Firm Dynamics and Growth with Soft Budget Constraints

Philippe Aghion Antonin Bergeaud Mathias Dewatripont Johannes Matt*

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Abstract

We develop a model of endogenous growth and firm dynamics with soft budget constraints, where firms differ in their innovation speed and slower firms need additional financing in order to eventually innovate. As creditors cannot anticipate refinancing needs in advance nor credibly commit to withholding future refinancing, a *Soft Budget Constraint Syndrome* emerges, causing excessive entry by slow firms and crowding out potentially more efficient innovators. The resulting trade-off between the positive effects of budget constraint softening on innovation by incumbents and slow-type entrants and its negative effects on entry by fast innovators, generates a hump-shaped relationship between refinancing costs and aggregate growth. Calibrating the model to French firm-level data, we show that the budget constraint softening associated with the decline in interest rates in the aftermath of the Global Financial Crisis accounts for 54% of the observed drop in the aggregate growth rates post-crisis. Although the softening in budget constraints has had a positive effect on incumbent innovation, this was more than offset by the resulting decrease in the entry rates of good firms (by 61% relative to the pre-crisis steady state).

JEL codes: O30, O43, E44, E50.

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*Aghion: Collège de France, INSEAD, and London School of Economics; Bergeaud: HEC Paris, CEP, POID, and CEPR; Dewatripont: Université Libre de Bruxelles; Matt: London School of Economics and CFM. The authors would like to thank, without implicating, Ufuk Akcigit, Maarten de Ridder and John Van Reenen for valuable comments. We thank Nicholas Tokay for excellent research assistance.

1 Introduction

The slowdown in economic growth in Europe following the Great Financial Crisis has triggered extensive theoretical and empirical research on the impact of credit constraints on productivity growth and innovation (e.g. Aghion et al., 2012; Duval et al., 2020). On the one hand, research and development (R&D) activities require investment and reducing the cost of credit fosters innovation by incumbent firms. On the other hand, lower financing costs may lead to significant misallocation by channeling R&D resources toward less efficient firms (Gopinath et al., 2017), thereby undermining business dynamism and growth (Aghion et al., 2019).

This paper develops a general equilibrium model of growth and firm dynamics. We analyze the aggregate growth implications of the *softness* of firms' budget constraints, that is, the ease with which firms can refinance their investments in situations in which creditors lack certainty about whether future refinancing will be necessary to complete projects. The analysis builds on the model in Dewatripont and Maskin (1995), who demonstrate that asymmetric information about future financing needs can naturally lead to the proliferation of unprofitable investments. Creditors who initially fund investment projects later regret their decision when they realize that the net present value is negative. They may still, however, find it profitable to continue financing these projects, as successful completion minimizes their overall losses.

We incorporate this mechanism into an endogenous growth model with firm dynamics, entry, and exit. Incumbent firms borrow from competitive creditors to finance their payroll labor investment in R&D. As in the canonical model of Klette and Kortum (2004), R&D projects allow firms to grow by taking over a competitor's product line. In our model, firms vary both in the success probability of their projects and also in the speed at which they can be completed. Fast borrowers invest in R&D and, when successful, innovate within one period. Slow borrowers also invest in R&D, but they do not innovate immediately. Instead, additional resources are required at an interim stage. If the slow borrower cannot secure refinancing, their innovation attempt fails. If they can, they innovate with the success probability associated with their project.

The *Soft Budget Constraint Syndrome* (henceforth SBC), in turn, arises from adverse selection. When extending credit at the beginning of the period, the creditor cannot observe the borrower's type, which he only learns at an interim date in the period. At this point, the creditor may grant additional funding based on the trade-off between refinancing costs and expected cash flow. As in Dewatripont and Maskin (1995), we assume that slow incumbents only ask for initial financing if they anticipate to be refinanced. For a given success probability, they are therefore more encouraged to ask for initial financing the softer the budget constraint. Such behavior in turn soaks up resources in the labor market and therefore drives up the wage faced by potential entrants. This baseline trade-off between the behavior of slow incumbents on the one hand, and entry on the other hand, is what drives the relationship between aggregate growth and the cost of refinancing.

In Section 2 we develop a simple growth model where both the speed of the firm – fast or slow – and the success probability of its project are i.i.d over time. Solving this model yields the following three predictions: First, lower refinancing costs lead to a rise in the equilibrium wage rate, which deters new innovating firms from entering the market. Second, the aggregate growth effect of lower refinancing costs, governed by the trade-off between the reduced entry of new innovating firms and the enhanced incumbent innovation, is hump-shaped. Third, lower refinancing costs, i.e. a softening of the budget constraint, leads to a fatter tail for the firm size distribution.

In Section 3 we extend our model to allow for persistence in firms' speed. More precisely, the dynamics of project types are governed by a Markovian transition matrix. Meanwhile, we assume that creditors observe firms' credit histories and also have perfect information on the transition matrix. Moving from i.i.d to persistent firm speed makes the aggregate growth rate also depend upon the firm size distribution. We derive an expression for the aggregate growth rate and the equilibrium firm size distribution, which allows us to decompose the growth and welfare effects of a lower cost of refinancing into the relative contribution of incumbent and entrant firms.

In Sections 4 and 5 we calibrate the model to French firm-level data and use it to estimate the growth impact of the softening of firms' budget constraints, which we trace back to policy changes in the aftermath of the Global Financial Crisis (GFC). We decompose this aggregate impact into its various components.[1] To be precise, we calibrate our model using pre-crisis data keeping all parameters identical apart from the parameter governing firms' refinancing costs, which we use to match the observed decline in interest expenses post crisis. The calibrated model implies a reduction in refinancing costs for slow firms of approximately 45%. Given the parameter values, this translates into a decline in the average annualized growth rate from 1.70% pre-crisis to 1.51% post-crisis – compared to 1.35% in the data. Thus, our exercise can replicate 54% of the observed decline in growth rates.

The overall growth effect of budget constraint softening can be decomposed as follows. First, it has a positive effect on incumbent innovation. However, this increase is more than offset by the induced reduction in the entry rates of good firms. More precisely, softening incumbent firms' budget constraints has two counter-acting effects on firm entry. On the one hand, cheaper refinancing implies that slow incumbents require fewer resources to innovate, which releases labor resources for firm entry (resource cost effect). On the other hand, the prospect of lower refinancing costs encourages more slow incumbent firms to innovate, which reduces the amount of labor available to entrants (selection effect). Our simulation suggests that the latter – negative - effect on entry strongly dominates. In total, entry rates fall by 61% relative to the pre-crisis steady state, which is equivalent to a reduction in the aggregate growth rate of 0.23 percentage points. The selection effect alone would have resulted in a reduction in growth of 0.29 percentage points, but the resource effect and the endogenous changes in the firm size distribution partly offset this decline.

Connection to the literature. This paper links two different strands of literature: on the one hand, the effect of imperfect contracts in corporate financing and how they can introduce soft budget constraints, and on the other hand, a body of research on firm dynamics with endogenous growth which incorporates financial market frictions.

[1] We voluntarily remain agnostic about the source of the magnitude of these unobserved refinancing costs, which may stem from the nature of innovation. For example, innovation may vary in its degree of uncertainty (OECD, 2021), or firms may prioritize faster projects (Caicedo and Pearce, 2024), or from monetary and macro-prudential policies (Blattner et al., 2023).

First, our model builds on corporate finance perspectives of the soft-budget constraint. The SBC concept was originally introduced by Kornai (1986) to explain why resource shortages could persist in centralized systems, but Dewatripont and Maskin (1995) later showed that the refinancing of unprofitable firms can also occur in a capitalist system under suitable informational assumptions. Kornai et al. (2003) and Maskin (1999) offer a comprehensive review of this literature and explain why the notion of soft budget constraint proved to be quite helpful to understand the transition from centrally planned economies to market economies (see e.g. Roland, 2000) but also the broader role of financial discipline in corporate governance. A key takeaway from this body of work is that restricting access to refinancing exerts restraint on entrepreneurs who might otherwise continue loss-making projects. If managers expect additional funds despite poor performance, their incentive to discontinue failing ventures diminishes. From the perspective of Bolton and Scharfstein (1990), for instance, debt contracts lose their disciplinary power if managers anticipate bailouts, since subpar outcomes no longer carry the credible threat of termination or other punitive actions. Aghion and Bolton (1992) further propose that allocating contingent control rights to investors can counteract this hazard by enabling them to replace failing management, a principle documented empirically by Hellmann and Puri (2002), who show how venture capitalists use such rights to intervene before losses become irretrievable. Likewise, in decentralized credit markets, Qian and Roland (1998) argue that having multiple competing lenders hardens a firm's budget constraint by making bailouts more difficult to coordinate, thereby reinforcing the disciplining effect of refinancing constraints. While these mechanisms can mitigate the SBC problem by reducing overinvestment in underperforming projects, empirical evidence confirms that soft budget constraints remain widespread in some contexts. Prolonged bank–firm relationships in Japan, for instance, have led to "zombie lending" — where banks continue to finance unprofitable firms (Caballero et al., 2008). Similarly, implicit government guarantees in advanced banking systems create moral hazard for institutions deemed "too big to fail," as highlighted by Farhi and Tirole (2012). Altogether, these findings underscore the importance of robust bankruptcy laws (Hart, 1995), effective resolution frameworks, and carefully designed financial instruments (Tirole, 2010), all of which help maintain a credible threat of failure and prevent underperforming projects from proliferating.

Second, our paper relates to the Schumpeterian growth literature that integrates firm dynamics and endogenous innovation (see, for example, Klette and Kortum, 2004; Aghion et al., 2015; Akcigit and Kerr, 2018). A common theme in these models is a trade-off between supporting incumbent firms and encouraging the entry of new firms: stronger support for incumbents can raise barriers to entry and dampen overall innovation (Acemoglu et al., 2018). Introducing credit constraints into these frameworks naturally emphasizes that trade-off. For instance, Aghion et al. (2019) embed R&D financing frictions into a Klette and Kortum (2004) model and find that lowering the cost of credit can have a net negative impact on productivity growth by allowing less efficient incumbents to remain active. More recently, Akcigit et al. (2022), Geelen et al. (2022), and Keuschnigg et al. (2022) also incorporate this feature into Schumpeterian growth models. For example, Akcigit et al. (2022) examine how bankruptcy processes, reallocation delays, and default decisions shape both short- and long-run growth outcomes. A parallel stream of research studies how firm dynamics, innovation, and various types of financial frictions interact more broadly, see for example Bustamante and Zucchi (2023) on discount-rate fluctuations, Corbae and D'Erasmo (2021) on bankruptcy institutions, Kalemli-Özcan and Saffie (2023) on the impact of aggregate shocks in an open economy, and Malamud and Zucchi (2019) on liquidity hoarding and long-term growth.[2] Finally, Ates and Saffie (2021) propose a model in which interest rate shocks tighten bank lending and give rise to entry selection, thus generating a trade-off between the quantity and the quality of innovators.

We bridge these two strands of the literature by embedding the SBC problem into a Schumpeterian growth model with entry, exit, and firm dynamics, and by analyzing how budget constraint softening affects aggregate growth.[3] Our analysis therefore offers a microfoundation to speak to the broader debate on how easing financial barriers may

[2]Meanwhile, Celik (2023) and Ottonello and Winberry (2024) show how borrowing constraints can lead to persistent misallocation in innovation models with firm dynamics, while Sui (2024) looks at how size-dependent financial frictions shape investment decisions.

[3]On the link between soft budget constraints and innovation, see Qian and Xu (1998), who note that creditors may be deterred from financing innovation if they fear an inability to terminate failing projects.

contribute to the productivity slowdown observed in the United States and other advanced economies, particularly in Europe (e.g., Banerjee and Hofmann, 2018; Gopinath et al., 2017; Gropp et al., 2022). Indeed, evidence suggests that persistently low real interest rates can inhibit aggregate productivity growth through various channels (Liu et al., 2022).

A series of empirical studies lends credence to these concerns: Caballero et al. (2008) and Peek and Rosengren (2005) document how Japanese banks extend credit to insolvent borrowers to hide losses, thereby reducing the profitability of healthy entrants. Blattner et al. (2023) show that undercapitalized banks in Portugal continue lending to non-performing firms, fueling capital misallocation and explaining around a quarter of the country's productivity slowdown in 2012. On a broader scale, undercapitalized financial systems distort macro outcomes by inducing cash hoarding, dampening investment, and occasionally generating deflationary pressures (Acharya et al., 2019, 2024; Adalet McGowan et al., 2018; Andrews and Petroulakis, 2019; Bonfim et al., 2023). The aggregate magnitude of this effect is however debated. For example, Schivardi et al. (2022) maintain that while additional lending to weaker firms indeed occurs, the effect on overall productivity is modest. Meanwhile, Becker and Ivashina (2022) demonstrate how high insolvency costs can push lenders to forgo restructuring in favor of perpetuating "zombie" relationships.

Our framework naturally contributes to this debate, we develop a fully-fledged quantitative model of innovation-based growth with firm dynamics and credits which allows us to decompose the overall effect of budget constraint softening on aggregate growth. In our model, the softening of the budget constraint will be parametrized and we develop counterfactual simulations using this parameter to assess the aggregate impact on output and growth. Such an approach is reminiscent of Aghion et al. (2023) who look at the growth and welfare impact of the French labor regulation using a similar approach.

The remainder of this paper is organized as follows: Section 2 develops and solves the baseline model. Section 3 extends the baseline model to allow for persistence in types. Section 4 calibrates the extended model. Section 5 uses the calibrated model to assess and then decompose the aggregate growth effect of the budget constraint softening which occurred in response to the global financial crisis. And Section 6 concludes.

2 The baseline model

Our model is an extension of the Klette and Kortum (2004) model of endogenous growth with firm dynamics with credit and soft budget constraints.

2.1 Overview

There are three groups of agents: Households consume and save, firms produce and innovate, and a competitive creditor intermediates funds between them. As in Klette and Kortum (2004), firms, in turn, are split between a final good producer, intermediate producers, and prospective entrants. Only intermediate firms and entrants can innovate. Incumbents borrow from the creditor to finance their innovation attempt. Entrant firms receive equity funding from households.[4]

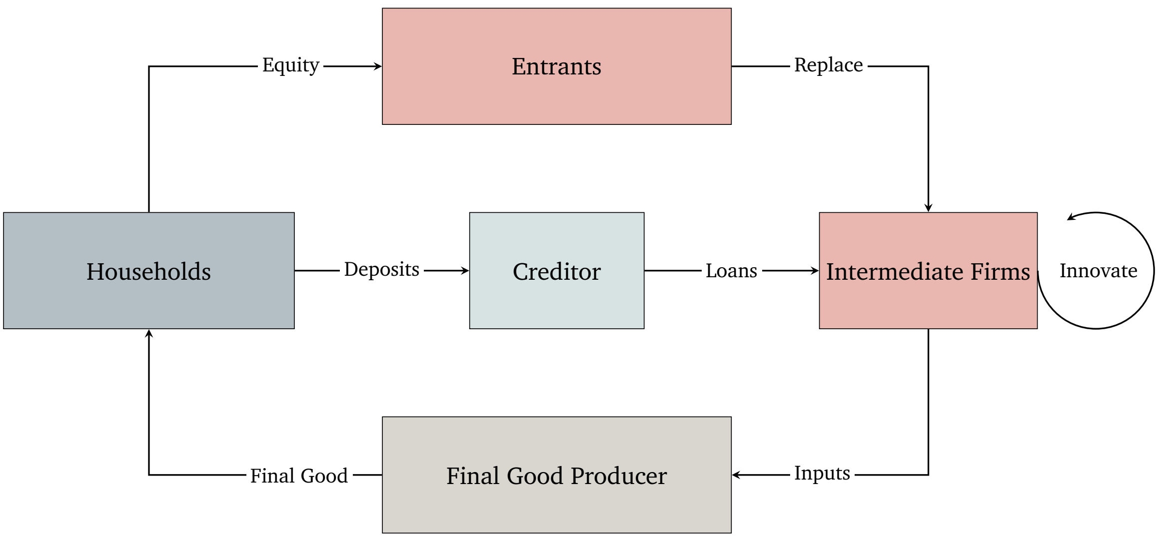

Figure 1: The economy.

The creditor operates under perfect competition, collects funds from households and lends them out to incumbent firms who can be either fast or slow. Slow firms' innovation attempts do not succeed without additional financing, which gives rise to the *soft budget*

[4]We do not assume that entrants receive funding from the stock market, because its disclosure requirements impose such a high fixed cost that an Initial Public Offering makes sense mostly for larger firms. Instead, we are talking about inside equity, with the view that in early stages innovative firms often receive money either from family and friends, or from business angels and other specialized non-bank investors, before transitioning to bank credit (for most firms) and then to stock markets (for a minority of firms). In the model, our entrants are therefore less impacted by bank-related soft budget constraints than more established firms, as highlighted by the examples of Japan in the 1990s and, subsequently, of those countries affected by the GFC.

constraint syndrome. The creditor can provide additional capital to slow firms if the continuation value is sufficiently high. Although entrants are not directly affected by the SBC problem, it affects their resource cost of innovation. Figure 1 contains a stylized overview of the environment. We now provide a detailed description of the model.

