

We decompose the welfare effects of a particular tax, starting from the stochastic steady state of the economy without tax, into the following three components

$$\Delta W \approx \Delta W_{SR} + \Delta W_{LR} + \Delta W_{VAR}. \quad (22)$$

¹⁷ The median sale price of a house in the United States in 2024 was \$419,200, according to data from U.S. Census Bureau/U.S. Department of Housing and Urban Development.

¹⁸ See e.g. Coeurdacier, Rey, and Winant (2011).

where ΔW is the total welfare effect, ΔW_{SR} is the welfare component reflecting the transition to the new steady state of consumption (short-run transition in mean consumption), ΔW_{LR} is the welfare component reflecting changes in long-run mean consumption (long-run mean level) and ΔW_{VAR} is the welfare component reflecting the changes in variance associated with a particular tax policy (volatility effect).¹⁹

We define the short-run welfare changes associated with the transition component of the tax as:

$$\Delta W_{SR} = \sum_{t=0}^T \beta^t \log E_0(c_t^{tax}) - \sum_{t=0}^T \beta^t \log E_0(c_t^{no-tax}), \quad (23)$$

where c_t^{tax} is the equilibrium consumption in the economy with the active tax system, and c_t^{no-tax} is consumption in the economy without taxes. This formula measures the change in utility associated with the new consumption path for the first $T = 80$ periods.²⁰ For the policy experiment described in the text, such calculation returns $\Delta W_{SR} = 0.18\%$, $\Delta W'_{SR} = -0.03\%$, respectively, for the borrower and the saver. Analogously, the long-run mean-level component of the tax is

$$\Delta W_{LR} = \sum_{t=T+1}^{\infty} \beta^t \log E_0(c_t^{tax}) - \sum_{t=T+1}^{\infty} \beta^t \log E_0(c_t^{no-tax}), \quad (24)$$

yielding $\Delta W_{LR} = -0.06\%$ and $\Delta W'_{LR} = 0.06\%$.

We now move to the volatility effects associated with the change in the tax. These changes can be approximated — up to second order — by the change in the variance of log consumption associated with the tax change. That is:

$$\Delta W_{VAR} = - \left(\sum_{t=0}^{\infty} \beta^t \frac{E_0 (c_t^{tax} - E_0 c_t^{tax})^2}{2(E_0 c_t^{tax})^2} - E_0 \sum_{t=0}^{\infty} \beta^t \frac{E_0 (c_t^{no-tax} - E_0 c_t^{no-tax})^2}{2(E_0 c_t^{no-tax})^2} \right), \quad (25)$$

yielding $\Delta W_{VAR} = 0.06\%$ and $\Delta W'_{VAR} = -0.01\%$. The welfare decomposition is summarized in Table 2.

The decomposition results in Table 2 can be understood by referring again to the transition to the new tax regime plotted in Figure 5. The response of consumption to introducing the tax rule is particularly informative regarding the welfare decomposition. In the short term, borrowers increase consumption. However, with higher house prices and lower precautionary motives, borrowers' debt increases in the long run, and consumption settles at a lower level in the new steady state. These trajectories explain the short-run welfare gains and long-run welfare losses of average consumption,

¹⁹ A similar decomposition in welfare between average and volatility components can be found in Obstfeld (1994), Bénabou (2002), and Flodén (2001). Relative to those papers, we further decompose the average effect into the short-run mean transition and the long-run mean level. All these welfare changes are expressed in percent consumption equivalents. Appendix A.4 provides further details.

²⁰ We set T based on the inspection of the transitions. Different transition periods T do not change the results qualitatively.

that is, $\Delta W_{SR} > 0$ and $\Delta W_{LR} < 0$. Consumption of the saver is akin to the mirror image of the borrower's.²¹ For the saver, consumption initially declines following the introduction of the tax, resulting in $\Delta W'_{SR} < 0$, but eventually increases due to the higher asset accumulation in the new steady state, resulting in $\Delta W'_{LR} > 0$.

Turning to the effects of volatility, this policy leads to a reduction in the volatility of consumption. Given the enhanced opportunities that borrowers have to obtain funds, the consumption policy function is flatter. These features translate into a welfare gain due to the borrower's lower consumption volatility.

5 Extensions and Discussion

In this section, we present two extensions: asymmetric taxes and taxes on borrowing.

5.1 Asymmetric Taxes

We consider two asymmetric policies. In one, the housing tax is only in effect during expansions, and in the other, the subsidy is active during recessions. More specifically, we consider a tax-only policy μ_t^h given by:

$$\mu_t^h = \begin{cases} \varepsilon^h(A_t - 1), & \text{if } A_t \geq 1, \\ 0, & \text{if } A_t < 1, \end{cases} \quad (26)$$

and a subsidy only policy σ_t^h given by:

$$\sigma_t^h = \begin{cases} 0, & \text{if } A_t \geq 1, \\ \varepsilon^h(A_t - 1), & \text{if } A_t < 1. \end{cases} \quad (27)$$

Studying the effects of these asymmetric tax policy rules allows us to understand how much of the welfare improvements are due to taming house prices and borrowing during expansions with positive taxes on housing as with the tax policy μ_t^h — a more macroprudential nature of the tax policy — and how much are due to an ex-post intervention mopping up after the crash as under the subsidy policy σ_t^h .²²

Figure 6 shows the welfare effects of introducing the tax-only policy rule μ_t^h . It shows that the tax always hurts both borrowers and savers. To better understand the mechanism, Figure 7 shows the policy functions. In this case, the tax intervention lowers house prices, leading to more stringent borrowing conditions. The consumption of the borrower is lower and that of the saver is higher.

²¹ Due to capital accumulation, the consumption patterns of borrowers and savers do not completely offset.

²² See also Benigno, Rebucci, and Zaretski (2024).

Figure 8 shows the welfare effects of introducing the subsidy policy σ_t^h . The Pareto gains from this policy are roughly as high as in our baseline policy. Figure 9 shows the policy functions. House prices increase even in expansions, borrowing increases mainly during recessions, and the consumption of borrowers increases for all levels of productivity.

Overall, the findings in this section suggest that the effectiveness of the tax policy is not primarily driven by its macroprudential role – namely, its ability to moderate house prices, borrowing, and consumption during economic booms. While traditional macroprudential policies focus on curbing excessive risk-taking in good times to prevent financial instability, the results here highlight a different mechanism. Specifically, the welfare gains associated with the asymmetric tax stem from its ability to mitigate the adverse consequences of economic downturns ex-post. By reducing the severity and persistence of house price declines, the policy helps stabilize the economy when it is most vulnerable. Regarding macroprudential regulation versus mopping up after the crash, the welfare gains in our model are in the latter.

5.2 Taxes on Borrowing

What are the results of a tax debt rule instead of a housing tax? In this section, we show that the two taxes are very close substitutes and deliver very similar effects. To see this, we first rewrite the optimality conditions for the borrowers to eliminate the Lagrangian multiplier. This shows that the two taxes affect the optimality condition in a very similar way. We then show quantitatively that optimizing on the elasticity of debt taxes yields results that are very similar to the ones obtained with the housing tax.