2.2 Environment

2.2.1 Households

Time is discrete and a unit measure of households derives logarithmic utility from consumption and discounts the future at β. Households earn interest income r_t and inelastically supply two types of labor: Research labor L^R at wage w_t and production labor L^P at wage w_t^P. Their Euler equation is $1 + g_{t+1} = \beta(1 + r_{t+1})$.

2.2.2 Production

A competitive final good producer aggregates a unit measure of intermediate inputs $y_{i,t}$ according to the production function:

$$Y_t = \exp\left(\int_0^1 \ln y_{i,t} di\right). \tag{1}$$

Inputs are produced by the firm who has innovated last on a given product line. Each producer operates a technology that turns $1/A_{i,t}$ units of production labor into one unit of its intermediate good. For a production wage w_t^P, marginal costs are $w_t^P/A_{i,t}$.

Given the production function in (1) and a price $p_{i,t}$, intermediate producers face isoelastic demand $y_{i,t} = Y_t/p_{i,t}$. Competition is Bertrand. If the gap between the leader's productivity and that of the second most productive firm (the previous innovator) is λ, the leader will set her price at marginal costs. Equilibrium prices, quantities and profits are respectively given by:

$$p_{i,t} = \frac{\lambda w_t^P}{A_{i,t}}, \qquad y_{i,t} = \left(\frac{A_{i,t}}{\lambda w_t^P}\right) Y_t, \qquad \text{and} \qquad \pi_{i,t} = \left(\frac{\lambda - 1}{\lambda}\right) Y_t \equiv \pi Y_t. \tag{2}$$

A firm itself is a collection of n product lines. Exploiting symmetry across lines, firm-level profits are $\pi_t(n) = \pi n Y_t$. We index a firm by its number of product lines n. See Appendix A.1 for the details.

2.2.3 Innovation

At the beginning of any period, an incumbent firm draws an *innovation project*. If successful, the project improves a randomly selected product line by an exogenous step $\lambda > 1$ and thus allows the firm to advance from n to $n+1$ products. The innovation project can be one of two types. With probability $1-\alpha$ the project is fast and with probability α the project is slow. We refer to firms with fast projects as *fast firms* and similarly to firms with slow projects as *slow firms*, and index them by $j \in \{f, s\}$. Fast and slow projects differ in two dimensions:

1. A fast project requires an investment of $\zeta z n$ units of R&D labor at wage w_t and results in an innovation with probability $\kappa z n$.

2. A slow project differs from its fast counterpart in two dimensions: First, after an initial investment of $\zeta z n$ units of labor at wage w_t, slow projects require refinancing of $\delta z n w_t$ at an interim stage in the period. The project only generates an innovation with probability $q z n$ if it is refinanced. Otherwise it fails with certainty. The success probability q is drawn from the cumulative distribution function $\mathcal{Q}(q)$ with support $[0, 1]$. One might think of the refinancing need as an additional investment (Dewatripont and Maskin, 1995) a negative liquidity shock (Aghion et al., 2010; Matt, 2025), or heterogeneous cost of information acquisition (Mazet-Sonilhac, 2024). Second, slow projects also suffer from a contracting friction. Whenever a slow project is successful, a fraction θ of its value directly accrues to the innovator and is inalienable in the sense of Hart and Moore (1994) such that only a fraction $1-\theta$ can be transferred to outside the boundaries of the firm.[5]

[5]An emerging literature documents the relationship between financial contracting and innovation. Patents are often collateralized in debt contracts, making debt financing a widely used source of financing for innovation (e.g. Chava et al., 2017; Hochberg et al., 2018). According to Mann (2018), 40% of innovating firms in the United States use some form of debt financing with patents as the primary collateral. These

In the baseline model, we take the realization of firm speed to be i.i.d over time, an assumption we shall relax in the next section where we add persistence to firm types. As for the success probability q of a slow firm, we take it to be i.i.d both, in the baseline and in the extended model. Finally, we denote the value of a firm of speed-type j with n product lines is $V_t(n, j)$ and the expected change in its value due to a successful innovation is $\mathbb{E}_{j,q}[\Delta V_{t+1}(n)]$.

2.2.4 Credit

Innovation by an incumbent firm is financed by a creditor operating on a competitive credit market, so that all what is required from a creditor is that her participation – or individual rationality – constraint be satisfied. Credit market imperfection arises from the fact that a creditor cannot observe the firm's speed-type but only knows the overall distribution between fast and slow firms. However, she can infer the firm's size n from observing the firm's desired loan amount. Finally, in this baseline model where speed-types are reassigned in i.i.d. fashion each period, a firm's credit history is uninformative about its current speed-type.

The creditor has two decisions to make. First, she offers a menu of pooling contracts consisting of a loan $\zeta z n w_t$ at a lending rate $\tilde{R}_t(n)$. Second, at the interim stage when slow firms reveal their type, the creditor decides whom to refinance. We characterize the creditor's problem by working backwards starting from the refinancing decision at the interim stage before determining the lending rate at the beginning of the period.

At the refinancing stage, a slow firm faces a liability $\tilde{R}_t(n)\zeta z n w_t$ and requires $\delta z n w_t$ units of labor to avert project failure. Continuing the project promises the creditor a cash flow of $zn(1-\theta)\mathbb{E}_{j,q}[\Delta V_{t+1}(n)]$ with probability q next period, and the creditor decides to refinance if:

$$\frac{qzn(1-\theta)\mathbb{E}_{j,q}[\Delta V_{t+1}(n)]}{1+r_{t+1}} \geq \delta z n w_t. \tag{3}$$

firms account for more than a quarter of aggregate R&D. Finally, Kermani and Ma (2023) and Ma et al. (2022) show that the liquidation value of patent collateral is substantial and that creditors frequently seize and resell patents during Chapter 11 liquidations.

Note that at this point in time, the initial investment is sunk. Keeping the project afloat therefore only depends on the continuation cost and the expected future cash flow, but not the initial investment. While the total cost of financing the project is $(\zeta + \delta)znw_t$, the dynamic commitment problem arising from (3) implies that projects might be refinanced even if their ex-ante net present value is negative. This gives rise to the soft budget constraint syndrome.

We think of the parameter δ as capturing the creditor's cost of refinancing and therefore the hardness of the constraint. The refinancing condition (3) defines a threshold success probability $q_t^* \equiv \delta(1 + r_{t+1}) / \left((1-\theta)\mathbb{E}_{j,q}[\Delta V_{t+1}(n)]\right)$ such that slow projects with $q < q_t^*$ are terminated while projects with $q \geq q_t^*$ can continue. For later use we define

$$\underline{q}(q_t^*) \equiv \int_{q_t^*}^{1} q d\mathcal{Q}(q). \tag{4}$$

Anticipating the refinancing decision at the interim stage, the creditor sets a menu of loan rates $\tilde{R}_t(n)$ at the beginning of the period. Because of the i.i.d. structure there are $1 - \alpha$ fast firms and α slow firms for every size bin n.

Slow firms with success probability below q_t^* anticipate that they will not receive refinancing at the interim stage, and therefore do not participate in the credit market such that the share of active slow firms is only $\alpha[1 - \mathcal{Q}(q_t^*)]$. Regardless of type, a firm whose innovation attempt fails defaults. The creditor's break-even condition is:

$$\underbrace{(1-\alpha)\kappa zn\frac{\tilde{R}_{t+1}\zeta znw_t}{1+r_{t+1}}}_{\text{Expected cash-flow on fast firms}} + \underbrace{\alpha\underline{q}(q_t^*)zn\frac{\mathbb{E}_{j,q}[(1-\theta)\Delta V_{t+1}(n)]}{1+r_{t+1}}}_{\text{Conditional cash flow on slow firms}} = \underbrace{\Big((1-\alpha)\zeta + \alpha\left(\zeta+\delta\right)\left[1-\mathcal{Q}\left(q_t^*\right)\right]\Big)znw_t}_{\text{Creditor's cost}}. \tag{5}$$

The first term is the expected cash flow if the firm turns out to be fast. The second term is the cash flow if the firm is slow, receives refinancing, and innovates successfully. The third term is the creditor's cost of funds.

Note that since speed-types are re-assigned without memory every period, a firm's credit history, that is, its past speed-type, is uninformative about its current speed-type. In fact, a share α of firms in each size bin will always be slow and a share $1 - \alpha$ will always be fast. As such, the interest rate $\tilde{R}_t(n)$ does not depend on the firm's credit history either.

In the full quantitative model in Section 3 where types are persistent in a Markovian way, having observed last period's type becomes informative about the firm's current type.[6]

2.2.5 Firm's activity constraint

Given the creditor's interest rate policy (5), fast and slow firms decide whether to participate in the credit market. As described above, slow firms with a draw $q \leq q_t^*$ anticipate that they will not receive refinancing and do not try to innovate as a consequence. Fast firms are active whenever the following constraint is satisfied:

$$\kappa z n \left(\frac{\mathbb{E}_{j,q}[\Delta V_{t+1}(n)] - \tilde{R}_t(n)\zeta z n w_t}{1 + r_{t+1}} \right) \geq 0, \tag{6}$$

which defines a cutoff interest rate above which the market breaks down.

2.2.6 Entry

Prospective entrants invest ψz_t^e units of research labor at wage w_t to innovate on a random product line with probability z_t^e. A successful entrant becomes an incumbent next period. With probability α the new incumbent is slow and with $1 - \alpha$ he is fast. Entrants are financed directly by households and therefore are not directly impacted by the soft budget problem before entry. The free-entry condition is:

$$z_t^e \left(\frac{\alpha \mathbb{E}_q[V_{t+1}(1,s)] + (1-\alpha)\mathbb{E}_q[V_{t+1}(1,f)]}{1 + r_{t+1}} \right) = \psi z_t^e w_t. \tag{7}$$

2.2.7 Market clearing

There are six markets in the economy: Intermediate goods, credit, firm equity, production and research labor, and the final good. Intermediate good and credit market clearing respectively follow from (2) and (5). Equity and production labor market clearing conditions

[6] In the quantitative model extension, we assume that credit markets are segmented across credit history to better fit the data. In that case, one specialist creditor caters to firms with credit history j each. Since types are uninformative in the baseline present model, whether the market is segmented or not does not matter for creditor behavior.

are in Appendix A.5. The market for research labor clears if:

$$L^R = \psi z_t^e + (1-\alpha)\zeta z + \alpha\left(\zeta+\delta\right)\left[1-\mathcal{Q}(q_t^*)\right]z, \tag{8}$$

and the output market clears if $Y_t = C_t$.

2.2.8 Growth

Creative destruction is generated both by entrants and incumbent firms. A unit measure of entrants innovates with probability z_t^e, a measure $(1-\alpha)$ of fast incumbents innovates with probability κzn, and a measure $\alpha[1-\mathcal{Q}(q_t^*)]$ of slow incumbents innovates with qzn given that $q \geq q_t^*$. The creative destruction rate is:

$$x_t = z_t^e + (1-\alpha)\kappa z + \alpha \underline{q}(q_t^*)z. \tag{9}$$

As shown in Appendix (A.3), the growth rate is proportional to the rate of creative destruction:

$$g_{t+1} \approx \ln(\lambda)x_t. \tag{10}$$

2.2.9 Firm size distribution

A share α of firms draws a slow project every period, while $1-\alpha$ are fast. We denote by $\mu_t(n)$ the fraction of firms that have n product lines. Because of the i.i.d. assignment of speed-types, $\mu_t(n,s) = \alpha\mu_t(n)$ is the density of slow firms of size n.

We normalize $\sum_{n=1}^{\infty}\mu_t(n)n = 1$. The stationary distribution equates inflows into state n to outflows into states $n+1$ and $n-1$:

$$\underbrace{\left[\alpha\underline{q}(q_t^*)z + (1-\alpha)\kappa z + x_t\right]\mu_t(n)n}_{\text{Flow-out of state n}} = \underbrace{x_t(n+1)\mu_t(n+1)}_{\text{Flow-in state n from n+1}} + \underbrace{\left[\alpha\underline{q}(q_t^*) + (1-\alpha)\kappa\right]z(n-1)\mu_t(n-1)}_{\text{Flow-in state n from n-1}}, \tag{11}$$

for $n \geq 2$. The term on the left-hand side captures, in order, the outflow from n product lines to $n+1$ coming from α slow firms who innovate successfully with probability $\underline{q}(q_t^*)zn$, the outflow from $1-\alpha$ fast firms who innovate successfully with probability κzn, and the outflow from both fast and slow firms who are hit by creative destruction on one of their lines with probability $x_t n$. The first term on the right-hand side captures the inflow of firms from $n+1$ due to creative destruction. The final term are the inflows from successful innovators with $n-1$ product lines, again split between fast and slow firms.

At $n = 1$ inflows and outflows need to be adjusted for entrant innovation: $\mu_t(1)x_t = z_t^e$ and $[\alpha \underline{q}(q_t^*)z + (1-\alpha)\kappa z + x_t]\mu_t(1) = 2x_t\mu_t(2) + z_t^e$.

2.3 Equilibrium

To simplify the exposition of the results and key insights of the analytical model, we assume that $Q(q)$ is a uniform distribution. In Appendix B.4 we show that the results carry over to the case when q is drawn from a more general beta-type distribution, which we also use in the quantitative model in Section 3.

Assumption 1. *Let $Q(q) = q$ with support $[0,1]$.*

Throughout the remainder of the paper, we focus on a balanced growth path on which all key variables grow at a common and constant growth rate and the firm size distribution is stationary. To indicate a balanced growth path, we drop time subscripts. We define a balanced growth path as follows:

Definition 1 (Balanced growth path)**.** *On a balanced growth path, all aggregate variables, output, consumption, profits, as well as production and research wages, grow at a common and constant growth rate g, the growth rate of the aggregate stock of technology as defined in (10). Moreover:*

1. *Intermediate firms set prices and earn profits according to (2).*
2. *Creditors finance fast and slow firms by setting a pooling rate $\tilde{R}(n)$ consistent with the zero-profit condition (5). They refinance slow firms according to (3). Fast firms' participation constraint (6) is satisfied.*
3. *Intermediate goods, credit, equity, production and research labor, and the final good markets clear.*

4. *Firm entry satisfies the free-entry condition* (7). *The stationary firm size distribution is given by* (11).

Given definition 1, we now proceed to characterize the close-form, analytical solution. The strategy is to first solve for credit market equilibrium and then derive entry, creative destruction, and the firm size distribution given the creditor's refinancing policy.

2.3.1 Credit

As in Klette and Kortum (2004), we guess and check that the expected value of a firm is linear in the number of product lines, vnY. The free entry condition (7) and the Euler equation pin down $\beta v = \psi\omega$, where $\omega \equiv w/Y$ is the research wage over output.

Creditors refinance those slow firms with sufficiently high success probability. With a linear value function the refinancing condition (3) collapses to a simple cut-off rule $q \geq q^*$ with the cutoff given by:

$$q^* \equiv \frac{\delta}{(1-\theta)\psi}. \tag{12}$$

A measure $1 - q^*$ of slow firms receives refinancing every period, while a fraction q^* have their refinancing request refused, and therefore do not participate in the credit market in the first place. As can be seen from (12), a harder budget constraint, in the form of higher refinancing costs δ, shifts the cutoff q^* upwards, that is, it reduces the number of firms who are refinanced. Similarly, stronger contracting frictions θ result in a smaller share of the new product line that can be seized by the creditor and hence less refinancing. Finally, the same logic applies for a higher value of an additional product line ψ.

Given the refinancing threshold q^*, the creditor sets lending rates according to (5), which gives rise to a *size-dependent* interest rate schedule:

$$R(n) = \frac{1}{\kappa z n}\left[1 + \frac{\alpha k(q^*)}{(1-\alpha)\zeta}\right], \tag{13}$$

where $k(q^*) \equiv (\zeta + \delta)\left[1 - \mathcal{Q}(q^*)\right] - \underline{q}(q^*)(1-\theta)\psi$ captures the creditor's loss from lending to slow firms. That loss is increasing in the refinancing cost δ and decreasing in the value of a new product line ψ. As a higher share of slow types leads to more refinancing, and hence a larger loss for the creditor, the interest rate is increasing in the share of slow types α, who must be cross-subsidized by fast firms. Fast firms participate in the credit market as long as lending rates are not too high. See Appendix A.2 for details on refinancing.

Finally, note that the interest rate (13) is decreasing in the firm's number of product lines n, as the success probability zn is increasing in the number of product. Large firms are less likely to default and therefore pay lower interest rates.

2.3.2 Innovation and entry

Both incumbent firms and entrants use research labor to generate innovations. Labor market clearing (8) pins down the entry rate as a function of model parameters:

$$z^e = \frac{L^R - (\zeta + \alpha\delta)z}{\psi} + \frac{\alpha z}{\psi}(\zeta + \delta)q^*, \tag{14}$$

the entry rate is increasing in the amount of residual labor available to entrants. The refinancing cost affects entry directly through the residual amount of labor available to entrants and through the refinancing cutoff q^*.