Note that if we use equation (11) to substitute for λ in equation (12), we get an optimality condition for saving given by:

$$1 = \beta E_t \frac{c_t}{c_{t+1}} \left[\frac{1}{1 - \frac{m}{1+\tau_t^h}} \frac{R_{t+1}^h}{(1 + \tau_t^h)} - \frac{\frac{m}{1+\tau_t^h}}{1 - \frac{m}{1+\tau_t^h}} R_t (1 + \tau_t^b) \right], \quad (28)$$

where $R_{t+1}^h = \frac{\gamma \frac{y_{t+1}}{h_t} + q_{t+1}}{q_t}$ are the returns on housing investment net of housing taxes. Equation (28) shows that the borrower will always optimize the consumption-saving decision to equate the marginal cost of saving to the marginal return to its leverage investment in housing, given by $\frac{1}{1 - \frac{m}{1+\tau_t^h}} \frac{R_{t+1}^h}{(1 + \tau_t^h)} - \frac{\frac{m}{1+\tau_t^h}}{1 - \frac{m}{1+\tau_t^h}} R_t (1 + \tau_t^b)$. The borrowing constraint will affect the portfolio decision by limiting the amount of leverage that the borrower takes in their housing investment. In particular, when the constraint is binding, $b_t = m q_t h_t$ and the borrower will have positive excess returns on housing investment:

$$E_t \frac{c_t}{c_{t+1}} \left[\frac{R_{t+1}^h}{(1 + \tau_t^h)} - R_t (1 + \tau_t^b) \right] > 0, \quad (29)$$

as we discuss further below.

When the constraint is slack, these excess returns are arbitrated away:

$$E_t \frac{c_t}{c_{t+1}} \left[\frac{R_{t+1}^h}{(1 + \tau_t^h)} - R_t (1 + \tau_t^b) \right] = 0. \quad (30)$$

Rewriting the optimality conditions for the borrower as in equations (28)-(30) also shows that taxes on housing and taxes on debt are substitute instruments that affect excess returns on housing investment, and hence the borrowers' portfolio choices, in a very similar manner.

Figure 10 shows the impulse response of the economy to positive and negative productivity shocks in the model without taxes and in the model with taxes on borrowing (for a value of the tax $\tau^b = 0.03$). The results are analogous to those with a housing tax shown in Figure 1. The tax is positive after the increase in aggregate productivity and is a subsidy when productivity declines. The subsidy on borrowing after a negative productivity shock dampens the drop in debt and, therefore, dampens the drop in housing investment and house prices. The mechanism is similar to that in the baseline. As house prices and housing investment decline less, the constraint is not as tight, allowing the borrower to borrow more and dampen the decrease in consumption. The increase in taxes on borrowing after a positive technology shock follows an analogous logic to the baseline but operates in the first instance through borrowing.

Figure 11 shows the effects on welfare. The patterns are again similar to our baseline. For intermediate values of the tax elasticity, both agents are better off. In Figure 12, we artificially shut down aggregate shocks, but agents expect the possibility of shocks; this is the same procedure as in Figure 5. The economy first settles in the risk-adjusted steady state, and then in period 20, the borrowing tax is introduced. The magnitudes of the responses after the introduction of the tax differ from those of Figure 5, but qualitatively, the responses are similar. Consumption of the borrower increases in the short run as the collateral constraint is immediately relaxed given the increase in house prices; as time passes, debt service payments kick in, and consumption settles at a lower level than that in the old regime. Consumption of the saver is the opposite. House prices and debt settle at a higher level. In summary, an appropriately designed borrowing tax also improves welfare for both agents.

6 Conclusions

This paper examines the optimal credit market policy in a stochastic, quantitative general equilibrium framework with collateral constraints. Our analysis shows that a state-contingent housing tax can generate Pareto improvements by mitigating financial frictions. Specifically, the proposed tax system follows a procyclical pattern: it taxes housing in economic expansions and subsidizes it during downturns. This policy reduces the link between collateral prices and consumption, increasing

average asset prices and generating welfare gains for both borrowers and savers.

The welfare benefits from implementing the optimal tax can be understood through the following mechanism. First, by stabilizing house prices, the tax alleviates borrowing constraints during downturns, reducing the incidence and severity of fire sales, which raises expected future prices and relaxes borrowing constraints further. Second, the tax facilitates intertemporal trade by giving borrowers increased credit access, which reduces consumption volatility.

A key result of our quantitative analysis is that the welfare gains from the optimal housing tax are comparable to those from eliminating business cycles. Decomposing these welfare effects, we show that borrowers benefit primarily from short-term consumption gains and reduced volatility, whereas savers gain in the long run due to increased asset positions. The tax achieves these improvements while remaining revenue-neutral, ensuring that gains are not driven by redistributive transfers but rather by efficiency enhancements in the credit market. We also showed that the welfare gains of our tax are mainly attributable to a “mopping up after the crash” rather than a macroprudential aspect.

Our findings contribute to the broader literature on macroprudential policy and credit market interventions by highlighting the role of asset price stabilization in improving welfare. Unlike conventional fiscal or monetary policy tools, the proposed tax directly targets the pecuniary externalities arising from collateral constraints.

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Table 1: Calibration

Parameter	Value	Description
β	0.985	Discount factor of borrowers
β'	0.99	Discount factor of savers
α'	0.2	Capital share in production
γ	0.3	Housing share in borrowers' production
γ'	0.1	Housing share in savers' production
δ	0.025	Depreciation rate
m	0.8	Collateral requirements parameter
n	0.5	Share of borrowers
ρ	0.95	Persistence of shock
σ	0.0165	Standard deviation of shock

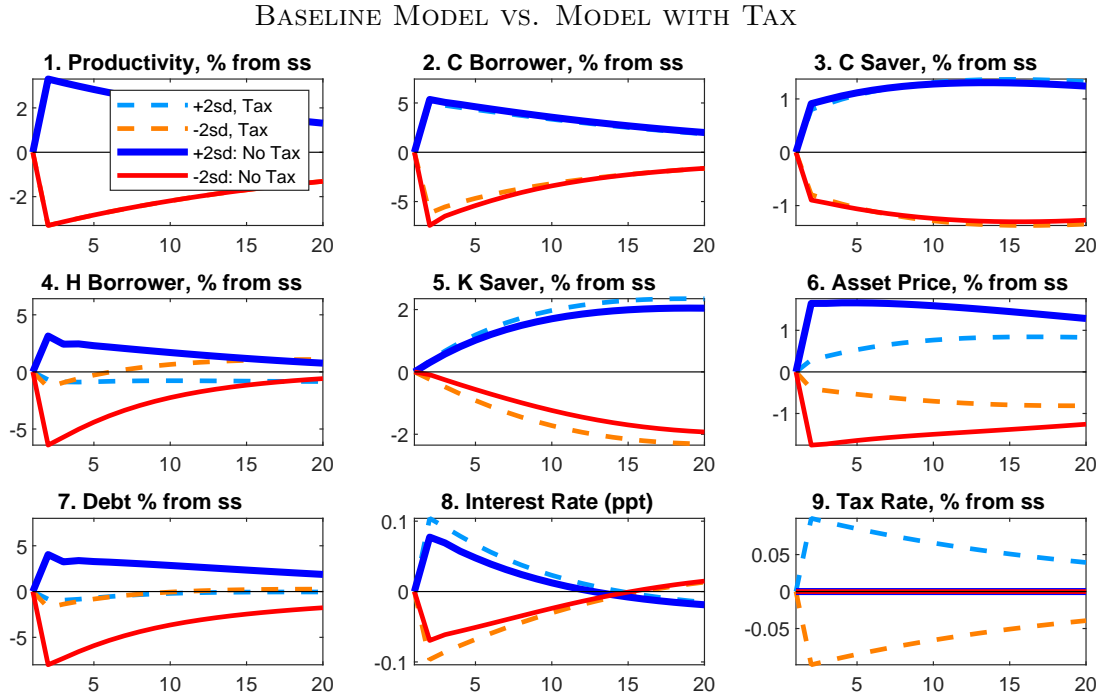
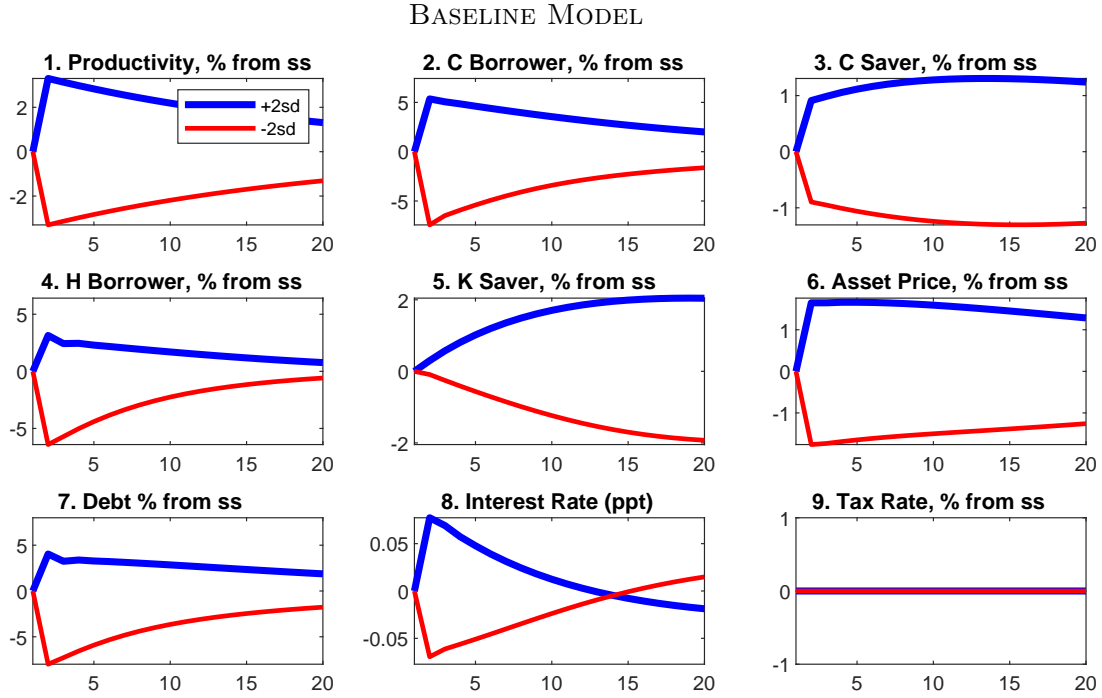
Note: Parameter calibration for the benchmark model.