The economy's growth rate is proportional to the rate of creative destruction:

$$x = z^e + (1-\alpha)\kappa z + \alpha \underline{q}(q^*)z, \tag{15}$$

where the first term is incumbent innovation, the second term is innovation from the $1-\alpha$ fast firms, and the final term captures innovation from the subset of slow firms who receive refinancing. The approximate growth rate in the economy is $g = x \ln \lambda$.

2.3.3 Firm size distribution

Finally, the equilibrium size distribution of firms is simply equal to: $\mu(n,s) = \alpha\mu(n)$ and $\mu(n,f) = (1-\alpha)\mu(n)$ with its usual geometric form:

$$\mu(n) = \frac{\tilde{z}}{n\,(1+\tilde{z})^n}, \qquad \text{where} \qquad \tilde{z} \equiv \frac{z^e}{(1-\alpha)\kappa z + \alpha \underline{q}(q^*)z}, \tag{16}$$

is the ratio of entrant to incumbent innovation. The distribution is right-skewed. In the remainder of this section we will analyze the effect of changes in the hardness of the budget constraint δ on entry, creative destruction, and the firm size distribution.

2.4 Comparative statics

The key parameter in the model is δ, the cost of refinancing a slow firm's project at the interim stage within a given period. Following Dewatripont and Maskin (1995) and Kornai et al. (2003), we interpret the parameter δ as an implicit measure of the severity of the SBC syndrome. After all, the relative magnitude of δ compared to value of an innovation determines the creditor's financial limit on supporting the firm's innovation effort.

A low cost of refinancing allows slow firms with a relatively low success probability q to obtain the necessary funds, which – anticipating the creditor's refinancing decisions – encourages these firms to enter the credit market in the first place and borrow to finance payroll labor outlays for research. In general equilibrium, δ therefore affects the wage on research labor and hence the cost of entry.

2.4.1 Entry

We start by discussing the effect on entry. Differentiating the free-entry condition (14) with respect to δ reveals two opposing effects of changes in the refinancing cost for slow firms on the entry rate:

$$\frac{dz^e(\delta)}{d\delta} = \frac{\alpha z}{\psi}(\zeta + \delta) \cdot \frac{dq^*}{d\delta} - \frac{\alpha z}{\psi}(1 - q^*). \tag{17}$$

The first term captures the fact that an increase in the cost of refinancing $d\delta$ raises the cutoff q^* and therefore frees up $\alpha z \zeta \cdot dq^*$ units of labor previously used for initial financing for incumbent slow firms, plus an additional $\alpha z \delta \cdot dq^*$ units of labor used for refinancing. To start an innovation attempt, each prospective entrant requires ψ units of labor. The selection effect of a change in the cutoff q^* therefore results in a net addition of $\alpha z(\zeta + \delta)/\psi \cdot d\delta$ entrant firms to the economy.

The second term in (17) captures changes in the labor cost due to the refinancing of those $\alpha(1-q^*)$ slow firms that still receive refinancing after the change in δ. Each of these incumbent slow firms uses up an additional $z \cdot d\delta$ units of labor in the refinancing process, reducing the amount of labor available for entry by $\alpha z(1-q^*)/\psi \cdot d\delta$.

Given these two opposing effects on entry, the net effect can be ambiguous. Using the definition for the refinancing cutoff q^* in (12), expression (17) above can be re-written in terms of model parameters:

$$\frac{dz^e}{d\delta} = \frac{\alpha z}{\psi}\left[\frac{\zeta + 2\delta}{(1-\theta)\psi} - 1\right]. \tag{18}$$

In the trivial case when the entry rate is decreasing in the cost of refinancing δ, the optimal policy is always to refinance more incumbent slow firms. In that case, there is no trade-off between refinancing and growth.[7]

Anticipating the results in our quantitative model in Section 3, we restrict attention to the case when entry is increasing in the cost of refinancing incumbent slow firms. The following Proposition 1 formalizes our assumption and the effect of a change in the refinancing cost δ on firm entry.

Proposition 1. *Assume that parameter values satisfy $\zeta + 2\delta > (1-\theta)\psi$. Then, an increase in the cost of refinancing δ reduces labor demand and raises entry z^e.*

Proof. Follows from (18). □

2.4.2 Growth

When entry is decreasing in the cost of refinancing, the effect of budget constraint hardening on growth is unambiguously negative. When entry is increasing in δ, however, there

[7] We show in Appendix B.4 that in the more general case when $\mathcal{Q}(q^*)$ is an arbitrary, regular distribution, our intuition for the selection and resource cost effects carries over. In particular, entry will increase in response to an increase in δ when the labor released by not refinancing the mass of slow incumbents at the threshold exceeds the additional labor cost for those that still receive refinancing.

As we are using a uniform distribution for q in this section, both effects are equally weighted. In the general case with an arbitrary $\mathcal{Q}(q)$, the selection effect is weighted by the hazard rate $[1-\mathcal{Q}(q^*)]/\mathcal{Q}'(q^*)$ at the threshold q^*. Equation (18) generalizes to the condition that, on net, the change in labor demand as a result of an increase in δ is negative when adjusted for the shape of the distribution:

$$\zeta + (1-\theta)\psi\phi(q^*) > 0,$$

where $\phi(q^*) \equiv q^* - [1-\mathcal{Q}(q^*)]/\mathcal{Q}'(q^*)$.

are two competing effects. To see this, we differentiate the creative destruction rate in (15) with respect to δ:

$$\frac{dx}{d\delta} = \frac{dz^e}{d\delta} - \alpha z q^* \frac{dq^*}{d\delta}. \tag{19}$$

On the one hand, a harder budget constraint encourages more innovating entry, but on the other hand it reduces innovation by slow incumbents. The growth-maximizing rate of refinancing trades off the relative contribution of entrants and slow incumbent firms.

Balancing these two effects implies that there exists a hump-shaped relationship between the cost of refinancing δ and the growth rate when the entry effect is positive. Proposition 2 formalizes this trade-off.

Proposition 2. *Assume parameter values satisfy the assumption in Proposition 1. Moreover, assume that the incentive friction is sufficiently strong, that is, $\theta > 1/2$. Then, there exists a hump-shaped relationship between the refinancing cost δ and economic growth. That is, for $\delta < \delta^*$ we have $dx/d\delta > 0$, and for $\delta \geq \delta^*$ we have $dx/d\delta < 0$. The maximum is given by:*

$$\delta^* = (1-\theta)\left[\frac{\zeta - (1-\theta)\psi}{2\theta - 1}\right]. \tag{20}$$

Proof. Follows from differentiating (15) with respect to δ. □

Refinancing a larger number of slow incumbents has a positive and a negative effect on growth. On the one hand, it allows more incumbent innovation. On the other hand, it crowds out entry through a higher wage rate. As with the effects on entry, in the more general case where the distribution of q is non-uniform, the weighting of the two counteracting effects in (19) depends on the mass close to the cutoff q^*. We discuss that case in details in Appendix B.4.

Our baseline model thus suggests a trade-off between refinancing incumbent slow firms and encouraging prospective entrants. The refinancing trade-off arises because of scarce research labor, similar to the models in Acemoglu et al. (2018) and Aghion et al. (2019) in which less efficient firms absorb labor that could have otherwise gone to entrants. As such, our model relates to a broader set of papers on reallocation and growth along with

Hopenhayn (1992) and Melitz (2003) that emphasize the connection between entry and exit through factor prices.[8]

2.4.3 Firm size distribution

The trade-off between entry and incumbent innovation also comes up when looking at the firm size distribution. Differentiating the ratio of entrant to incumbent innovation $\tilde{z}$ with respect to the cost of refinancing, we find that the share of entrant innovation is increasing in the cost of refinancing, namely:

$$\frac{d\tilde{z}}{d\delta} = \frac{1}{\tilde{z}} \cdot \frac{dz^e}{d\delta} + \frac{\alpha q^* z\tilde{z}}{(1-\alpha)\kappa z + \alpha \underline{q}(q^*)z} \cdot \frac{dq^*}{d\delta} > 0. \tag{21}$$

Recall that although firm speeds are re-assigned in i.i.d. fashion at the beginning of a new period – which is why the equilibrium share of fast and slow firms for each size bin remain constant at $1-\alpha$ and α, respectively – the cost of refinancing affects the shape of the firm size distribution. In particular, a lower cost of refinancing has two distinct effects on the the size distribution. First, it allows more slow incumbent firms to innovate, increasing the average growth rate of incumbent firms and hence their size. Second, it drives up the wage rate in the labor market and therefore reduces firm entry. Both effects imply that a reduction in δ will lead to fewer small firms and more large firms. As can be seen from (16), the firm size distribution will have a fatter tail whenever the refinancing costs fall.

3 The extended model with persistent types

In the baseline model, we have assumed that firms' speeds were redrawn every period in i.i.d. fashion. Consequently, observing last period's refinancing choice was uninformative about the firm's current speed-type. We now extend the baseline model so as to allow for persistence in firms' speeds.

[8]Even though we focus on skilled (research) labor in this paper, the reallocation mechanism is quite general and arises whenever input factors are scarce. The bottle neck can be physical capital (Cui, 2022), skilled labor (Acemoglu et al., 2018), bank deposits (Keuschnigg et al., 2022), managerial talent, or product market space.

We do so for two reasons, one empirical and one theoretical. Empirically, various episodes of loose monetary policy have deemed to have favored the rise of so-called *zombie firms*, for example in Japan (e.g. Caballero et al., 2008; Peek and Rosengren, 2005), or in other developed countries post financial crisis (e.g. Acharya et al., 2019; Adalet McGowan et al., 2018), and post Covid-19 (e.g. Acharya et al., 2022). Zombie firms are characterized as having low innovation potential while soaking up large amounts of resources to stay afloat and various empirical studies have shown that the refinancing needs of these firms are quite persistent over time (e.g. Banerjee and Hofmann, 2018).

On the theory side, adding persistence in speed-types introduces an additional effect of refinancing on growth through the firm size distribution. Not only will the speed-type persistence matter for the firm's innovation potential in any given period, it will also matter for future innovation activity via the firm's credit history. Creditors can observe a firm's speed-type last period, which is revealed when firms ask for additional funds at the interim stage, and are able to condition the interest rate accordingly. When speed-types are persistent, firms that have required refinancing last period will face higher interest rates next period, even if they are fast. These tighter financial conditions quell innovation for firms with bad credit histories and therefore alter their expected growth rate. As such, changes in the cost of refinancing δ will not only affect firm entry, but also the equilibrium composition of firms' speed-types and therefore lead to an endogenous feedback between the quality of the pool of firms and the refinancing decision.

After introducing persistent speed-types in subsection 3.1, we will proceed to quantify the effect of budget constraint softening on the growth slowdown in France, and decompose the effect into the various constituent parts.

3.1 Persistent types

We assume that the evolution of firms' speed-types over time is governed by the Markov transition matrix:

$$\mathbf{\Phi} = \begin{bmatrix} \phi_s(s) & \phi_s(f) \\ \phi_f(s) & \phi_f(f) \end{bmatrix}, \tag{22}$$

where $\phi_j(j')$ is the probability that a firm moves from speed-type j to speed-type j' at the start of the period, and $\phi_j(f) + \phi_j(s) = 1, \forall j \in \{s, f\}$. In this section we occasionally refer to a firm's speed type in the previous period as the firm's *credit history*. In the baseline model, we were focusing on the special case when the credit history was not informative about a firm's future type, $\phi_s(s) = \phi_f(s) = \alpha$.

3.2 Creditors

The creditor observes the firm's credit history. She also knows the transition probabilities across types as defined in (22). The creditor offers a menu of interest rates $\tilde{R}(n, j)$ given firm size n and last period's type j.

As already discussed in the baseline model section, we assume that there are two distinct loan markets, one for firms with a history of being fast types $j = f$, and one for firms with a history of being slow types $j = s$. With i.i.d. speed-types, this distinction did not matter for creditor behavior as a firm's past speed-type was uninformative about its future speed-type. The break-even conditions are:

$$\begin{aligned}
&\sum_{n=1}^{\infty} \frac{\kappa z n}{1 + r_{t+1}} \phi_j(f) \mu_{t-}(n, j) \tilde{R}_t(n, j) \zeta z n w_t + \sum_{n=1}^{\infty} \phi_j(s) \mu_{t-}(n, j) \frac{q(q_t^*) z n}{1 + r_{t+1}} (1 - \theta) \mathbb{E}_{j',q}[\Delta V_{t+1}(n)] \\
&\quad = \sum_{n=1}^{\infty} \Big(\phi_j(f) \mu_{t-}(n, j) \zeta + \phi_j(s) \mu_{t-}(n, j) (\zeta + \delta) [1 - Q_t^*] \Big) z n w_t,
\end{aligned} \tag{23}$$

for $j \in \{f, s\}$. Here the term $\mu_{t-}(n, j)$ denotes the density at the beginning of period t just before speed-types are re-assigned. As before, the refinancing threshold is determined at an interim stage when the speed-type realization becomes known. See Section 3.4 for more details on the firm size distribution. As for (23), the refinancing cutoff is defined by:

$$q_t^* = \min \left\{ q \in [0, 1] \;\middle|\; \frac{q z n (1 - \theta) \mathbb{E}_{j',q}[\Delta V_{t+1}(n)]}{1 + r_{t+1}} = \delta z n w_t \right\}, \tag{24}$$

where $\Delta V_t(n)$ is the change in value when moving from n to $n + 1$ product lines, and $\mathbb{E}_{j',q}$ is the expected value over next period's speed-type $j' \in \{f, s\}$ given today's speed-type j and the success probability of innovation q.

Note that the refinancing threshold q_t^* does not depend on the firm's history j. The reason is that types are Markovian and transition probabilities only depend on one previous period. At the point when refinancing is granted, the firm is revealed to be slow and the expected value $\mathbb{E}_{j',q}[\Delta V_{t+1}(n)]$ is of course the same for all firms given that they are slow this period.

3.3 Entry and growth

Entry is the same as in the baseline model. A new firm is fast with probability $1-\alpha$ and slow with probability α. Transitions for incumbent firms are governed by the Markov switching matrix (22). The free entry condition is

$$z_t^e \left(\frac{\alpha \mathbb{E}_q[V_{t+1}(1,s)] + (1-\alpha)\mathbb{E}_q[V_{t+1}(1,f)]}{1+r_{t+1}} \right) = \psi z_t^e w_t, \tag{25}$$

which equates the expected value of successful entry, split between entering as a slow and as a fast firm, to the cost of entry ψz_t^e at wage rate w_t on research labor.

Given a distribution $\mu_t(n,j)$ for $j \in \{f,s\}$ the labor market clearing condition pins down the entry rate:

$$L^R = \psi z_t^e + \sum_{n=1}^{\infty} \Big(\zeta z \mu_t(n,f) n + z[1-\mathcal{Q}(q_t^*)](\zeta+\delta)\mu_t(n,s)n \Big), \tag{26}$$

where the first term is labor demand from entrants, and the term in parentheses is labor demand from fast incumbents as well as slow incumbents who receive refinancing. Slow firms with $q < q^*$ anticipate that they will be refused refinancing at the interim stage. Hence these firms do not try to innovate and do not demand any research labor in this period. From that, the creative destruction rate is simply:

$$x_t = z_t^e + \sum_{n=1}^{\infty} \Big(\kappa z \mu_t(n,f) n + \underline{q}(q_t^*) z \mu_t(n,s) n \Big). \tag{27}$$

Equation (27) reveals that there is now a third, compositional effect on growth, which works through the firm size distribution. More refinancing increases the share of slow firms in the long run as they grow larger. Fewer slow firms are destroyed by creative destruction. If fast firms innovate with a higher average probability, the compositional effect decreases growth rates from one steady state to the next.

3.4 Firm size distribution

As before, we can derive the steady-state firm size distribution by equating inflows to outflows.[9] Starting with slow firms, for the state (n, s) with $n > 1$ we have:

$$\begin{aligned}
[\phi_s(f) + & \phi_s(s)(\underline{q}z + x)n]\mu_-(n,s) \\
&= \phi_f(s)(1 - xn - \kappa zn)\mu_-(n,f) \\
&\quad + \phi_s(s)\underline{q}z(n-1)\mu_-(n-1,s) + \phi_f(s)\kappa z(n-1)\mu_-(n-1,f) \\
&\quad + \phi_s(s)x(n+1)\mu_-(n+1,s) + \phi_f(s)x(n+1)\mu_-(n+1,f).
\end{aligned} \tag{28}$$

The left-hand side collects all outflows from n to $n+1$ and $n-1$ due to successfully innovation or creative destruction, as well as type switching from being slow to fast. The right-hand side terms capture, in order, inflows from type switchers who were fast and become slow but did not innovate; from successful innovators who were slow and retained their speed-type as well as from successful innovators who switched their speed-type; and finally inflows from creative destruction, again accounting for speed-type switching.

Similar to (28) we can define the stationary firm size distribution for (n, f) and $n > 1$ by the following equation:

$$\begin{aligned}
[\phi_f(s) + & \phi_f(f)(\kappa z + x)n]\mu_-(n,f) \\
&= \phi_s(f)(1 - xn - \underline{q}zn)\mu_-(n,s) \\
&\quad + \phi_f(f)\kappa z(n-1)\mu_-(n-1,f) + \phi_s(f)\underline{q}z(n-1)\mu_-(n-1,f) \\
&\quad + \phi_f(f)x(n+1)\mu_-(n+1,f) + \phi_s(f)x(n+1)\mu_-(n+1,s).
\end{aligned} \tag{29}$$

Again, the left-hand side contains the terms relating to outflows from n to $n-1$ and $n+1$

[9] We write $\mu_-(n,j)$ for the stationary distribution of product lines $n \geq 1$ and types $j \in \{f, s\}$ at the beginning of a period before types are re-assigned, or equivalently at the end of the previous period, and $\mu(n,j)$ for the stationary distribution after types have been reassigned. Note that $\mu(n,j') = \sum_{j=\{f,s\}} \phi_j(j')\mu_-(n,j)$.

as well as speed-type switchers. The right-hand side shows the inflows into (n, f) from speed-type switchers who did not innovate, from successful innovators, and from firms that have been hit by creative destruction.

Finally, at $n = 1$ a stationary distribution with a unit mass of firms requires that inflows into the economy match outflows for each speed-type, $x\mu(1, s) = \alpha z^e$ and $x\mu(1, f) = (1 - \alpha)z^e$, as well as for slow firms:

$$\begin{aligned}
&[\phi_s(f) + \phi_s(s)(x + \underline{q}z)]\mu_-(1, s) \\
&\quad = \phi_f(s)(1 - \kappa z - x)\mu_-(1, f) + \alpha z^e \\
&\qquad + \phi_s(s)2x\mu_-(2, s) + \phi_f(s)2x\mu_-(2, f).
\end{aligned} \tag{30}$$

The left-hand side are the the outflows from $(1, s)$ due to speed-type switchers, creative destruction and successful innovation. On the right-hand side, we have the usual terms described above plus the adjustment for the share α of the measure of entrant firms z^e who turn out to be slow after entry into the economy. Similarly, for fast firms with one product line we have:

$$\begin{aligned}
&[\phi(s|f) + \phi(f|f)(x + \kappa z)]\mu_-(1, f) \\
&\quad = \phi(f|s)(1 - \underline{q}z - x)\mu_-(1, s) + (1 - \alpha)z^e \\
&\qquad + \phi(f|f)2x\mu_-(2, f) + \phi(f|s)2x\mu_-(2, s).
\end{aligned} \tag{31}$$

The left-hand side are the the outflows from $(1, f)$ due to speed-type switchers, creative destruction and successful innovation. On the right-hand side, we have the usual terms described above plus the adjustment for the share $1 - \alpha$ of the measure of entrant firms z^e who turn out to be fast. For later use we can define the weighted and unweighted share of fast and slow firms in the economy as

$$S^{\Omega}(j) = \frac{\sum_{n=1}^{\infty} \Omega_n \mu_-(n, j)}{\sum_{n=1}^{\infty} \sum_{j \in \{f,s\}} \Omega_n \mu_-(n, j)}, \tag{32}$$

where we use weights $\Omega_n = \{1, n\}$ to compute equally-weighted and size-weighted averages. Having laid out the extended model with persistent speed-types, we describe our methodology to calibrate in the next section.

4 Calibration

We calibrate the extended model to French firm-level data at annual frequency. The model features 14 structural parameters. We identify these parameters using a mixture of external calibration, estimation, and indirect inference. Whenever possible, we rely on data that cover the pre-GFC period which will allow us to use our model to inform about the quantitative impact of the policy mix that followed the financial crisis and, among other things, reduced the cost of refinancing. Table 1 contains an overview of the calibrated parameters and calibration methods.

Table 1: Calibrated parameters.

Parameter	Description	Identification	Value
L	Unskilled labor	Normalization	1.00
L^R	Skilled labor	External calibration	0.58
λ	Innovation step size	External calibration	1.40
θ	Contracting friction	External calibration	0.68
ξ	Distribution parameter	External calibration	0.24
α	Share of slow firms at entry	Indirect inference	0.62
β	Discount factor	Indirect inference	0.99
δ	Cost of refinancing	Indirect inference	0.79
$\phi_f(f)$	Persistence fast firms	Indirect inference	0.92
$\phi_s(s)$	Persistence slow firms	Indirect inference	0.86
ψ	Entry cost	Indirect inference	7.28
χ	Interest semi-elasticity/size	Indirect inference	0.30
ζ	Cost parameter	Indirect inference	6.23
z	Innovation cost	Indirect inference	0.11

4.1 External calibration

We externally calibrate five structural parameters $\left(L, L^R, \lambda, \theta, \xi\right)$ using a mixture of macroeconomic data, estimation and references in the literature.[10]

[10]Without loss of generality we set the success probability for fast firms $\kappa = 1/2$. The parameters κ and z are indistinguishable for fast firms. For slow firms, normalizing κ is equivalent to re-scaling the mean of

First, we interpret the mass L workers who are employed in the production of intermediate goods as unskilled labor and the mass L^R who are hired to perform R&D as skilled workers. We normalize the former to $L = 1$ such that baseline steady-state output is just $Y = A_0$, see Appendix A.1. With $A_0 = 1$, the normalization allows us to interpret all quantities as *per-capita* ratios. Data from Eurostat (2023) suggest that the share of skilled workers in France is $L^R/(L + L^R) = 0.368$ on average for the period from 2000 to 2006. We set $L^R = 0.58$ accordingly.

Second, in the baseline model, step sizes are homogeneous. We calibrate the innovation step size λ to match the unweighted average mark-up for French firms over the period 1994-2006, which we estimate using the methodology outlined in (Berlingieri et al., 2024) and data from `FICUS`, a database produced by the French statistical office with balance sheet information on all firms in the private sector. We set $\lambda = 1.4$, or equivalently $(\lambda - 1)/\lambda \approx 0.286$.

Third, the share $1 - \theta$ quantifies how much of the value of a new product line a creditor can seize from slow firms after successful innovation. We interpret θ as the severity of the contracting friction between the parties. As is standard in the literature, we interpret a new product line as a patent. Kermani and Ma (2023) estimate that the average recovery values for book intangible capital – excluding goodwill – is 32% across two-digit SIC industries in Compustat. Similar estimates are also obtained in Ma et al. (2022) for the average resell value of patents in Chapter 11. Bolton et al. (2019) assume that 80% of firm-specific knowledge is inalienable and cannot be appropriated by creditors. Following the empirical estimates we set $\theta = 0.68$.[11]

Finally, we assume that the success probabilities for slow firms are drawn from the beta-type distribution:

$$\mathcal{Q}(q) = 1 - (1 - q)^{\frac{1}{\xi}}. \tag{33}$$

the distribution of success probabilities for by a constant factor. Hence our normalization does not affect the relative R&D productivity of fast and slow firms in the model.

[11]Higher values for θ will amplify the soft budget constraint's effect on economic growth. A calibration using the recovery value for all book intangibles, including goodwill, as reported in Kermani and Ma (2023) would imply $\theta > 0.9$. We prefer to stick with the more conservative estimate of 0.68 to avoid overstating the effect of contracting frictions.

The tail parameter is $d \ln(1 - \mathcal{Q}(q)) / d \ln(1-q) = 1/\xi$. To measure ξ, we assume that the number of patents filed by a given firm every year is equal to nqz. This is effectively associating the value of q to the probability that a patent get granted and z will be a common shifter. We retrieve such information from `FICUS` and a matching of patents to firms constructed by Bergeaud et al. (2022). Given that z and py is common across firms, then $nqz/npy = qz/py$ will follow the same distribution as q and can be measured in the data by looking at the average number of patents filed per year divided by sales. We then compute the empirical distribution of this quantity and estimate ξ from:

$$\log(1 - Q(X)) = \alpha + \frac{1}{\xi} \log(1 - X) + \varepsilon. \tag{34}$$

We estimate this model using the OLS for each firm with at least one patent from 1994 to 2006 and measure X using the ratio of the average number of patents filed per year over average sales over this period. This yields a value of $\xi = 0.24$.

4.2 Indirect inference

We use the remaining nine structural parameters $\Gamma = \{\alpha, \beta, \delta, \phi(f|f), \phi(s|s), \psi, \chi, \zeta, z\}$ to jointly match nine moments from firm-level data using indirect inference inside the model.

Our targets are (i) the tail parameter of the firm size distribution, (ii) the average annual growth rate, (iii) the real interest rate, (iv) the entry rate as measured by entrants' contribution to creative destruction, (v) firms' average innovation spending relative to sales, (vi) the share of non-performing loans (NPLs), (vii) firm's operating profit (EBIT) relative to sales, (viii) the correlation between firm size and interest rates, and (ix) average interest payments normalized by value added. Table 2 contains the list of targeted moments.

To match the data, we minimize the distance between model-generated moments $\mathcal{M}(\Gamma)$ and their empirical counterparts $\mathcal{M}_0$ by searching over the parameter space Γ with a generalized pattern search (GPS) algorithm. The objective function is:

$$\min ||\boldsymbol{W}\left(\mathcal{M}(\Gamma) - \mathcal{M}_0\right)||_p, \tag{35}$$

Table 2: Summary of target moments.

Moment	Model	Data	Method
Share skilled workers	36.8%	36.8%	External
Average markup	1.40	1.40	External
Tail firm size distribution	−2.52	−2.48	Internal
Growth rate	1.70%	1.70%	Internal
Risk-free rate	2.50%	2.48%	Internal
Entrants' contribution to creative destruction	1.1%	6.1%	Internal
Innovation intensity/ sales	22.4%	9.6%	Internal
Share of NPLs	6.5%	4.0%	Internal
Correlation firm size interest rates	−0.59	−0.61	Internal
Operating profits/ sales	6.2%	4.6%	Internal
Average interest expenses/ value added	5.6%	6.1%	Internal

where $\boldsymbol{W}$ a diagonal weighting matrix and $||\cdot||_p$ denotes the p^{th} norm. In the baseline calibration, we weight towards matching the growth rate and the average interest rate spread, and set $p = 2$ (Euclidean norm). We select our target moments as follows:

1. To match the empirically observed unconditional firm size distribution we target the slope coefficient on the firm size distribution, which we obtain from running the regression:

$$\ln\left(\mu(n,s) + \mu(n,f)\right)_n = \eta_0^L + \eta_1^L \ln(n) + \varepsilon_n^L. \tag{36}$$

   Data on the firm size distribution comes from the *Enquête Annuel de Production*, a firm-product level dataset only available for the manufacturing sectors which provides information on values and quantities sold yearly by each firmt from a very detailed product category (over 4000 different product codes). We estimate $\hat{\eta}_1^L = -2.69$ in 2009, the earliest year for which product level dataset is available.

2. We target an average annual growth rate of GDP per capita in France of 1.7% for the the period from 1995 to 2006.

3. Given our target for the growth rate, we pick household discount factor $\beta = (1+g)/(1+r)$ to achieve a real interest rate of 2.52% corresponding to the annualized average rate of return on the 10-year French government bond over the period 2000-2006 minus average inflation over the same period.

4. We pin down the entry rate z^e, defined in (25), by measuring entry as the share of innovator that is filing a patent for the first time in a given year. We then take the average of this share over the period 2000-2006. Because the matching between patents and firm starts in 1994, we drop the first 5 years that are likely to overstate the number of entrants. We find a value of 6.1% which is close to the results in Berlingieri et al. (2024) (4.4%).

5. We target incumbent firms' average innovation intensity. Our preferred measure is the innovation wage bill normalized by sales:

$$\overline{RD} = \frac{\sum_{n=1}^{N}[\zeta\mu(n,f) + (\zeta+\delta)[1-\mathcal{Q}(q^*)]\mu(n,s)]znw}{PY}. \tag{37}$$

   We compute the unweighted average of this ratio over all firms with positive R&D wage bill over the period 2000-2006. This yields a value of 9.6%.

6. Another target is the average share of non-performing business loans in France for 2012-2019. We define a loan to be non-performing if the debtor is in arrears for more than thirty days (Antonin et al., 2018). We take the data on non-performing loans from the World Banks's Global Financial Development data base and restrict the sample to all loans to French non-financial corporations. The data suggest an average share of 4.0% for 2000-2006 for the sub-sample of loans to non-financial firms only. In the model, we define NPLs as the share of slow projects that receive refinancing at the interim stage. We assume that the remainder is in default and written off to zero. The share of non-performing loans therefore is:

$$\overline{NPL} = \frac{[1-\mathcal{Q}(q^*)]\sum_{n=1}^{N}\mu(n,s)\zeta znw}{\sum_{n=1}^{N}[\mu(n,s)+\mu(n,f)]\zeta znw}. \tag{38}$$

7. We match firms' average operating profit, that is, profits from all its operations before taxes and interest payments, relative to sales. Although the firm pays an interest rate $\tilde{R}(n,j)$ on its borrowing, it will account for R&D expenditure before interest as R&D spending. Interest payments are treated as financing activity on the profit-and-loss statement. In the model this profitability measure is:

$$\overline{PM} = \frac{\pi Y - \sum_{n=1}^{N} \left[\zeta\mu(n,f) + (\zeta+\delta)[1-\mathcal{Q}(q^*)]\mu(n,s)\right] znw}{PY}. \tag{39}$$

The data from `FICUS` suggest an average operating profit relative to sales of $\overline{PM} = 4.6\%$, on average from 2000 to 2006.

We calibrate two additional moments related to firm's interest expenses. In the quantitative model extension the creditor breaks even only across her two separate loan portfolios. Yet, the balance sheet condition (23) does not fully pin down the menu of interest rate $\tilde{R}_t(n,j)$ for each credit history j. Hence, we specify that the creditor's menu of loan contract takes an affine form:

$$\tilde{R}_t(n,j) = \rho_{0,t} + \rho_t(n,j), \tag{40}$$

consisting of a fixed component $\rho_{0,t}$ and a variable part $\rho_t(n,j)$ which can be conditioned on size n and the type j. In the baseline model in Section 2 we have assumed that the creditor breaks for each loan type (n,j) separately. In other words, there is no segmentation across the loan market. We deviate from this assumption for two reasons. First, the functional form in (40) gives us greater flexibility in matching the cross-sectional distribution of interest rates. Second, a piece-wise break-even setup as in (5) implies that interest rates are strongly convex in firm size n, resulting in unrealistically high spreads for small firms and unrealistically low spreads for large firms under standard parameter values.[12]

Equation (40) specifies a contract consisting of a fixed part and a variable size-dependent spread, $\tilde{R}_t(n,j) = \rho_{0,t} + \rho_t(n,j)$. We set the constant part $\rho_{0,t} = 1 + r_{t+1}$ to equal the risk-

[12] One explanation for the functional form in (40) is that the creditor cannot perfectly price default probabilities based on firm size and type. Instead she resorts to a *linear credit risk model*, which are popular in the financial industry (e.g. Ackerer and Filipović, 2020).

free rate, such that we can interpret $\rho_t(n, j)$ as a risk-premium. We allow the risk-premium to decline with size according to the power function $\rho_t(n, j) = \rho(j)n^{-\chi}$ such that interest rate spread over the risk-free rate is simply:[13]

$$\overline{SP}(j) = 1 + \frac{\rho(j)}{(1+r)n^{\chi}}. \tag{41}$$

Given the functional form for the spread (41), the parameters $\rho(j)$ are pinned down by the creditor's break-even conditions (23). The parameter χ, which is common across credit histories j, is determined through indirect inference. As outlined in Table 2, we target the following set of moments related to interest rates:

(viii) We target the empirical correlation between firm size and gross interest rates. Using (41) we run a regression of the form:

$$\tilde{R}_n = \eta_0^R + \eta_1^R \ln(n) + \varepsilon_n^R, \tag{42}$$

The data suggest a correlation of $\hat{\eta}_1^R = -0.61$.

(ix) Lastly, we target a measure of interest rate expenses relative to firm's profitability. As the level of interest rates indirectly relates to the creditor's expected cost of refinancing δ, this moment is crucial to identify the softness of the creditor's budget constraint. To account for firm size, we normalize interest expenses by value added and average:

$$\overline{IE} = \sum_{n=1}^{N} \sum_{j \in \{f,s\}} \left[\frac{\kappa z n \tilde{R}(n,j) z \zeta n w}{nPY} \right] \phi_j(f) \mu_-(n,j). \tag{43}$$

As higher refinancing costs are associated with higher interest rates across the creditor's loan portfolio, a fall in corporate interest expenses, for instance due to cheap capital in the banking sector, should capture a softening of the budget constraint.

[13] A model in which the creditor breaks even piece-wise on each segment of the loan market (n, j) implies spreads over the risk-free rate that are highly convex in firm size. The model-implied interest rates of such a set-up decline too fast with firm size relative to the empirically observed spreads. (41) achieves a better fit to the data.