Table 2: Welfare Change Decomposition

	Welfare Change Decomposition			
	Total	SR Mean (1)	LR Mean (2)	Variance (3)
Borrower	0.19	0.18	-0.06	0.06
Saver	0.02	-0.03	0.06	-0.01

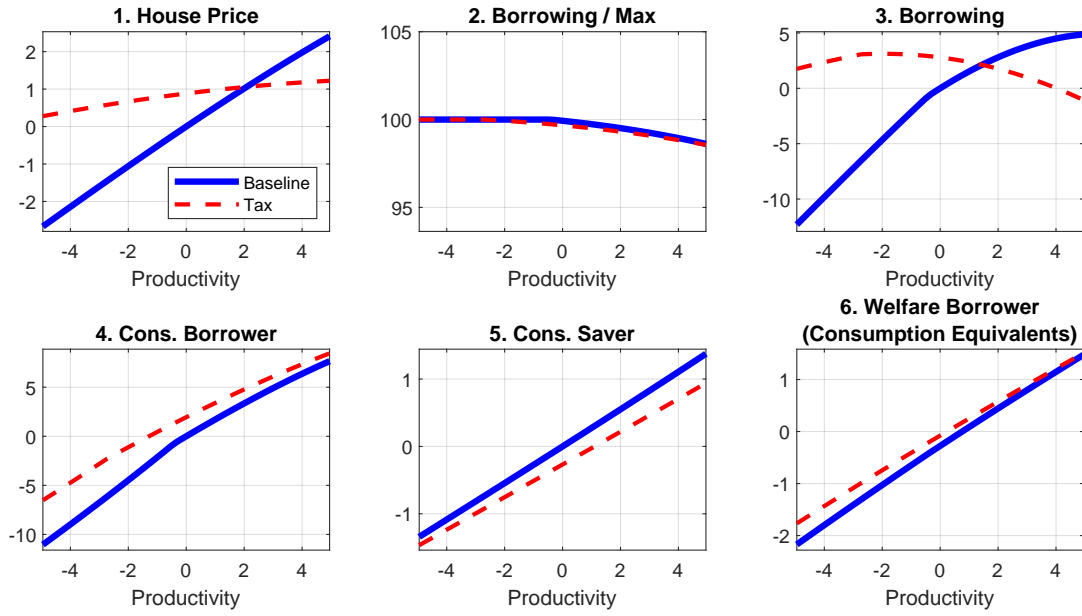
Note: Total change in conditional welfare (measured in % change in consumption equivalents) from introducing a tax. Welfare is calculated starting at the risk-adjusted steady state of the model without tax.

Figure 1: IMPULSES RESPONSES



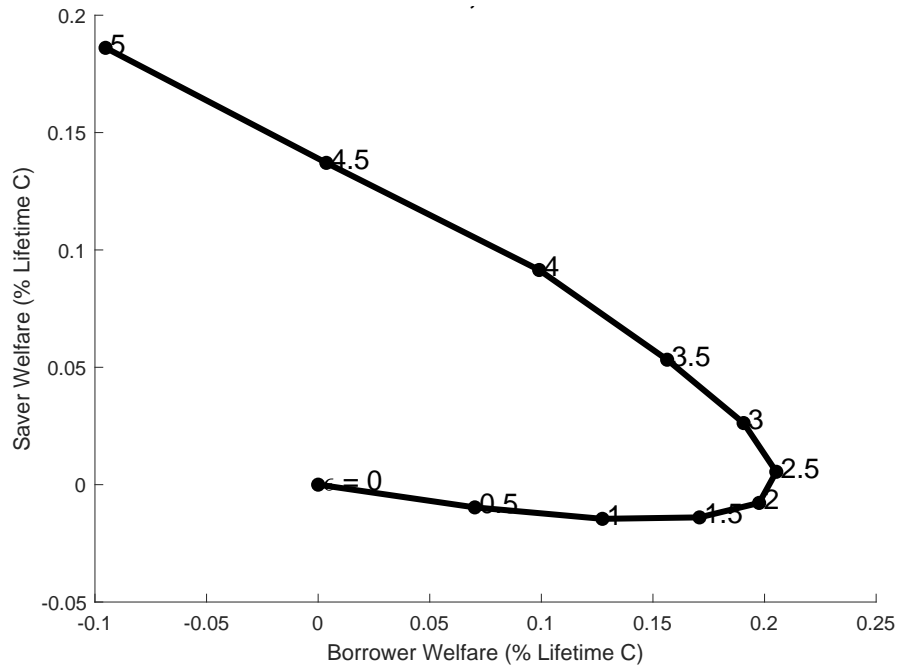
Note: Impulse Response without and with tax. The top 9 panels show the effects of positive and negative two standard deviation productivity shocks without the tax. The economy starts from the risk-adjusted steady state of the economy without tax. The lower 9 panels show responses to positive and negative productivity shocks, with and without the optimal tax (the economies start from respective risky steady states).