4.3 Equilibrium

The share of fast and slow firms is unobservable in the data. Slow firms can be both more productive and less productive than good firms in our model, depending on their realization of q and the parameter values ζ and δ. As such, we do not explicitly target the share of slow firms in the economy using some proxy. Instead, our simulation gives rise to an implied distribution of fast and slow firms. As discussed in the previous section, for this calibration exercise the only moment that we use which relates to the firm size distribution is the tail parameter of the unconditional firm size distribution. All other moments do not not impose any structure on the distribution itself. The left panel of Figure 2 shows the unconditional stationary firm size distribution, $\mu(n)$. The right panel shows the unweighted share of slow firms, $S^{uw}(s)$, as defined in equation (32).

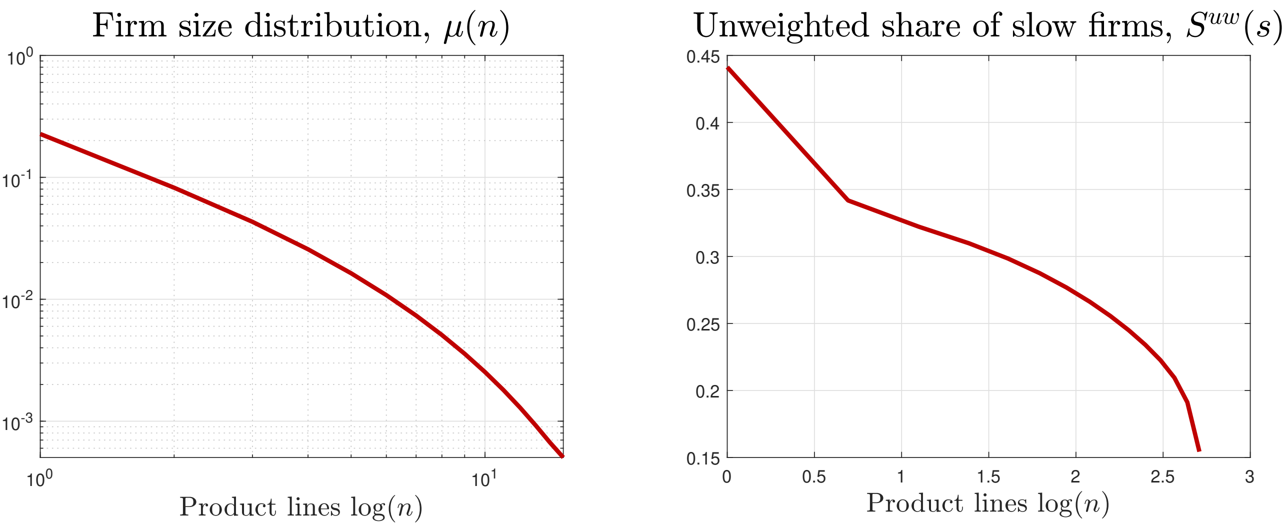

Figure 2: Equilibrium firm size distribution.

The firm size distribution has a long right tail. Our simulation suggests that the ratio of fast-to-slow firms is roughly 44%. The share of fast to slow firms is increasing as firms become larger. At $n = 1$ roughly 60% of firms are slow. Most of this is driven by new entrants, who predominantly tend to be slow firms as well ($\alpha = 0.61$). As slow firms innovate with a lower success probability than fast firms, and types are very persistent, a slow firm's expected growth rate and size are lower, and its average lifetime is shorter than a fast firm's. Consequently, the share of slow firms decreases among larger firms – falling below 10% for firms with $n > 12$.

The distribution of success probabilities is right-skewed. Although the cutoff value q^* that determines refinancing for slow firms is low, the skewness of the distribution implies that only around one quarter of slow firms receive refinancing. Given that around 40% of firms are slow, this implies that only 70% of all firms in our economy try to innovate. These slow firms soak up a large amount of labor, as their average labor cost per *realized* innovation $(\zeta + \delta)/\zeta$ is around 13% higher compared to fast firms. At the same time, the subset of slow firms that receives refinancing and tries to innovate is very productive: Around 14% of aggregate creative destruction comes from slow innovators compared to 66% from fast firms and the remaining 20% from entrants.

In the next section, we will discuss the impact of a change refinancing cost on entry, growth, and the firm size distribution in detail.

5 The growth effects of budget constraint softening

In this section we use our calibrated model to decompose the aggregate growth effects of the budget constraint softening that occurred in response to the Global Financial Crisis (GFC). As discussed in the introduction, we posit that a range of macro-prudential and monetary policy intervention in the aftermath of the GFC have led to a *de-facto* softening in firm's budget constraints.

We will first discuss how changes in the refinancing cost δ affect the entry rate, the growth rate, and the firm size distribution under our current calibration, before evaluating the effect of policies that have led to a softening of budget constraints in more detail.

5.1 The refinancing trade-off

5.1.1 Entry and growth

To study the relationship between refinancing and growth in the quantitative framework, we vary the parameter δ and solve the model for each of these counterfactual values. Our calibration suggests that the French economy in the run-up to the GFC is already to the left of the hump in the inverse-U relationship between the growth rate and the cost of refinancing, which we have characterized in Proposition 2. In this scenario, more stringent refinancing rules reduce innovation by slow incumbents but foster innovation through firm entry, with a net positive effect on growth.

Entry. Starting with the effect on entry, Figure 3 shows the equilibrium entry rate z^e as a function of the refinancing cost δ. The dashed line in the figure indicates the calibrated steady state.

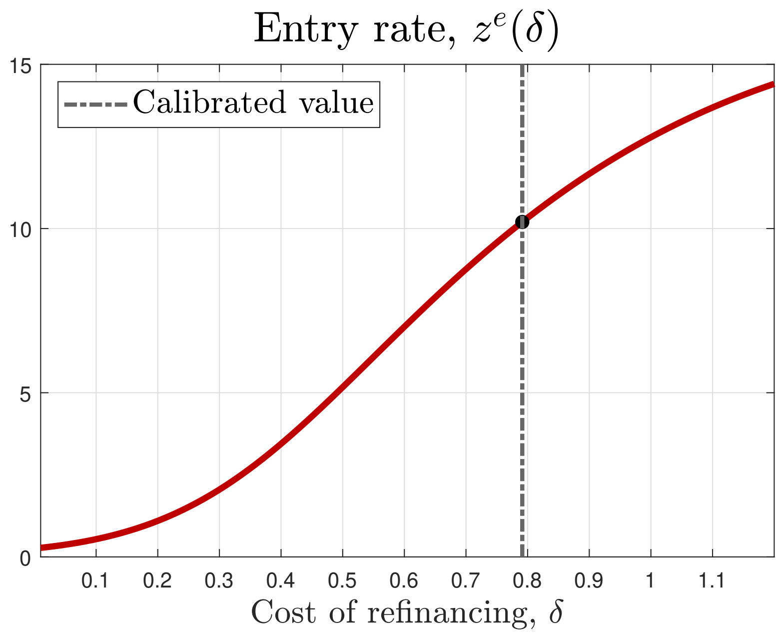

Figure 3: Firm entry as a function of refinancing costs.

One can see that an increase in the cost of refinancing δ has a highly non-linear effect on entry: When refinancing costs are relatively low, a small decrease in δ can have seizable effects on the entry rate. The curvature of the entry rate as a function of δ depends indirectly on the distribution of success probabilities for slow incumbents.

Compared to the analytical model in Section 2 where we have used a uniform distribution, our quantitative results are based on a more general distribution function, the modified beta in (33) with tail parameter $1/\xi \approx 4$. This calibration generates a relatively high right (positive) skew of success probabilities for slow types, which also drives the non-linearity in Figure 3.[14]

[14] In the uniform distribution case, a change in δ always has an equally sized effect through the labor market and through aggregate innovation. With a general distribution function, the mass close to the threshold,

Given the distribution of slow firms has a fat tail, the effect of changes in δ on entry weakens when refinancing costs are high. When a large number of incumbents clusters close to the cutoff value q^*, small changes in δ can result in a large cut in the number of firms that are refinanced, and hence release large quantities of labor for entry.

Growth. Moving to growth rates, Figure 4 shows the equilibrium growth rate as a function of δ. Note that under our calibration, the French economy is to the left of the optimal refinancing cost in the pre-GFC period, as indicated by the dashed line.

Starting from the calibrated steady state, an increase in the cost of refinancing pushes out the most productive slow firms. These firms have higher success probabilities than the average entrant, which means that these firms are on average more productive than entrants and should be refinanced. Conversely, when δ is low, an increase in entry can compensate for the fall in incumbent innovation due to tighter refinancing conditions. At some point the entry effect weakens leading to a fall in growth, and a hump-shaped relationship between refinancing and growth.

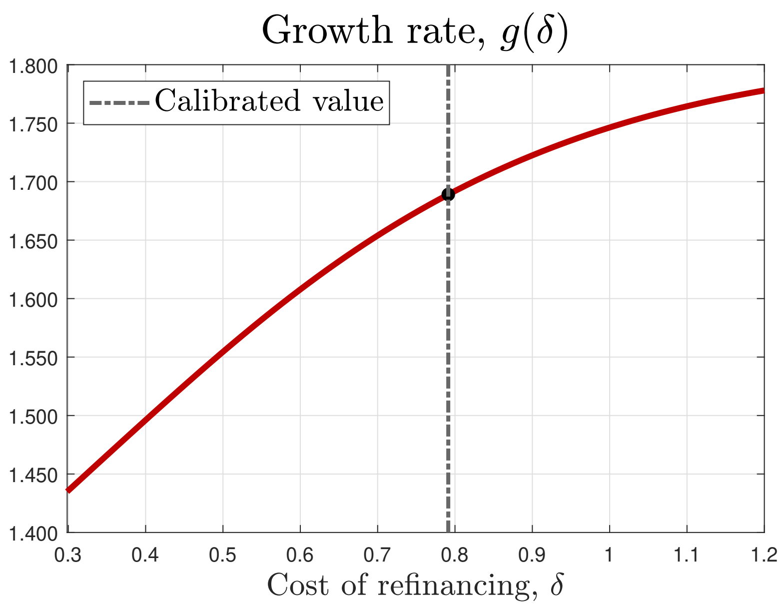

Figure 4: Growth rate as a function of refinancing costs.

$\mathcal{Q}'(q^*)$, scales the innovation effect of a change in δ relative to the labor market effect. See Appendix B.4 for a detailed discussion.

5.1.2 Non-performing loans

As refinancing costs rise, only the most productive slow firms receive additional funding and the refinancing threshold shifts upward. Slow firms with low q, who anticipate that their projects will not receive refinancing, select out of the credit market reducing the total amount of loans that will become non-performing at the interim date in the period. As discussed in (38), we define non-performing loans as the ratio of slow firm loans over the total initial loan volume, both defined at the beginning of the period. In other words, the share of NPLs is the fraction of the creditor's balance sheet that will require refinancing at the interim stage of the period.

The right panel of Figure 5 shows the the share of non-performing loans as a function of δ. The left panel shows the equilibrium threshold q^* as a function of δ.

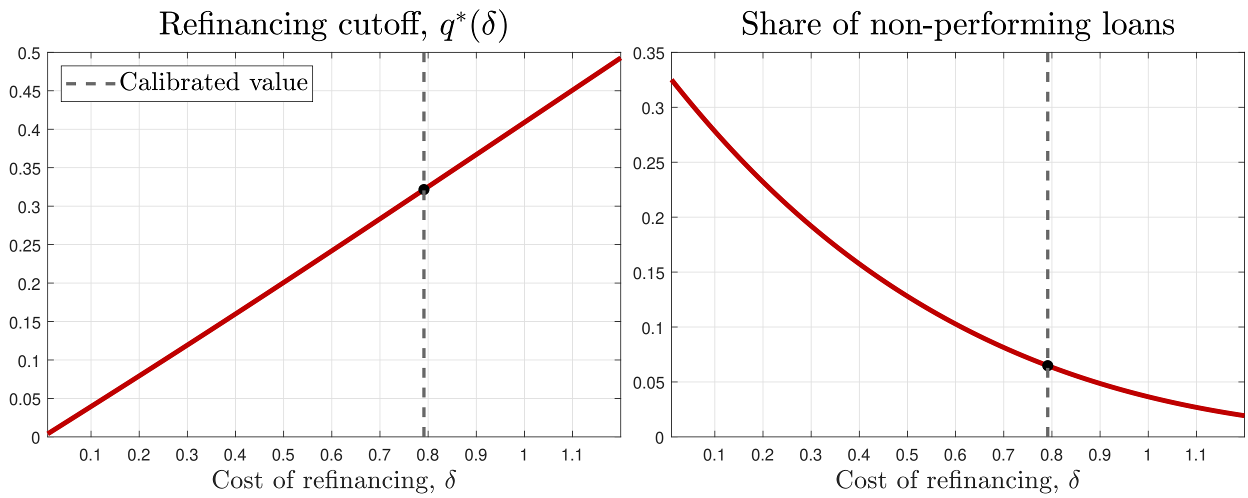

Figure 5: Refinancing rate and NPLs as a function of refinancing costs.

5.2 Declining refinancing costs

The average annual growth rate of GDP per capita in France fell from 1.70% for the period 2000-2006 before the Global Financial Crisis to 1.35% per year for 2014-2019. In this section, we investigate how much of this decline our model can generate from a softening of the creditors' budget constraints and associated changes in their refinancing decisions.

5.2.1 Approach

One major obstacle to answering these questions is that the creditor's cost of refinancing δ is, of course, unobservable in the data. Our reduced form measure in the model depends in reality not only on the physical cost of refinancing struggling firms but also on implicit, regulatory and potentially even non-monetary costs that the creditor may face such as, for instance, the prospect of impairing her own equity capital when forcing struggling debtors into default and having to recognize the associated accounting losses.

Among others, recent research by Blattner et al. (2023) shows that banks with relatively low capitalization rates are more willing to provide additional financing to prop up struggling borrowers in order to avoid having to write-off unrealized losses on non-performing loans against their equity buffer, an effect whose significant consequences on productivity Caballero et al. (2008) had highlighted in the case of Japan since 1990. In this spirit, one might naturally think that banks that feel, due to adverse shocks, "too close" to regulatory capital requirements can exacerbate the soft budget syndrome (this question has been discussed in the case of transition economies too : see e.g. Kornai et al. (2003), as well as Dewatripont and Roland (2000) who stressed the adverse effect of the soft budget constraint on entry of new firms).[15]

To tackle the problem of an unobservable δ, we resort to an approach that tries to implicitly capture the changes in the softness of creditor's budget constraints: In the French firm level data, we are able to observe firm's interest expenses. Taking our model literally, these interest payments implicitly capture the refinancing cost δ and the creditor's refinancing decision q^* through her interest setting policy. *Ceteris paribus*, we aim to match the observable changes in average interest payments in the data with a decline in the refinancing cost δ in the model.

Our theoretical model suggests that on the left-side of the hump, a decrease in refinancing costs δ lowers the creditor's loss from lending to slow firms. Via the creditor's break-even condition (23) the decline in refinancing costs implies that lending rates $\tilde{R}(n, j)$

[15] Similarly, we think that monetary policy intervention, and in particular quantitative easing, have led to a significantly fall in banks' cost of capital.

should fall. In the French data, we indeed observe that normalized interest payments have indeed fallen pre- and post-financial crisis by about 2 percentage points from 6.1% to 4.0% of value added.

5.2.2 Results

We run the following exercise: keeping all other parameters of the model constant, we change δ to simulate a decline in interest expenses (43) of 2 percentage points, in line with the data.

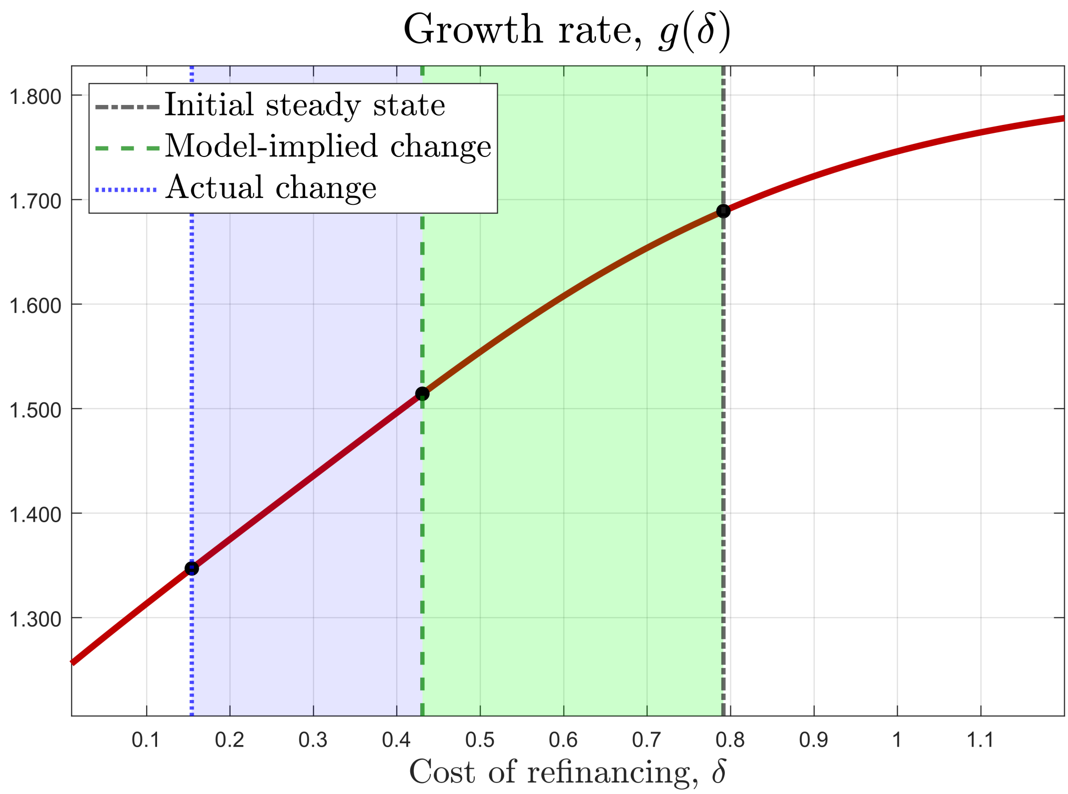

Figure 6: Model-implied change in the growth rate.