Figure 2: POLICY FUNCTIONS



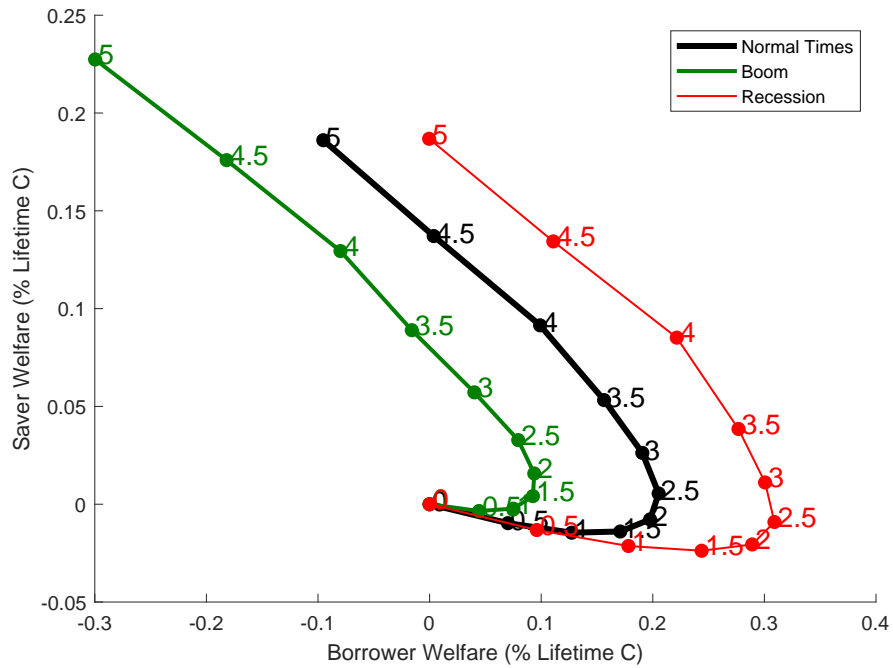
Note: Policy Functions. The blue lines denote policy function without tax. The red lines denote policy functions with the housing tax. The x-axis refers to debt.

Figure 3: PARETO FRONTIER FOR THE BENCHMARK MODEL



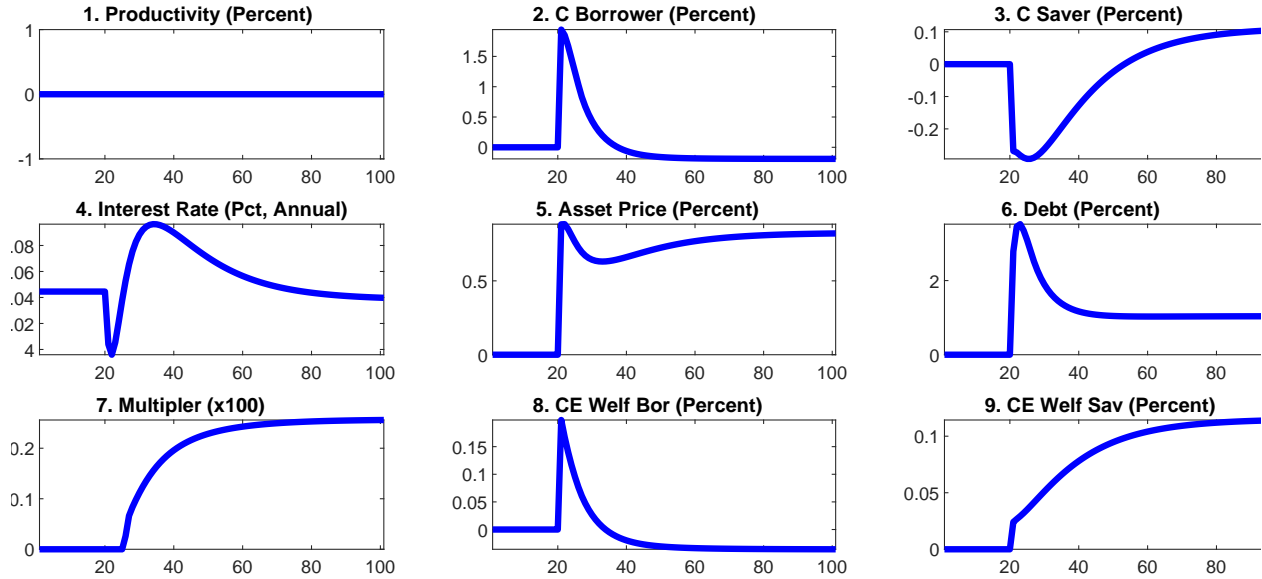
Note: Welfare Calculations. The panel shows the Pareto frontier of the economy for various combinations of housing taxes (the origin corresponds to the competitive equilibrium without taxes). The dots indicate allocation for various values of the elasticity of taxes to productivity defined as 100τ .

Figure 4: PARETO FRONTIER FOR THE BENCHMARK MODEL: BOOM VS RECESSION



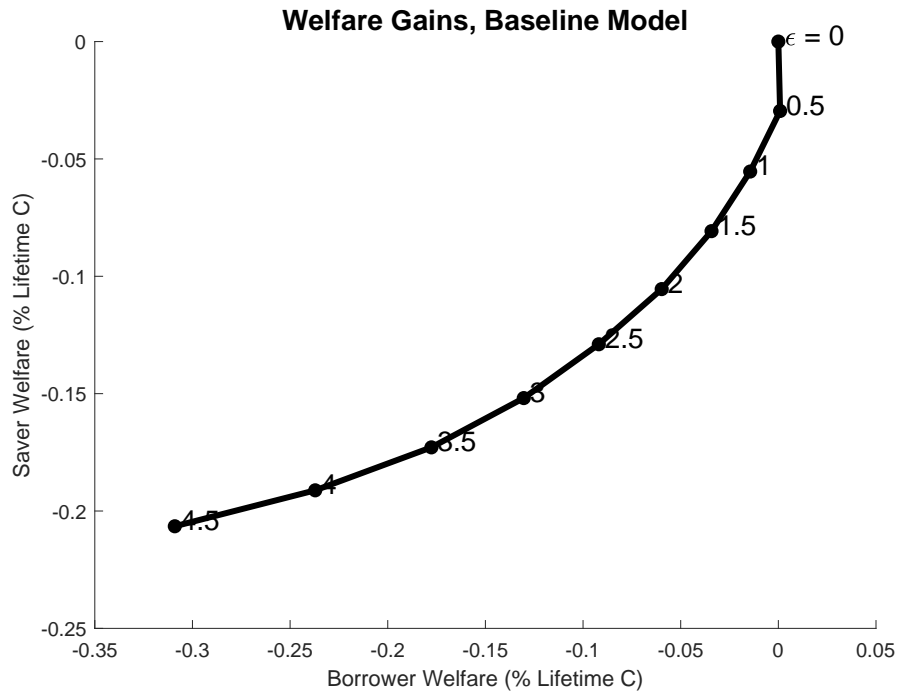
Note: Welfare Calculations. See notes to Figure 3. The black line refers to normal times when the shock A_t is initialized at steady state. In the green and red lines, the economy is respectively above and below average.

Figure 5: TRANSITION TO NEW REGIME



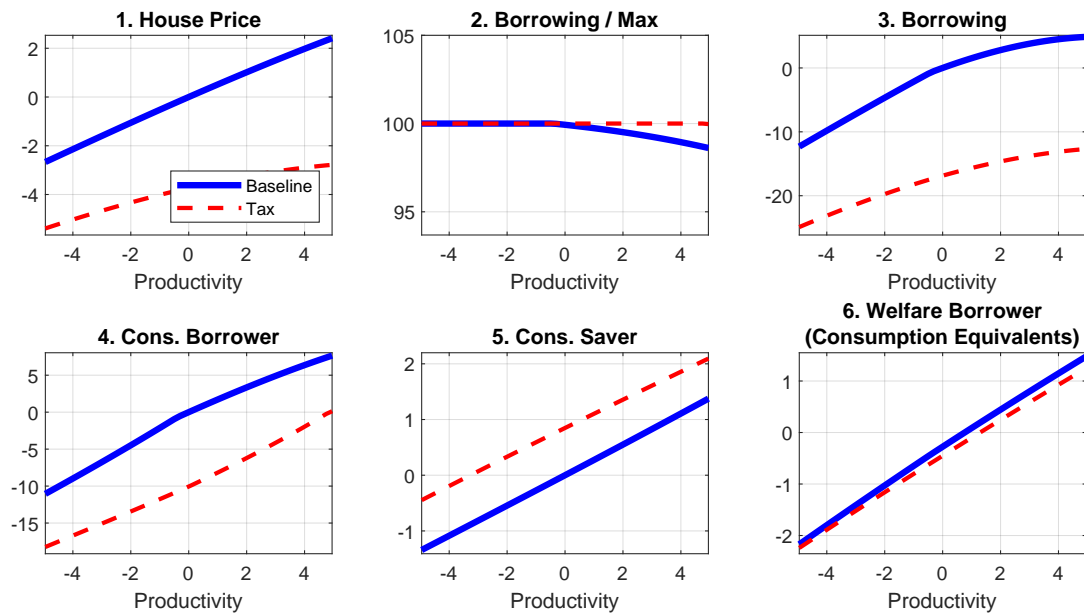
Note: The figure shows the transition to the new regime. More specifically, the figure plots the evolution over time of the economy where we artificially shut down aggregate shocks, so that the economy settles around its risky steady state, but equilibrium prices and quantities fully reflect the possibility of shocks. In period 20, we artificially and unexpectedly move the no-tax economy to a new regime, in which agents still expect future shocks but at the same time anticipate the tax rule.

Figure 6: MODEL WITH TAX ONLY IN EXPANSIONS: FRONTIER



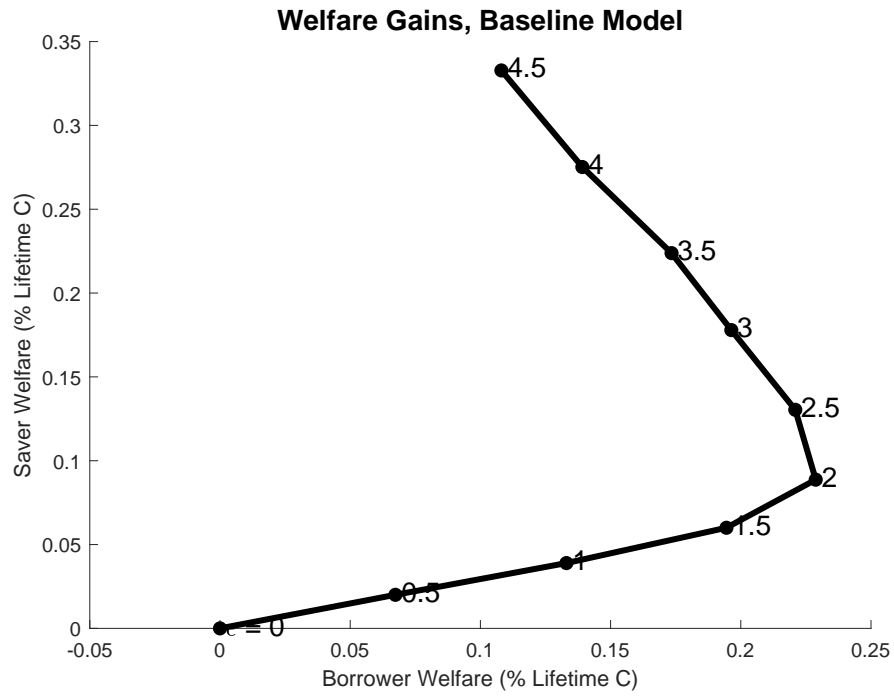
Note: Welfare gains for the model with tax only in expansions. See notes to Figure 3 for details.

Figure 7: MODEL WITH TAX ONLY IN EXPANSIONS: POLICY FUNCTIONS



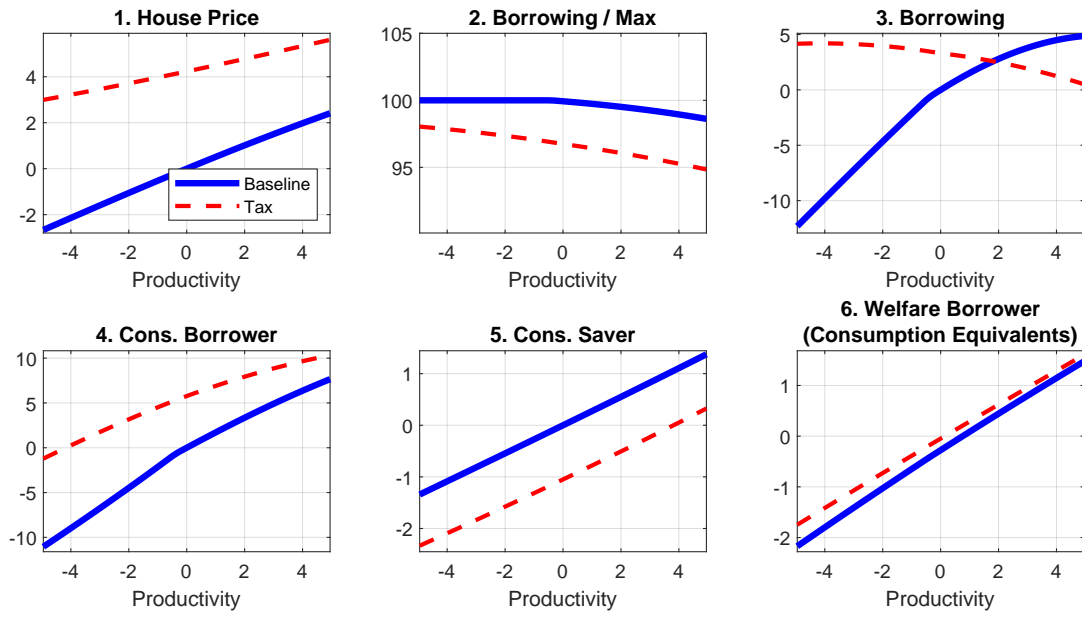
Note: Policy functions for the model with tax only in expansions. See notes to Figure 2 for additional details.

Figure 8: MODEL WITH TAX ONLY IN RECESSIONS: FRONTIER



Note: Welfare gains for the model with tax only in recessions. See notes to Figure 3 for details.

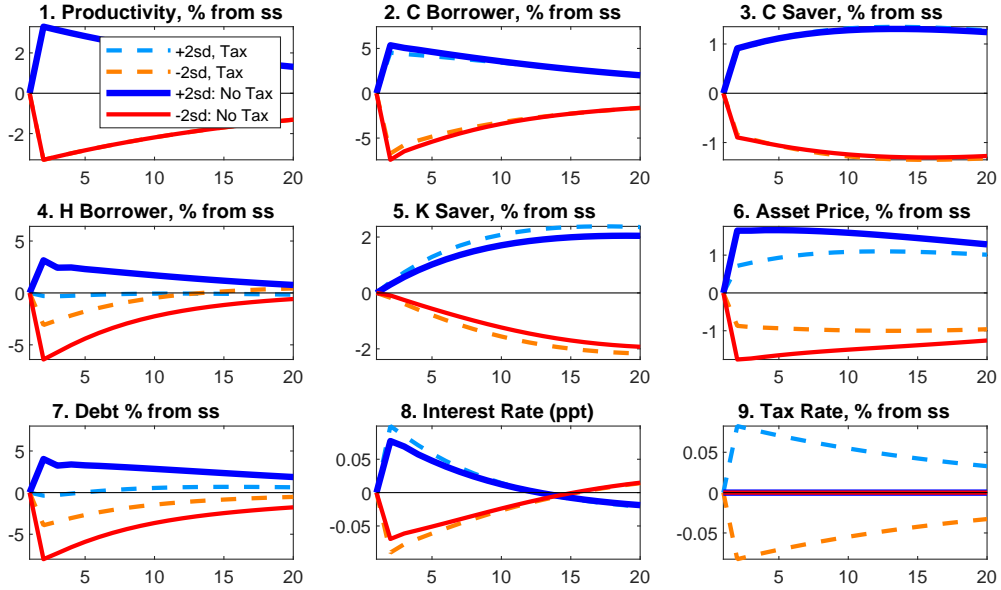
Figure 9: MODEL WITH TAX ONLY IN RECESSIONS: POLICY FUNCTIONS



Note: Policy functions for the model with tax only in recessions. See notes to Figure 2 for additional details.