Figure 6 shows the effect on growth. The dash-dotted gray line on the right side of the figure indicates the initial pre-GFC steady state with an average annual growth rate of 1.7%. The dotted blue line on the left side of the panel indicates the new steady state with an average annual growth rate of 1.35%, which corresponds to the post-crisis period 2014-2019. Finally, the green dashed line in the middle indicates the model-implied change in the growth rate if we decrease the cost of refinancing δ from its initially calibrated level such that we exactly match the decline in firms' interest expenses, as defined in (43), of 2 percentage points that we observe in the French firm-level data.

A decrease in interest expenses over value added by 2 percentage points implies a decrease in the unobservable cost of refinancing δ from 0.79 to about 0.43, that is, a decline in refinancing costs for creditors of about 45%. Given this decline, our model suggests that growth rates should have decreased to about 1.51% per year versus an observed decline to 1.35% in the data. Our model therefore explains about 53% of the decline in observed growth rates in France since the GFC.

Of course, this aggregate effect is the sum of a range of different effects on firm entry, incumbent innovation, and the firm size distribution. In the next section, we decompose the model-implied change in the growth rate into its constituent channels. In particular, we are interest in the fraction of the total effect driven by feedback loops between the creditor's refinancing decision and the cross-sectional distribution of firms, that is, general equilibrium effects.

5.3 Decomposition

We decompose the effect of a change in δ, the cost of refinancing, on growth in its constituent parts. Throughout we will focus on a first-order approximation of the effect that these changes have on entry, incumbent innovation, and creative destruction. The details and algebraic derivations can be found in Appendix C.

For entry we can decompose the effect into three channels. First, a direct *selection effect* (SE) on the number of slow firms that invest in innovation. Second, a direct *resource cost effect* on entry through the cost of innovation for active slow firms. And third, an additional indirect *distribution effect* through changes in the firm size and type distribution that arise only in the model with persistent firm types.

Denoting values in the new steady state by a superscript tilde and the change from steady state to steady state by Δ, we can write (26) as:

$$\Delta z^e = \overbrace{\frac{1}{\psi}\sum_{n=1}^{\infty} \Delta \mathcal{Q}(q^*)(\zeta+\delta) zn\mu(n,s)}^{\text{Selection effect (SE)}} - \overbrace{\frac{1}{\psi}\sum_{n=1}^{\infty} [1-\mathcal{Q}(\tilde{q}^*)]\, \Delta\delta\, zn\mu(n,s)}^{\text{Resource cost effect (RE)}}$$
$$\underbrace{- \frac{1}{\psi}\sum_{n=1}^{\infty}\Big([1-\mathcal{Q}(\tilde{q}^*)]\,(\zeta+\tilde{\delta})zn\,\Delta\mu(n,s) + zn\,\Delta\mu(n,f)\Big)}_{\text{Distribution effect (DE)}}. \tag{44}$$

The creative destruction rate consists of entrant and incumbent innovation. We can further decompose incumbent innovation into changes in the selection margin of slow firms that innovate and an effect through the firm size and type distribution. The effect on incumbent innovation is:

$$\Delta z^i = \underbrace{\sum_{n=1}^{\infty} \Delta \underline{q}(\tilde{q}^*) zn\mu(n,s)}_{\text{Selection effect (SE)}} + \underbrace{\sum_{n=1}^{\infty}\Big(\kappa zn\,\Delta\mu(n,f) + \underline{q}(\tilde{q}^*) zn\,\Delta\mu(n,s)\Big)}_{\text{Distribution effect (DE)}}. \tag{45}$$

The total effect on the rate of creative destruction is $\Delta x = \Delta z^e + \Delta z^i$. Combining the effect on entry through the labor market (44) and the effect on incumbents (45), we have:

$$\Delta x = \overbrace{\sum_{n=1}^{\infty}\left(\Delta \underline{q}(\tilde{q}^*) + \frac{\Delta\mathcal{Q}(\tilde{q}^*)}{\psi}(\zeta+\delta)\right) zn\mu(n,s)}^{\text{Selection effect (SE)}} - \overbrace{\sum_{n=1}^{\infty}\frac{1-\mathcal{Q}(\tilde{q}^*)}{\psi}\Delta\delta zn\mu(n,s)}^{\text{Resource cost effect (RE)}}$$
$$\underbrace{+ \sum_{n=1}^{\infty}\left[\left(\kappa - \frac{\zeta}{\psi}\right) zn\Delta\mu(n,f) + [1-\mathcal{Q}(\tilde{q}^*)]\left(\frac{\underline{q}(\tilde{q}^*)}{1-\mathcal{Q}(\tilde{q}^*)} - \frac{\zeta+\tilde{\delta}}{\psi}\right) zn\Delta\mu(n,s)\right]}_{\text{Distribution effect (DE)}}. \tag{46}$$

A change in the cost of refinancing δ affects the creative destruction rate through three distinct channels: First, it directly affects the selection margin among slow innovators and entrants, as captured by the first term. Second, it affects entry through the resource cost of innovation. Third, there is a compositional effect on creative destruction because the equilibrium distribution of fast and slow firms changes.

Finally, for later use we define the relative contribution of an effects to the change in the entry rate, incumbent innovation, and the creative destruction rate as:

$$\varsigma^{\ell}(SE) = \frac{|\text{SE}|}{|\text{SE}| + |\text{RE}| + |\text{DE}|}, \qquad \text{for } \ell \in \{e, i, x\} \tag{47}$$

see Appendix C.1 for the detailed expression.

5.3.1 Direct versus indirect effects

First, starting with the effect on the entry rate z^e in (44), the overall change is negative. As a result of the change in δ, the contribution of entrants to creative destruction drops by about 0.68 percentage points in our simulation, which is equivalent to a drop in the annual growth rate of the economy of about 0.23 percentage points. Most of the effect on entry is explained through selection into the credit market by slow firms, which alone would have generated a decline in entry rates by about 0.87 percentage points and a decline in growth by 0.29 percentage points per year, but is partially offset by the lower resource cost of refinancing for these slow firms and through changes in the distribution of firms. We will discuss the distributional effect in detail below, but the lower number of entrants leads in the long run to fewer slow firms in the market, as entrants are on average slower than incumbent firms. This has a mildly positive feedback effect. In total, the relative contribution of the selection effect on entry, $\varsigma^e(SE)$ as define in (47), amounts to about 82% of the gross total effect on entry and growth, whereas resource cost and distribution effects account for about 7% and 11%, respectively.

Second, the overall effect of a reduction in δ on incumbent innovation in (45) is positive but small at 0.16 percentage points of creative destruction or 0.05 percentage points of annual growth. Here, the selection effect alone would have increased incumbent innovation by 0.21 percentage points and growth by 0.07 percentage points but is again partially offset by changes in the firm size distribution. In absolute value, the selection effect amounts to about 78% of the total effect on incumbent innovation while the effect through the distribution is around 22% of the total effect on incumbent innovation.

Finally, we focus on the effect on the aggregate creative destruction rate in (46), which drops by about 0.52 percentage points leading to the 0.17 percentage point reduction in growth explained by the model. Grouping incumbent and entrant selection effects together, these changes in credit access alone would have reduced creative destruction by 0.65 percentage points and growth by closer to 0.22 percentage points per year. The resource cost effect and the selection effect cushion this decline somewhat. In absolute value, the selection effect amounts to about 83% of observable changes in growth rates, while the other two effects explain 10% and 7%, respectively. Our simulation therefore suggests that the key channel through which softer refinancing rules affect firm entry is selection among incumbent firms – and not changes in the resource cost of these firms.

Although the size of the effect through the firm size-type distribution is relatively small, it echos the results in Aghion et al. (2023) who find that changes in the firm size distribution account for about 10% when analyzing the effect of size-based regulation on innovation in France.

5.3.2 Within versus between effects

The effect through the firm size distribution can be decomposed into three components: A selection effect that changes the ratio of fast and slow firms for each size bin (*within effect*), a size effect that captures shifts in the distribution of firm sizes (*between effect*), and an interaction effect between the two. The details are in Appendix C.2.

As before denoting the new distribution by $\tilde{\mu}(n,j)$, and the change in the distribution as $\Delta\mu(n,j) \equiv \tilde{\mu}(n,j) - \mu(n,j)$, we have:

$$\Delta\mu(n,j) = \underbrace{[\mu(n)\tilde{\mu}(j|n) - \mu(n,j)]}_{\text{Within effect, } WE(n,j)} + \underbrace{[\tilde{\mu}(n)\mu(j|n) - \mu(n)\tilde{\mu}(j|n)]}_{\text{Between effect, } BE(n,j)} + \underbrace{[\tilde{\mu}(n,j) - \tilde{\mu}(n)\mu(j|n)]}_{\text{Interaction effect, } IE(n,j)}. \tag{48}$$

The first term captures the effect of changes in types j given fixed firm size n. The second term captures the effect of changes in firm size n given fixed types j. The last term captures interaction effect.

We then add up changes for each size bin n and normalize by the size of the total effect. To account for the differences in size bins across the firm-size distribution, we also weight effects by firm size n and normalize by the total effect. For example, for the contribution of within effects we have:

$$WE(j) = \frac{\sum_{n=1}^{\infty} n \cdot WE(n,j)}{\sum_{n=1}^{\infty} n \cdot TE(n,j)} = \frac{\sum_{n=1}^{\infty} n \left[\mu(n)\tilde{\mu}(j|n) - \mu(n,j)\right]}{\sum_{n=1}^{\infty} n \left[\tilde{\mu}(n,j) - \mu(n,j)\right]}, \tag{49}$$

which captures changes in the share of fast and slow firms for each size bin. Similarly, we define the between effect $BE(j)$, that relates to changes in the average firm size, and an interaction effect $IE(j)$ between the two. Finally, we define the total effects $TE(j) = WE(j) + BE(j) + IE(j)$.

Taking as the basis the effect on the number of slow firms s (the effects are mirrored for fast firms), a decrease in the cost of refinancing δ leads to a net decrease in the share of slow firms across size bins. Within and between effect work in opposite directions, which explains the relatively small effect of changes in the size distribution on growth, that we have pointed out in the previous section.

Starting with the within effect, slow firms receive more refinancing, increasing their innovation output and allowing them to escape creative destruction more easily, within each segment of the firm size distribution there will be more slow firms. At the same time, the firm size distribution shifts towards larger firms as the number of entrant firms drops. As entrants are, on average, slower than incumbent firms, most of the between changes are concentrated at small firms with $n \leq 3$. The net effect of these shifts in the size distribution is positive but small. The interaction effect between within and between changes is small.

6 Conclusion

In this paper, we develop a model of endogenous growth and firm dynamics with soft budget constraints. In our model, firms differ in how quickly they can innovate, and some among the slow firms require additional financing in order to eventually innovate. Creditors cannot observe firms' refinancing needs ex-ante, nor can they commit not to refinance firms at the interim stage. The Soft Budget Constraint problem results in excessive activity by slow firms, thereby crowding out entry by potentially more efficient innovators. The trade-off between on the one hand the positive effect of budget constraint softening on innovation by incumbents, and on the other hand the negative effect of budget constraint softening on entry, generates a hump-shaped relationship between refinancing costs and aggregate and growth.

We calibrate a model version with persistent firm speed to French firm-level data to assess the aggregate growth effects of the budget constraint softening associated with the decline in observed interest payments which have followed the Global Financial Crisis. We find that our model can explain about half of the observed decline in aggregate growth rates following the crisis, most of which is driven by a selection effect whereby enhanced innovation by slow incumbents crowds out entry by good firms.

Our analysis sheds light on the policy debate on the relationship between growth and the design of monetary policy. Aghion et al. (2019) analyze the effects of the introduction by the European Central Bank of the so-called Additional Credit Claims (ACC) program in 2012, designed to avoid a recession by relaxing credit constraints for incumbent firms. Their findings reveal two counteracting effects of the ACC program on productivity growth. On the one hand, the program enhanced productivity growth and innovation among firms directly benefiting from it. On the other hand, it impeded the exit of less productive firms, thereby discouraging the entry of potentially more productive new firms into the market.

Using the methodology and analysis in the present paper, one could try to assess the direct and indirect growth effects of the ACC program on productivity growth in EU countries. Yet, a systematic analysis of the direct and indirect aggregate growth effects of monetary policy in an economy with innovation-led growth, entry, exit and firm dynamics, still lies ahead of us.

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Online Appendix

Firm Dynamics and growth with Soft Budget Constraints

P. Aghion, A.Bergeaud, M. Dewatripont, and J.Matt

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A Further proofs and derivations

A.1 Production

We define aggregate productivity in the economy as

$$A_t \equiv \exp\left(\int_0^1 \ln A_{i,t} di\right). \tag{A.1}$$

From (1), demand for each intermediate good is $y_{i,t} = Y_t/p_{i,t}$. Given productivity $A_{i,t}$, intermediate producers' marginal cost is $w_t^P/A_{i,t}$, where w_t^P is the wage on production labor. Prices are $p_{i,t} = \lambda w_t^P/A_{i,t}$, which yields equilibrium profits as in (2).

The labor cost of intermediate production per firm is $y_{i,t}/A_{i,t}$. Labor market clearing (A.18.4) together with $y_{i,t}$ in (2) yields:

$$L^P = \int_0^1 \left(\frac{y_{i,t}}{A_{i,t}}\right) di = \frac{Y_t}{\lambda w_t^P}, \tag{A.2}$$

where $1/\lambda$ is the factor share of final output accruing to production labor. The remainder is paid out to firm owners in the form of profits $\Pi_t = (\lambda - 1)/\lambda Y_t$. Adding up:

$$Y_t = \Pi_t + w_t^P L^P = \left(\frac{\lambda - 1}{\lambda}\right) Y_t + w_t^P \left(\frac{1}{\lambda w_t^P}\right) Y_t = Y_t. \tag{A.3}$$

Finally, we can solve for aggregate output. Take $y_{i,t} = A_{i,t}Y_t/(\lambda w_t^P)$ from (2) and replace the wage w_t^P by (A.2) to obtain $y_{i,t} = A_{i,t}L^P$, then substitute into equation (1) for $Y_t = \exp\left(\int_0^1 \left[\ln A_{i,t} + \ln L^P\right] di\right) = A_t L^P$. We normalize $L^P \equiv 1$ such that output is $Y_t = A_t$.

A.2 Refinancing

The market-clearing interest rate (13) is $R(n) = \frac{1}{\kappa z n}\left(1 + \frac{\alpha}{(1-\alpha)\zeta} k(q^*)\right)$, where $k(q^*) \equiv (\zeta + \delta)\left[1 - \mathcal{Q}(q^*)\right] - \underline{q}(q^*)(1-\theta)\psi$. Slow firms obtain $\zeta z n w$ units of credit. The creditor's maximum share in the debtor's innovation is $(1-\theta)\psi w$. If $R(n)\zeta z n w \geq (1-\theta)\psi w$, the creditor always seizes the maximum possible share. Substituting and rearranging yields:

$$(1-\alpha)\zeta + \alpha(\zeta+\delta)\left[1-\mathcal{Q}(q^*)\right] \geq \left[(1-\alpha)\kappa + \alpha\underline{q}(q^*)\right](1-\theta)\psi, \tag{A.4}$$

Fast firms participate in the credit market as long as $\psi w > R(n)\zeta znw$, which is equivalent to:

$$(1-\alpha)\kappa\psi > (1-\alpha)\zeta + \alpha(\zeta+\delta)\left[1-\mathcal{Q}(q^*)\right]. \tag{A.5}$$

The set defined by (A.4) and (A.5) is non-empty when α is sufficiently small or θ is large.