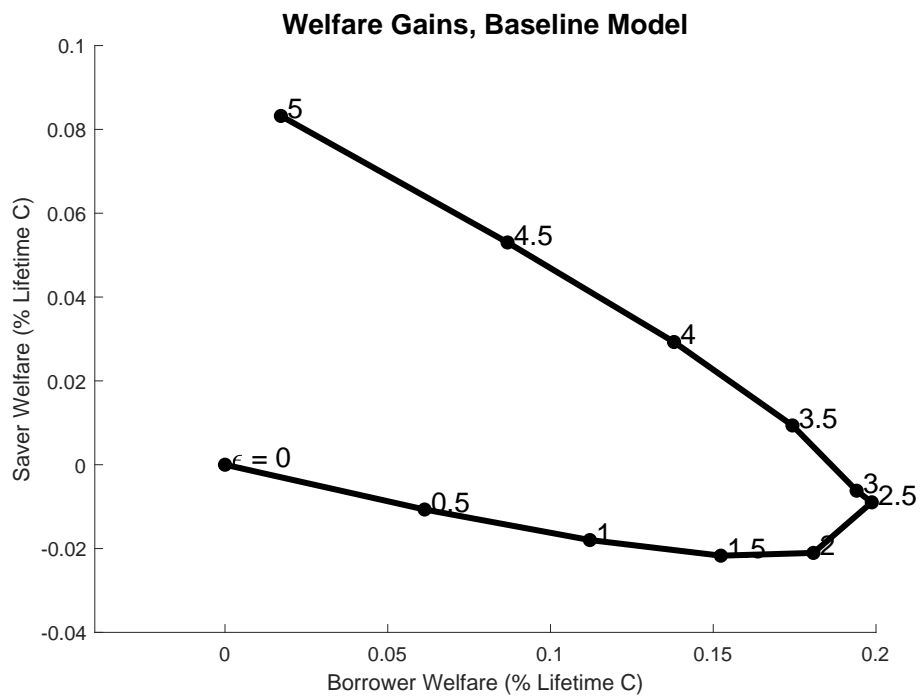
Figure 10: MODEL WITH BORROWING TAX: IMPULSE RESPONSE

Impulse Response, with and without Taxes



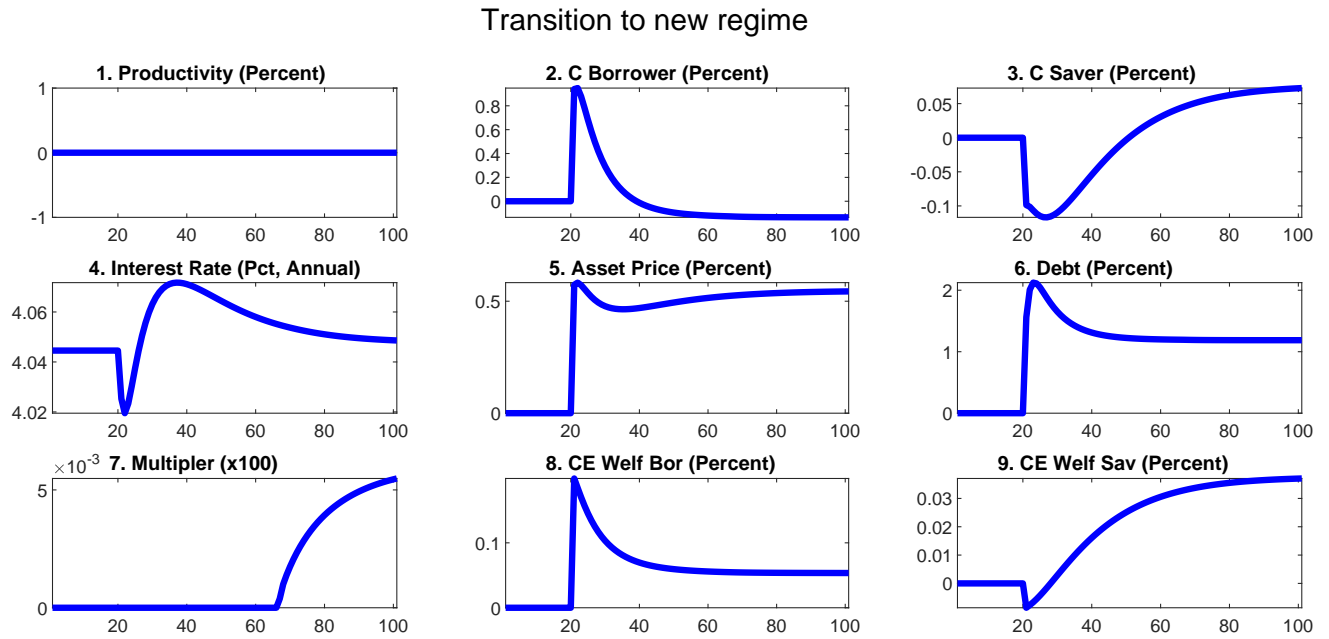
Note: See notes to 1. This figure shows, for the model with borrowing tax, the impulse responses with and without tax. Both economies start from the risky steady state of the economy without tax.

Figure 11: MODEL WITH BORROWING TAX: FRONTIER



Note: See notes to Figure 3. This figure shows the frontier for the model with borrowing tax.

Figure 12: MODEL WITH BORROWING TAX: TRANSITION TO NEW REGIME



Note: See notes to Figure 5. The figure shows the transition to the new regime in the model with borrowing tax.

Figure 13: EULER EQUATION ERRORS, BASELINE MODEL



Note: The figure shows Euler Equation errors in the Baseline Model.

A Technical Appendix

A.1 Model Equations

The equations for the natural savers are:

$$y'_t = A_t h_{t-1}^\gamma k_{t-1}'^\alpha, \quad (\text{A.1})$$

$$c'_t + k'_t + q_t h'_t - R_{t-1} b'_{t-1} = y'_t + q_t h'_{t-1} - b'_t + (1 - \delta) k'_{t-1}, \quad (\text{A.2})$$

$$\frac{1}{c'_t} = \beta' R_t E_t \frac{1}{c'_{t+1}}, \quad (\text{A.3})$$

$$\frac{1}{c'_t} = \beta' E_t \frac{1}{c'_{t+1}} \left(\alpha \frac{y'_{t+1}}{k'_t} + 1 - \delta \right), \quad (\text{A.4})$$

$$\frac{q_t}{c'_t} = \beta' E_t \frac{1}{c'_{t+1}} \left(q_{t+1} + \gamma \frac{y'_{t+1}}{h'_t} \right). \quad (\text{A.5})$$

The equations for the natural borrowers are:

$$y_t = A_t h_{t-1}^\gamma, \quad (\text{A.6})$$

$$c_t + q_t h_t (1 + \tau_t) = b_t - R_{t-1} b_{t-1} - y_t + q_t h_{t-1} + T_t, \quad (\text{A.7})$$

$$b_t \leq m q_t h_t, \quad (\text{A.8})$$

$$\frac{1}{c_t} = \beta R_t E_t \frac{1}{c_{t+1}} + \lambda_t, \quad (\text{A.9})$$

$$\frac{q_t (1 + \tau_t)}{c_t} = \beta \frac{1}{c_{t+1}} \left(q_{t+1} + \gamma \frac{y_{t+1}}{h_t} \right) + \lambda_t m q_t, \quad (\text{A.10})$$

$$\lambda_t \geq 0, \quad b_t \leq m q_t h_t, \quad \lambda_t (b_t - m q_t h_t) = 0. \quad (\text{A.11})$$

Market equilibrium is:

$$n b_t = (1 - n) b'_t, \quad (\text{A.12})$$

$$n h_t + (1 - n) h'_t = 1. \quad (\text{A.13})$$

The shock process is:

$$A_t - 1 = \rho (A_{t-1} - 1) + \sigma \varepsilon_t, \quad \varepsilon \sim N(0, 1). \quad (\text{A.14})$$

The tax equals $\tau_t = \varepsilon (A_t - 1)$ and is rebated to the borrower lump-sum, so that in equilibrium $T_t = \tau_t q_t h_t$.

A.2 Steady State Analysis

Assuming that the saver is never at the constraint and using market clearing conditions (dropping b' and h'), the steady state is characterized by:

$$\lambda = 1 - \beta R, \quad (\text{A.15})$$

$$R = \frac{1}{\beta'}. \quad (\text{A.16})$$

In steady state, the housing tax is zero, so $\tau = T = 0$

The steady-state capital-output ratio is given by:

$$\frac{k'}{y'} = \frac{\alpha\beta'}{1 - \beta'(1 - \delta)}. \quad (\text{A.17})$$

The production relationships are:

$$y = Ah^\gamma, \quad (\text{A.18})$$

$$y' = A\kappa'^\alpha h'^\gamma. \quad (\text{A.19})$$

This implies:

$$\frac{qh'}{y'} = \frac{\beta'\gamma}{1 - \beta'}, \quad (\text{A.20})$$

$$\frac{c'}{y'} = 1 - \delta\frac{k'}{y'} + (R - 1)\frac{n}{1 - n}\frac{b}{y'}. \quad (\text{A.21})$$

For the borrower, the three key ratios are:

$$\frac{qh}{y} = \frac{\beta\gamma}{1 - \beta - (1 - \beta R)m}, \quad (\text{A.22})$$

$$\frac{c}{y} = 1 - \delta\frac{k}{y} - (R - 1)\frac{b}{y}, \quad (\text{A.23})$$

$$\frac{b}{y} = m\frac{qh}{y}. \quad (\text{A.24})$$

With the housing market clearing condition:

$$nh + (1 - n)h' = 1 \quad (\text{A.25})$$

A.3 Solution Method

The heterogeneous agents macroeconomics literature has witnessed large and recent advancements in computational techniques. [Reiter \(2009\)](#) introduces a solution method combining projection

and perturbation to handle high-dimensional state spaces. [Auclert, Bardczy, Rognlie, and Straub \(2021\)](#) build on this with the Sequence-Space Jacobian, enabling fast and efficient computation of general equilibrium models. [Grand and Ragot \(2024\)](#) extend the literature to fiscal policy, analyzing optimal debt and capital taxation in heterogeneous agents settings.

These methods, however, do not take aggregate uncertainty into account and cannot be used for our analysis. In our model, and unlike [Iacoviello \(2005\)](#), we do not impose that the collateral constraint is always binding.²³ We take into account that an appropriate tax regime can make the collateral constraint slack; it is key that the tax policy can change the stochastic steady state. As we mentioned earlier, the tax policy leads to less volatile house prices, which in turn raises the average house price and alleviates the collateral constraint. As a consequence, the methods mentioned above would not be sufficient.

A.3.1 Overview of the Parameterized Expectations Algorithm

We solve the model with global solution methods. More precisely, we use the Parameterized Expectations Algorithm approach introduced by [den Haan and Marcet \(1990\)](#). Recent applications and developments can be found in [Judd, Maliar, and Maliar \(2011\)](#), [Faraglia, Marcet, Oikonomou, and Scott \(2019\)](#) and [Valaitis and Villa \(2024\)](#).

To solve the model using the Parameterized Expectations Algorithm (PEA), we transform the system to be invertible with respect to current variables. The PEA approximates conditional expectations as a function of the state variables. This method is particularly useful for handling occasionally binding constraints, as in our case with borrowing limits.

A.3.2 Auxiliary Functions

Define the maximum borrowing allowed as $\bar{b}_t = mq_t h_t$. We define the following auxiliary functions to represent conditional expectations:

$$\Psi_{1,t} = E_t \frac{1}{c_{t+1}} \frac{\bar{b}_t}{b_t} \tag{A.26}$$

$$\Psi_{2,t} = E_t \frac{1}{c_{t+1}} (q_{t+1} + \gamma \frac{y_{t+1}}{h_t}), \tag{A.27}$$

$$\Psi_{3,t} = E_t \frac{1}{c'_{t+1}}, \tag{A.28}$$

$$\Psi_{4,t} = E_t \frac{1}{c'_{t+1}} (q_{t+1} + \gamma \frac{y'_{t+1}}{h'_t}), \tag{A.29}$$

$$\Psi_{5,t} = E_t \frac{1}{c'_{t+1}} (\alpha \frac{y'_{t+1}}{k'_t} + 1 - \delta). \tag{A.30}$$

²³ See also [Christiano and Fisher \(2000\)](#).

A.3.3 System Under Binding Constraint

Define $d_t = R_t b_t$ to be the beginning of period debt. The system of equilibrium equations becomes:

$$\frac{1}{c_t} = \beta R_t \Psi_{1,t} \frac{b_t}{\bar{b}_t} + \lambda_t \quad (\text{A.31})$$

$$\frac{q_t(1 + \tau_t)}{c_t} = \beta \Psi_{2,t} + \lambda_t m q_t \quad (\text{A.32})$$

$$\frac{1}{c'_t} = \beta' R_t \Psi_{3,t} \quad (\text{A.33})$$

$$\frac{q_t}{c'_t} = \beta' \Psi_{4,t} \quad (\text{A.34})$$

$$\frac{1}{c'_t} = \beta' \Psi_{5,t} \quad (\text{A.35})$$

$$b_t = c_t + q_t h_t + d_{t-1} - y_t - q_t h_{t-1} \quad (\text{A.36})$$

$$n c'_t = n y_t + (1 - n) y'_t - (1 - n) (k'_t - (1 - \delta) k'_{t-1}) - n c_t. \quad (\text{A.37})$$

If the constraint is binding, $b_t = \bar{b}_t$. Accordingly, this system can be solved recursively as follows:

$$c'_t = \frac{1}{\beta' \Psi_{5,t}}, \quad (\text{A.38})$$

$$R_t = \frac{\Psi_{5,t}}{\Psi_{3,t}}, \quad (\text{A.39})$$

$$q_t = \frac{\Psi_{4,t}}{\Psi_{5,t}}. \quad (\text{A.40})$$

To solve for λ_t , we use:

$$\lambda_t = \frac{\beta \Psi_{5,t} (\Psi_{2,t} \Psi_{3,t} - \Psi_{4,t} (1 + \tau_t) \Psi_{1,t})}{(1 + \tau_t - m) \Psi_{3,t} \Psi_{4,t}}, \quad (\text{A.41})$$

where we have already set $b_t = \bar{b}_t$.