A.3 Growth rate

As in Aghion et al. (2023) we assume that the probability that two firms innovate on the same line and the probability that a firm innovates more than once are zero. There are $\mu_t(n,f)$ fast firms who innovate with κzn, and $\mu_t(n,s)$ slow firms who innovate with probability $\underline{q}(q_t^*)zn$. A unit measure of entrants innovate with probability z_t^e. The creative destruction rate is:

$$x_t = z_t^e + \sum_{n=1}^{\infty}\Big(\kappa zn\mu_t(n,f) + \underline{q}(q_t^*)zn\mu_t(n,s)\Big). \tag{A.6}$$

A successful innovator lowers the marginal cost on the product line he innovates on by a fixed step size $\lambda > 1$. (A.1) evolves according to

$$\begin{aligned} A_{t+1} &= \exp\left(\int_0^1 [x_t \ln(\lambda A_{i,t}) + (1-x_t)\ln A_{i,t}]di\right) \\ &= \exp\left(x_t \ln\lambda + \int_0^1 \ln A_{i,t} di\right) \\ &= A_t \exp\left(x_t \ln\lambda\right). \end{aligned} \tag{A.7}$$

Defining the growth rate $g_{t+1} \equiv (A_{t+1}-A_t)/A_t$ we have $1+g_{t+1} = \exp(x_t \ln\lambda)$. For small growth rates, $g_{t+1} \approx x_t \ln\lambda$. In the continuous time limit, the approximation is exact.

A.4 Value functions

Firm values depend on up to four state variables: Time t, the number of product lines n, the type $j \in \{f, s\}$, and the success probability $q \sim \mathcal{Q}(q)$ for slow and κ for fast firms' current project. Fast firms share the same success probability κ, such that we write their value simply as $V_t(n, f)$.

Slow firms are heterogeneous in their success probability q. Equation (12) defines a threshold q^* such that slow firms with $q \geq q^*$ receive refinancing and try to innovate, while slow firms with $q < q^*$ do not. Accordingly, we denote the value function as $V_t(n, j, q)$ for slow firms who try to innovate and as $\underline{V}_t(n, s)$ for slow firms who do not.

A.4.1 Transitions

Between period, firms move across product lines, types and draw new success probabilities. While movements across product lines depend on the current type, success probabilities and new types are drawn independently in the baseline model. We write:

$$\mathbb{E}_{j,q}\left[V_{t+1}(n)\right] \equiv \alpha \left(\mathcal{Q}(q_t^*)\underline{V}_t(n,s) + \int_{q_t^*}^{1} V_{t+1}(n,s,q) d\mathcal{Q}(q) \right) + (1-\alpha)V_{t+1}(n,f). \tag{A.8}$$

The first term is the expected value if the firm's type next period is slow, the second term is the expected value if the type is fast. Since types are re-assigned in i.i.d. fashion the expected value next period does not depend on this period's type.

A.4.2 Fast firms

All fast firms try to innovate with the same success probability κ. The value of a fast firm with n product lines is given by:

$$\begin{aligned} V_t(n,f) = \pi n Y_t &+ \frac{1-\kappa z n - x_t n}{1+r_{t+1}} \mathbb{E}_{j,q}\left[V_{t+1}(n)\right] + \frac{x_t n}{1+r_{t+1}} \mathbb{E}_{j,q}\left[V_{t+1}(n-1)\right] \\ &+ \frac{\kappa z n}{1+r_{t+1}} \left(\mathbb{E}_{j,q}\left[V_{t+1}(n)\right] - \tilde{R}_t(n) \zeta z n w_t \right), \end{aligned} \tag{A.9}$$

where the last term captures the value of a firm that successfully innovates, in which case the equity holders repay interest $\tilde{R}_t(n)\zeta z n w_t$ next period, which is set this period. Fast firms, whose innovation attempt fails, default as explained in the main text.

A.4.3 Slow firms

Slow firms never repay but have a fraction $1-\theta$ of the value of their new product line seized by creditors. Whether a slow firm is able to innovate depends on its realization of q. When $q \geq q_t^*$, the creditor provides refinancing and the firm will try to innovate. The value function in that case is:

$$\begin{aligned} V_t(n,s,q) = \pi n Y_t &+ \frac{1-qzn-x_t n}{1+r_{t+1}}\mathbb{E}_{j,q}[V_{t+1}(n)] + \frac{x_t n}{1+r_{t+1}}\mathbb{E}_{j,q}[V_{t+1}(n-1)] \\ &+ \frac{qzn}{1+r_{t+1}}\Big(\mathbb{E}_{j,q}[V_{t+1}(n+1)] - (1-\theta)\mathbb{E}_{j,q}[\Delta V_{t+1}(n)]\Big), \end{aligned} \tag{A.10}$$

where the last term captures the value of the new product line being seized by the creditor. In case $q < q_t^*$ the value function is simply:

$$\underline{V}_t(n,s) = \pi n Y_t + \frac{1-x_t n}{1+r_{t+1}}\mathbb{E}_{j,q}[V_{t+1}(n)] + \frac{x_t n}{1+r_{t+1}}\mathbb{E}_{j,q}[V_{t+1}(n-1)]. \tag{A.11}$$

Defining the average value across $q \sim \mathcal{Q}(q)$ as $V_t(n,s) \equiv \mathcal{Q}(q_t^*)\underline{V}_t(n,s) + \int_{q_t^*}^1 V_t(n,s,q)d\mathcal{Q}(q)$ and adding up across realizations of q:

$$\begin{aligned} V_t(n,s) = \pi n Y_t &+ \frac{1-x_t n-\underline{q}(q_t^*)zn}{1+r_{t+1}}\mathbb{E}_{j,q}[V_{t+1}(n)] + \frac{x_t n}{1+r_{t+1}}\mathbb{E}_{j,q}[V_{t+1}(n-1)] \\ &+ \frac{\underline{q}(q_t^*)zn}{1+r_{t+1}}\Big(\mathbb{E}_{j,q}[V_{t+1}(n+1)] - (1-\theta)\mathbb{E}_{j,q}[\Delta V_{t+1}(n)]\Big), \end{aligned} \tag{A.12}$$

where $\underline{q}(q_t^*) \equiv \int_{q_t^*}^1 q d\mathcal{Q}(q)$ is the conditional success probability defined in (4). For the remainder of the section we will use average values across q, as defined in (A.12).

A.4.4 Aggregate values

As in the standard model, we guess that the value function (in the case of slow firms for the average firm across realizations of q) is linear in the number of product lines, $V_t(n,j) =$

$v_t(j)nY_t$ for $j \in \{f, s\}$. Using the fact that there are always α and $1-\alpha$ firms of each type per size bin, we arrive at $V_t(n) = \alpha V_t(n,s) + (1-\alpha)V_t(n,f)$ and write $V_t(n) = v_t n Y_t$.

On the balanced growth path, all aggregate variables grow at rate g_t. Using our guess for the value function together with the Euler equation in (A.9) and (A.12) we obtain:

$$\begin{aligned} v_t(f) &= \pi + \beta(1+\kappa z - x_t)v_t - \kappa\zeta z^2 n R_t(n)\omega_t \\ v_t(s) &= \pi + \beta[1+\underline{q}(q_t^*)z - x_t]v_t - \underline{q}(q_t^*)z(1-\theta)\beta v_t, \end{aligned} \tag{A.13}$$

where $\omega_t \equiv w_t/Y_t$ is the wage-to-output ratio and $R_t(n) = \tilde{R}_t(n)/(1+r_{t+1})$. Using the same guess, $V_t = vnY_t$, in the creditor's break-even condition (5) yields an interest rate $R_t(n) = \frac{1}{\kappa z n}\left(1 + \frac{\alpha}{(1-\alpha)\zeta}k(q_t^*)\right)$, see (13). Substituting the expression for the creditor's lending rate back into the above, we obtain:

$$\begin{aligned} v_t(f) &= \pi + \beta(1+\kappa z - x_t)v_t - \zeta z\left(1 + \frac{\alpha}{(1-\alpha)\zeta}k(q_t^*)\right)\omega_t \\ v_t(s) &= \pi + \beta[1+\underline{q}(q_t^*)z - x_t]v_t - \underline{q}(q_t^*)z(1-\theta)\beta v_t. \end{aligned} \tag{A.14}$$

For the average value across types $v_t = \alpha v_t(s) + (1-\alpha)v_t(f)$, we have:

$$v_t = \pi + \beta(1-z_t^e)v_t - z\Big([(1-\alpha)\zeta + \alpha k(q_t^*)]\,\omega_t + \alpha\underline{q}(q_t^*)(1-\theta)\beta v_t\Big), \tag{A.15}$$

where we used (9) for $z_t^e = x_t - \Big[(1-\alpha)\kappa z + \alpha\underline{q}(q_t^*)z\Big]$. Next, use $k(q^*) = (\zeta+\delta)[1-\mathcal{Q}(q^*)] - \underline{q}(q^*)(1-\theta)\psi$ and write

$$\begin{aligned} v_t = \pi + \beta(1-z_t^e)v_t - z\Big((1-\alpha)\zeta + \alpha(\zeta+\delta)[1-\mathcal{Q}(q^*)]\Big)\omega_t \\ - \alpha z\underline{q}(q^*)(1-\theta)\Big(\beta v_t - \psi\omega_t\Big). \end{aligned} \tag{A.16}$$

The last term is zero because of the free-entry condition (7), $\beta v_t = \psi\omega_t$. The penultimate term is just the liability side of the creditor's break-even condition (5). Via labor market clearing (8), $(1-\alpha)\zeta + \alpha(\zeta+\delta)[1-\mathcal{Q}(q^*)] = L^R - \psi z_t^e$ such that $v_t = \pi + \beta(1-z_t^e)v_t - \left(L^R - \psi z_t^e\right)\omega_t$ and thus $(1-\beta)v_t = \pi - L^R\omega_t$ from (7) again. Summing up across the size distribution:

$$V_t = \left(\frac{\pi - L^R \omega_t}{1-\beta}\right) Y_t. \tag{A.17}$$

A.5 Market clearing

A.5.1 Market clearing conditions

The full set of market clearing conditions for intermediate goods, credit, equity, production labor, research labor, and output is given by:

$$\frac{Y_t}{p_{i,t}} = \frac{A_{i,t} Y_t}{\lambda w_t^P} \qquad \text{for all } i \in [0,1], \tag{A.18.1}$$

$$\begin{aligned}(1-\alpha)\kappa z n \tfrac{\tilde{R}_{t+1}\zeta z n w_t}{1+r_{t+1}} + \alpha \underline{q}(q_t^*) z n \tfrac{\mathbb{E}[(1-\theta)\Delta V_{t+1}(n)]}{1+r_{t+1}} \\ = \Big((1-\alpha)\zeta + \alpha(\zeta+\delta)[1-\mathcal{Q}\left(q_t^*\right)]\Big) z n w_t,\end{aligned} \tag{A.18.2}$$

$$B_t = \frac{V_t}{1+r_t}, \tag{A.18.3}$$

$$L^P = \int_0^1 \left(\frac{y_{i,t}}{A_{i,t}}\right) di, \tag{A.18.4}$$

$$L^R = \psi z_t^e + (1-\alpha)\zeta z + \alpha(\zeta+\delta)[1-\mathcal{Q}(q_t^*)] z, \tag{A.18.5}$$

$$C_t = Y_t. \tag{A.18.6}$$

A.5.2 Walras' law

The household's budget constraint is

$$B_{t+1} = (1+r_t)B_t + w_t L^R + w_t^P L^P - C_t. \tag{A.19}$$

Noting that on the balanced growth path $B_{t+1} = (1+g_t)B_t$ and using the equity market clearing condition in (A.18.3) together with (A.17), we have $C_t = \pi Y_t + w_t^P L^P$. Via (A.3), we arrive at the market clearing condition for the final good (A.18.6),

$$C_t = Y_t. \tag{A.20}$$

A.6 Welfare

The household has logarithmic utility over consumption. Labor supply for both types of labor is inelastic. We define the perpetuity value of welfare as:

$$\mathcal{U} \equiv (1-\beta)\mathcal{U}_0 = (1-\beta)\sum_{t=0}^{\infty}\beta^t \ln C_t. \tag{A.21}$$

On the balanced growth path, consumption and output grow at a common and constant growth rate g such that $C_t = (1+g)^t C_0$. From output market clearing $C_0 = Y_0$ and thus $\mathcal{U} = (1-\beta)\left[\ln(1+g)\left(\sum_{t=0}^{\infty} t\beta^t\right) + \ln Y_0\left(\sum_{t=0}^{\infty}\beta^t\right)\right] = \ln Y_0 + \frac{\beta}{1-\beta}\ln(1+g)$.

Final good production is $Y_0 = A_0 L^P$. Welfare only depends on the growth rate and a constant. Normalizing $L^P \equiv 1$ and given $A_0 \equiv 1$ we have $\mathcal{U} = \frac{\beta}{1-\beta}\ln(1+g)$. Substituting $1+g = \exp(x\ln\lambda)$:

$$\mathcal{U}(x) = \left(\frac{\beta}{1-\beta}\ln\lambda\right)x. \tag{A.22}$$

B General case

B.1 Value functions

The notation follows Appendix A.4. $V_t(n,f)$ is the value function for fast firms, $V_t(n,s,q)$ is the value function for slow firms who try to innovate and $\underline{V}_t(n,s)$ is the value function for those who do not.

B.1.1 Expected value

Between periods, firms move across product lines, types, and success probabilities. While the movements across product lines and types depend on each other, success probabilities are drawn independently. We economize on notation and write:

$$\begin{aligned}\mathbb{E}_q[V_{t+1}(n,f)] &= V_{t+1}(n,f) \\ \mathbb{E}_q[V_{t+1}(n,s)] &\equiv \mathcal{Q}(q_t^*)\underline{V}_{t+1}(n,s) + \int_{q_t^*}^{1} V_{t+1}(n,j,q)d\mathcal{Q}(q),\end{aligned} \tag{B.1}$$

for the expected value next period. For ease of notation in the sections below, we also write an expected value for fast firms even though there is no uncertainty about the success probability which is just κ for all fast firms.

B.1.2 Transitions

Given the Markov transition matrix $\mathbf{\Phi}$ in (22) the transition probabilities across product lines and types (n,j) can be expressed as:

- $p_t(n-1,j',q|n,j) \equiv \phi_j(j')x_t n$ from $(n,j) \to (n,j')$,
- $p_t(n,j',q|n,s) \equiv \phi_s(j')(1-qzn-x_t n)$ from $(n,j) \to (n,j')$ if $j=s$ and $q \geq q_t^*$, and $p_t(n,j'|n,f) \equiv \phi_f(j')(1-\kappa zn - x_t n)$ form $(n,f) \to (n,j')$ if $j=f$,
- $p_t(n+1,j',q|n,s) \equiv \phi_s(j')qzn$ from $(n,s) \to (n+1,j')$ for $j=s$ and $q \geq q_t^*$, and $p_t(n+1,j'|n,f) \equiv \phi_f(j')\kappa zn$ from $(n,f) \to (n+1,j')$ for $j=f$,

and $j' \in \{f,s\}$. Additionally, firms that are slow next period draw a success probability q, which is independent of their state (n,j) next period and governed by the expected values in (B.1). Firms that are fast next period have success probability κ.