The variable c_t can be obtained from equation (A.31). The variable h_t can be obtained from equation (A.36). The variable k'_t can be obtained from equation (A.37).

A.3.4 System Under Non-Binding Constraint

When the constraint does not bind, the system of equations in equilibrium is:

$$\frac{1}{c_t} = \beta R_t \Psi_{1,t} \frac{b_t}{b_t} + \lambda_t \quad (\text{A.42})$$

$$\frac{q_t(1 + \tau_t)}{c_t} = \beta \Psi_{2,t} \quad (\text{A.43})$$

$$\frac{1}{c'_t} = \beta' R_t \Psi_{3,t} \quad (\text{A.44})$$

$$\frac{q_t}{c'_t} = \beta' \Psi_{4,t} \quad (\text{A.45})$$

$$\frac{1}{c'_t} = \beta' \Psi_{5,t} \quad (\text{A.46})$$

$$b_t = c_t + q_t h_t + d_{t-1} - y_t - q_t h_{t-1} \quad (\text{A.47})$$

$$n c'_t = n y_t + (1 - n) y'_t - (1 - n) (k'_t - (1 - \delta) k'_{t-1}) - n c_t. \quad (\text{A.48})$$

$$\lambda_t = 0. \quad (\text{A.49})$$

If the constraint is not binding, $\lambda_t = 0$. Accordingly, this system can be solved recursively as follows. As before in the binding case, use equations (A.38), (A.39) and (A.40) to solve for c'_t , R_t and q_t . We can then use equation (A.43) to solve for c_t . We can obtain b_t and h_t from (A.42) and (A.47). Finally from equation (A.49) we obtain k'_t . This completes the recursion of how all variables are obtained.

Also, note that if we had instead defined

$$\Psi_{1,t} = E_t \frac{1}{c_{t+1}}, \quad (\text{A.50})$$

and accordingly changed equation (A.42) to

$$\frac{1}{c_t} = \beta R_t \Psi_{1,t} + \lambda_t, \quad (\text{A.51})$$

the subsystem of equations (A.51) and (A.43) to (A.46) has five equations for four endogenous variables c_t , q_t , R_t , c'_t with $\lambda_t = 0$. This is the reason we instead use definition (A.26).

A.3.5 Numerical Implementation

In the numerical implementation, we approximate the conditional expectations $\Psi_{i,t}$ with $i \in \llbracket 1, 5 \rrbracket$ using polynomial functions of the state variables. The state variables of our problem include i) the previous period's housing stock (h_{t-1}), ii) the previous period's capital stock (k_{t-1}), iii) the previous period's debt level (b_{t-1}), iv) and the current period productivity shocks (A_t). We collect the state variables in the vector X_t .

The polynomial used in the PEA can be represented as:

$$\Psi_i(X_t; \eta), \tag{A.52}$$

where η is the vector of coefficients. For a given sequence of exogenous aggregate productivity shocks $\{A_t\}_{t=1}^T$ and an initial guess of the polynomials' parameters, the standard stochastic PEA aims to find parameters η that solve all Euler equations and all laws of motion.

The coefficients of these polynomials are solved for iteratively until convergence is achieved. The solution algorithm follows these steps:

1. Initialize the polynomial coefficients for the conditional expectations at a guess η^0 . We compute the initial values of the conditional expectations and the guesses by solving for the model's nonlinear policy functions using Dynare and OccBin.²⁴
2. Solve the model's equilibrium conditions using these approximations.
3. Simulate the model for T periods, where T is large.
4. Update the polynomial coefficients based on the simulation results to η^j , where j denotes the iteration. This step can be done by OLS.
5. Repeat steps 2-4 until convergence of the polynomial coefficients.

This iterative procedure continues until the maximum change in polynomial coefficients between iterations falls below a pre-specified tolerance level, that is $\|\eta^j - \eta^{j-1}\| < \zeta$, where $\|\cdot\|$ denotes a norm and $\zeta > 0$ is the tolerance level. The Euler equation errors are quite small and shown in Figure 13.

A.4 Welfare Decomposition

Consider period utility at a generic time t , the second-order Taylor expansion around the expected mean consumption in that period $E_0 c_t$ is:

$$U(c_t) \approx \log(E_0 c_t) + \frac{c_t - E_0 c_t}{E_0 c_t} - \frac{(c_t - E_0 c_t)^2}{2 (E_0 c_t)^2}. \tag{A.53}$$

Note that for different time periods t , the point of approximation of the Taylor expansion is potentially different since $E_0 c_t$ is not necessarily equal to $E_0 c_s$ for $t \neq s$. We don't want to approximate period utility but expected utility $E_0 [U(c_t)]$. In this case, the second term on the right in equation (A.53) drops and we obtain:

²⁴ See [Adjemian et al. \(2011\)](#) and [Guerrieri and Iacoviello \(2015\)](#). The solution using OccBin approximates the model's policy functions relatively well but, of course, cannot capture anticipation effects that are important for a full welfare evaluation of the model.

$$E_0 [U(c_t)] \approx \log(E_0 c_t) - \frac{E_0(c_t - E_0 c_t)^2}{2 (E_0 c_t)^2}. \quad (\text{A.54})$$

We do this for every time period t and sum with weights $1, \beta, \beta^2, \dots$ and obtain:

$$W = E_0 \sum_{t=0}^{\infty} \beta^t U(c_t) = \sum_{t=0}^{\infty} \beta^t E_0 [U(c_t)] \approx \sum_{t=0}^{\infty} \beta^t \log(E_0 c_t) - \sum_{t=0}^{\infty} \beta^t \frac{E_0(c_t - E_0 c_t)^2}{2 (E_0 c_t)^2}. \quad (\text{A.55})$$

This formula is exact without shocks, even with a transition from old to new steady state. Note that we covered all the utility terms for all states and periods. Consider the “matrix of simulation”:

	$t = 0$	$t = 1$	$t = 2$	\dots	$t = \infty$
State 1	$c_{1,0}$	$c_{1,1}$	$c_{1,2}$	\dots	$c_{1,\infty}$
State 2	$c_{2,0}$	$c_{2,1}$	$c_{2,2}$	\dots	$c_{2,\infty}$
\vdots	\vdots	\vdots	\vdots	\ddots	\vdots
State ∞	$c_{\infty,0}$	$c_{\infty,1}$	$c_{\infty,2}$	\dots	$c_{\infty,\infty}$

where time periods run across columns, and states of nature of the time-series of productivity shocks run across rows.²⁵ We do a Taylor approximation for each time period t , which are in the columns. For each column, the point of approximation is $E_0 c_t$. Afterwards, we sum the Taylor approximations with weights $1, \beta, \beta^2, \dots$ and so on. Visually, each Taylor approximation covers each column; we then sum the approximations with the correct weights of discounting, summing across columns, and in doing so, we cover the entire “matrix of simulations”.

To compute welfare differences between two economies A and B we obtain:

$$W_A - W_B \approx \sum_{t=0}^{\infty} \beta^t (\log(E_0 c_{A,t}) - \log(E_0 c_{B,t})) + \sum_{t=0}^{\infty} \beta^t \left(-\frac{E_0(c_{A,t} - E_0 c_{A,t})^2}{2 (E_0 c_{A,t})^2} + \frac{E_0(c_{B,t} - E_0 c_{B,t})^2}{2 (E_0 c_{B,t})^2} \right). \quad (\text{A.56})$$

Finally, we separate the first summation on the right into two summations from 0 to T and $T + 1$ to ∞ . This yields the expressions in the main text.

²⁵ Each row corresponds to a different sequence $\{A_t\}_{t=0}^{\infty}$.