B.1.3 Fast firms

The value function for a fast firm with states (n,j) is given by:

$$\begin{aligned}V_t(n,f) = \pi n Y_t + \frac{1}{1+r_{t+1}} \sum_{j'=\{f,s\}} \begin{bmatrix} p_t(n-1,j'|n,f) \\ p_t(n,j'|n,f) \\ p_t(n+1,j'|n,f) \end{bmatrix}' \begin{bmatrix} \mathbb{E}_q[V_{t+1}(n-1,j')] \\ \mathbb{E}_q[V_{t+1}(n,j')] \\ \mathbb{E}_q[V_{t+1}(n+1,j')] \end{bmatrix} \\ - \frac{1}{1+r_{t+1}} \sum_{j'=\{f,s\}} p_t(n+1,j'|n,f)\tilde{R}_t(n)\zeta z n w_t.\end{aligned} \tag{B.2}$$

Fast firms only repay when they successfully innovate, which happens before the new type is revealed. Noting that $\sum_{j'=\{f,s\}} p(n+1, j'|n, f) = \kappa z n$ as $\phi(f|f) + \phi(s|f) = 1$, we have:

$$V_t(n,f) = \pi n Y_t + \frac{1}{1+r_{t+1}} \sum_{j'=\{f,s\}} \begin{bmatrix} p_t(n-1,j'|n,f) \\ p_t(n,j'|n,f) \\ p_t(n+1,j'|n,f) \end{bmatrix}' \begin{bmatrix} \mathbb{E}_q[V_{t+1}(n-1,j')] \\ \mathbb{E}_q[V_{t+1}(n,j')] \\ \mathbb{E}_q[V_{t+1}(n+1,j')] \end{bmatrix} - \frac{\kappa z n}{1+r_{t+1}} \tilde{R}_t(n) \zeta z n w_t. \tag{B.3}$$

B.1.4 Slow firms

Slow firms never repay but have a fraction $1-\theta$ of the value of a new product line seized by the creditor. We assume that next period, the old creditor becomes an equity holder commensurate to the fraction seized or that he sells his claim. Whether a slow firm is able to innovate depends on the realization of q. When $q \geq q_t^*$, the value function for a slow firm is:

$$V_t(n,s,q) = \pi n Y_t + \frac{1}{1+r_{t+1}} \sum_{j'=\{f,s\}} \begin{bmatrix} p_t(n-1,j'|n,s) \\ p_t(n,j',q|n,s) \\ p_t(n+1,j',q|n,s) \end{bmatrix}' \begin{bmatrix} \mathbb{E}_q[V_{t+1}(n-1,j')] \\ \mathbb{E}_q[V_{t+1}(n,j')] \\ \mathbb{E}_q[V_{t+1}(n+1,j')] \end{bmatrix} - \frac{1}{1+r_{t+1}} \sum_{j'=\{f,s\}} p_t(n+1,j',q|n,s)(1-\theta)\mathbb{E}_q[\Delta V_{t+1}(n,j')], \tag{B.4}$$

where the last term denotes the fraction of the firm's value seized by the creditor after a successful innovation. Finally, for a slow firm with success probability $q < q_t^*$, such that it cannot innovate this period, the value function is:

$$\underline{V}_t(n,s) = \pi n Y_t + \frac{1}{1+r_{t+1}} \sum_{j'=\{f,s\}} \begin{bmatrix} \phi_s(j') x_t n \\ \phi_s(j')(1-x_t n) \\ 0 \end{bmatrix}' \begin{bmatrix} \mathbb{E}_q[V_{t+1}(n-1,j')] \\ \mathbb{E}_q[V_{t+1}(n,j')] \\ \mathbb{E}_q[V_{t+1}(n+1,j')] \end{bmatrix}. \tag{B.5}$$

Adding up across realizations of q, $V_t(n,s) = \mathcal{Q}(q_t^*)\underline{V}_t(n,s) + \int_{q_t^*}^1 V_t(n,s,q)d\mathcal{Q}(q)$, the total value of slow firms of size n is:

$$\begin{aligned} V_t(n,s) = \pi n Y_t + \frac{1}{1+r_{t+1}} \sum_{j'=\{f,s\}} \begin{bmatrix} p_t(n-1,j'|n,s) \\ p_t(n,j'|n,s) \\ p_t(n+1,j'|n,s) \end{bmatrix}' \begin{bmatrix} \mathbb{E}_q[V_{t+1}(n-1,j')] \\ \mathbb{E}_q[V_{t+1}(n,j')] \\ \mathbb{E}_q[V_{t+1}(n+1,j')] \end{bmatrix} \\ - \frac{\underline{q}(q_t^*)zn}{1+r_{t+1}} \sum_{j'=\{f,s\}} \phi_s(j')(1-\theta)\mathbb{E}[\Delta V_{t+1}(n,j')], \end{aligned} \tag{B.6}$$

where we used that the sum over transition probabilities is $p_t(n,j'|n,s) \equiv \mathcal{Q}(q_t^*)\phi_s(j')(1-x_t n) + \int_{q_t^*}^1 p_t(n,j',q|s)d\mathcal{Q}(q) = \phi_s(j')[1-x_t n - \underline{q}(q_t^*)zn]$, and $p_t(n+1,j'|n,s) \equiv \phi_s(j')\underline{q}(q_t^*)zn$, as well as trivially $p_t(n-1,j'|n,s) \equiv \phi_s(j')x_t n$.

B.1.5 Aggregate value

Given the distribution $\mu_t(n,j)$ across product lines and project types, the aggregate equity value of firms is

$$V_t = \sum_{n=1}^{\infty} \sum_{j\in\{f,s\}} \mu_t(n,j)V_t(n,j), \tag{B.7}$$

with the normalization $\sum_{n=1}^{\infty}\sum_{j\in\{f,s\}} \mu_t(n,j)n = 1$. Adding up (B.3) and (B.6):

$$\begin{aligned} V_t = \pi Y_t + \frac{1}{1+r_{t+1}} \sum_n \sum_j \sum_{n'} \sum_{j'} \mu_t(n,j)p_t(n',j'|n,j)\mathbb{E}_q[V_{t+1}(n',j')] \\ - \sum_n \frac{\kappa z n}{1+r_{t+1}} \mu_t(n,f)\tilde{R}_t(n)\zeta z n w_t \\ - \sum_n \sum_{j'} \frac{\underline{q}(q_t^*)zn}{1+r_{t+1}} \mu_t(n,s)\phi_s(j')(1-\theta)\mathbb{E}[\Delta V_{t+1}(n,j')], \end{aligned} \tag{B.8}$$

From the law of motion for the joint distribution, we know that $\sum_n \sum_j \mu_t(n,j)p_t(n',j'|n,j) = \mu_{t+1}(n',j_t'^e[\alpha \mathbb{1}(n'=1,j'=s) + (1-\alpha)\mathbb{1}(n'=1,j'=f)]$, where the last term captures creative destruction through entry. Rewriting the second term in (B.8):

$$\begin{aligned}&\sum_n \sum_j \sum_{n'} \sum_{j'} \mu_t(n,j) p_t(n',j'|n,j) \mathbb{E}_q[V_{t+1}(n',j')] \\ &= \sum_{n'} \sum_{j'} \mu_{t+1}(n',j') \mathbb{E}_q[V_{t+1}(n',j')] - z^e \Big(\alpha \mathbb{E}_q[V_{t+1}(1,s)] + (1-\alpha) \mathbb{E}_q[V_{t+1}(1,f)] \Big).\end{aligned} \tag{B.9}$$

Using the definition for the cumulative value in (B.7), equation (B.8) becomes:

$$\begin{aligned}V_t = \pi Y_t + \frac{1}{1+r_{t+1}} V_{t+1} - z^e \left(\frac{\alpha \mathbb{E}_q[V_{t+1}(1,s)] + (1-\alpha) \mathbb{E}_q[V_{t+1}(1,f)]}{1+r_{t+1}} \right) \\ - \sum_n \frac{\kappa z n}{1+r_{t+1}} \mu_t(n,f) \tilde{R}_t(n) \zeta z n w_t \\ - \sum_n \sum_{j'} \frac{\underline{q}(q_t^*) z n}{1+r_{t+1}} \mu_t(n,s) \phi_s(j')(1-\theta) \mathbb{E}_q[\Delta V_{t+1}(n,j')].\end{aligned} \tag{B.10}$$

B.2 Firm size distribution

B.2.1 Transitions

There are two states, product lines n and firm types, fast and slow $j \in \{f, s\}$. The joint distribution $\mu(n,j)$ over product lines and types satisfies:

$$\sum_{n=1}^{\infty} \sum_{j \in \{f,s\}} \mu(n,j) n = 1. \tag{B.11}$$

Firms transition between product lines and types. While firms move endogenously between product lines, type transitions are exogenous and follow the transition matrix in (22). For shorthand we write $\phi_s \equiv \phi_s(s)$ and $\phi_f = \phi_f(f)$.

B.2.2 Timing

We adopt the following convention: $\mu(n,j)$ *is the distribution of product lines and types at the end of each period after types have been re-assigned.* This is equivalent to saying $\mu(n,j)$ is the distribution of types before innovation in any given period has been realized. $\mu_-(n,j)$ is the distribution before types are re-assigned. We define the short-hand $\underline{q} \equiv \underline{q}(q^*)$ as per (4).

We need to distinguish inflow-outflow equations for $n > 1$ and $n < 1$ to account for entry. On the balance growth path and for $n > 1$ and $j = s$:

$$
\begin{aligned}
\mu_-(n,s)\left[(1-\phi_s)+\phi_s\left(nz\underline{q}+nx\right)\right] = {} & \mu_-(n,f)(1-\phi_f)\left(1-nx-nz\kappa\right) \\
& +\mu_-(n-1,s)(n-1)z\underline{q}\phi_s \\
& +\mu_-(n-1,f)(n-1)z\kappa(1-\phi_f) \\
& +\mu_-(n+1,s)(n+1)x\phi_s \\
& +\mu_-(n+1,f)(n+1)x(1-\phi_f).
\end{aligned}
\tag{B.12}
$$

And similarly for $n > 2$ and $j = f$:

$$
\begin{aligned}
\mu_-(n,f)\left[(1-\phi_f)+\phi_f\left(nz\kappa+nx\right)\right] = {} & \mu_-(n,s)(1-\phi_s)\left(1-nx-nz\underline{q}\right) \\
& +\mu_-(n-1,f)(n-1)z\kappa\phi_f \\
& +\mu_-(n-1,s)(n-1)z\underline{q}(1-\phi_s) \\
& +\mu_-(n+1,f)(n+1)x\phi_f \\
& +\mu_-(n+1,s)(n+1)x(1-\phi_s).
\end{aligned}
\tag{B.13}
$$

For $n < 2$ we need to take entry into account. We assume that a share α of entrants is bad, and a share $1-\alpha$ of entrants is good. For $n = 1$ and $j = s$ we have

$$
\begin{aligned}
\mu_-(1,s)\left[1-\phi_s+\phi_s(x+\underline{q}z)\right] = {} & \mu_-(1,f)(1-\phi_f)(1-x-\kappa z) \\
& +\alpha z^e+\mu_-(2,s)2x\phi_s+\mu_-(2,f)2x(1-\phi_f),
\end{aligned}
\tag{B.14}
$$

and similarly for $n = 1$ and $j = f$:

$$
\begin{aligned}
\mu_-(1,f)\left[1-\phi_f+\phi_f(x+\kappa z)\right] = {} & \mu_-(1,s)(1-\phi_s)(1-x-z\underline{q}) \\
& +(1-\alpha)z^e+\mu_-(2,f)2x\phi_f+\mu_-(2,s)2x(1-\phi_s).
\end{aligned}
\tag{B.15}
$$

Finally, for the mass of firms and the share of each type to remain stationary as defined in (B.11), all firms that are destroyed need to be replaced by entrants $x\mu_-(1,s) = \alpha z^e$ and $x\mu_-(1,f) = (1-\alpha)z^e$.

B.3 Market clearing

B.3.1 Market clearing conditions

The full set of market clearing conditions for intermediate goods, credit, equity, production labor, research labor, and output is given by:

$$\frac{Y_t}{p_{i,t}} = \frac{A_{i,t}Y_t}{\lambda w_t^P} \qquad \text{for all } i \in [0,1], \tag{B.16.1}$$

$$\begin{aligned}\sum_{n=1}^{\infty} \tfrac{\kappa z n \tilde{R}_t(n)}{1+r_{t+1}} \mu_t(n,f)\zeta z n w_t + \sum_{n=1}^{\infty} \tfrac{q(q_t^*)zn}{1+r_{t+1}} \mu_t(n,s) \sum_{j'} \phi_s(j')(1-\theta)\mathbb{E}_q[\Delta V_{t+1}(n)] \\ = \textstyle\sum_{n=1}^{\infty} \Big(\mu_t(n,f)\zeta + \mu_t(n,f)(\zeta+\delta)[1-\mathcal{Q}_t^*] \Big) z n w_t,\end{aligned} \tag{B.16.2}$$

$$B_t = \frac{1}{1+r_t} \sum_{n=1}^{\infty} \sum_{j\in\{f,s\}} V_t(n,j)\mu_t(n,j), \tag{B.16.3}$$

$$L^P = \int_0^1 \left(\frac{y_{i,t}}{A_{i,t}} \right) di, \tag{B.16.4}$$

$$L^R = \psi z_t^e + \sum_{n=1}^{\infty} \Big(\zeta z \mu_t(n,f) n \Big) + \sum_{n=1}^{\infty} \Big(z[1-\mathcal{Q}(q_t^*)](\zeta+\delta)\mu_t(n,s)n \Big), \tag{B.16.5}$$

$$C_t = Y_t. \tag{B.16.6}$$

B.3.2 Walras' law

Using the creditor's balance sheet (B.16.2) we can replace the last two terms in (B.10) with the labor market clearing condition in (26). Firm values are:

$$\begin{aligned} V_t = \pi Y_t + \frac{1}{1+r_{t+1}} V_{t+1} - z^e \left(\frac{\alpha \mathbb{E}_q[V_{t+1}(1,s)] + (1-\alpha)\mathbb{E}_q[V_{t+1}(1,f)]}{1+r_{t+1}} \right) \\ - \left(L^R - \psi z_t^e \right) w_t. \end{aligned} \tag{B.17}$$

From the free-entry condition (25), it follows that $V_t = \pi Y_t - L^R w_t + \frac{1}{1+r_{t+1}} V_{t+1}$. On the balanced growth path, the equity value of firms, consumption, wages and output all grow at a common and constant growth rate g_t. Using the Euler equation $1 + g_{t+1} = \beta(1 + r_{t+1})$ we can write:

$$(1-\beta)V_t = \pi Y_t - L^R w_t. \tag{B.18}$$

The household's budget constraint is $B_{t+1} = (1+r_t)B_t + w_t L^R + w_t^P L^P - C_t$. From equity market clearing (B.16.3) $V_t = (1+r_t)B_t$ such that $(g_t - r_t)/(1+r_t)V_t = (\beta - 1)V_t = w_t L^R + w_t^P L^P - C_t$. Substituting (B.18) yields $C_t = \pi Y_t + w_t^P L_t^P$, and via the factor share identity (A.3), the market clearing condition for the final good (B.16.6), $C_t = Y_t$.

B.4 General distribution function

We discuss how the main comparative statics result in Section 2.4 generalize to an arbitrary distribution function $\mathcal{Q}(q)$ with density $\mathcal{Q}'(q) > 0$ on its entire support $[0, 1]$. We then repeat the analysis for the special case of a modified beta distribution, which we use in the quantitative model in Section 3.

B.4.1 Entry

Differentiating the entry rate (8) with respect to the cost of refinancing:

$$\frac{dz^e}{d\delta} = \frac{\alpha(\zeta+\delta)z}{\psi}\mathcal{Q}'(q^*)\frac{dq^*}{d\delta} - \frac{\alpha z}{\psi}\left[1 - \mathcal{Q}(q^*)\right]. \tag{B.19}$$

The first term is the selection effect of a decrease in labor demand due to a higher refinancing threshold. The second term is the resource cost effect of an increase in labor demand coming from higher refinancing costs for all slow firms that continue to be refinanced. The entry effect is positive if:

$$(\zeta+\delta)\frac{dq^*}{d\delta} > \frac{1-\mathcal{Q}(q^*)}{\mathcal{Q}'(q^*)}. \tag{B.20}$$

The term on the left-hand side of (B.20) captures the decrease in labor demand due to a higher threshold q^*. The term on the right-hand side is the hazard rate, that is, by how much refinancing costs increase along refinanced firms. The effect on entry is positive whenever the refinancing threshold is sufficiently elastic, or when there is not "too much mass" to the right of the cutoff q^*. Noting that $q^* = \delta/[(1-\theta)\psi]$ we can re-write (B.20) as

$$\zeta + (1-\theta)\psi\phi(q^*) > 0, \tag{B.21}$$

where $\phi(q^*) \equiv q^* - [1 - \mathcal{Q}(q^*)]/\mathcal{Q}'^*)$ is the virtual valuation (Myerson, 1981). Similar to basic auction theory, one can think of $\phi(q^*)$ as an adjusted measure of labor demand close to the threshold q^*, taking into account the characteristics of the distribution function $\mathcal{Q}(q)$.

For the second order properties, differentiating (B.19) again with respect to δ yields:

$$\frac{d^2 z^e}{d\delta^2} = \frac{2\alpha z}{(1-\theta)\psi^2}\mathcal{Q}'(q^*) + \frac{\alpha(\zeta+\delta)z}{(1-\theta)^2\psi^3}\mathcal{Q}''(q^*). \tag{B.22}$$

Entry is strictly convex in the refinancing cost δ when $\mathcal{Q}''(q^*) > 0$.

B.4.2 Growth

The creative destruction rate is $x = z^e + (1-\alpha)\kappa z + \alpha \underline{q}(q^*)z$. Differentiate the creative destruction rate (9) with respect to the cost of refinancing:

$$\frac{dx}{d\delta} = \frac{dz^e}{d\delta} - \alpha z \mathcal{Q}'^*)q^* \frac{dq^*}{d\delta}, \tag{B.23}$$

where we used Leibniz' rule $\frac{\partial \underline{q}(q^*)}{\partial \delta} = \frac{\partial}{\partial \delta}\int_{q^*}^1 q d\mathcal{Q}(q) = -\mathcal{Q}'(q^*)q^*\frac{\partial q^*}{\partial \delta}$. Substituting (B.19), re-arranging, and using the definition of $\phi(q)$ gives:

$$\frac{dx}{d\delta} = \frac{\alpha z \mathcal{Q}'(q^*)}{\psi}\left[(\zeta - \psi q^*)\frac{dq^*}{d\delta} + \phi(q^*)\right]. \tag{B.24}$$

The growth rate is increasing if:

$$\phi(q^*) > \frac{\psi q^* - \zeta}{(1-\theta)\psi}. \tag{B.25}$$

The left-hand side is the increase in growth due to higher entry rates. The right-hand side is the fall in growth because of less incumbent innovation. To generate a hump-shaped relationship between growth and the cost of refinancing we require: $\phi(q^*) <$

$(\psi q^* - \zeta)/[(1-\theta)\psi]$ for low values of δ and $\phi(q^*) > (\psi q^* - \zeta)/[(1-\theta)\psi]$ for high values of δ. For curvature differentiate the above with respect to δ:

$$\begin{aligned} \frac{d^2x}{d\delta^2} &= \frac{\alpha z Q'(q^*)}{\psi}\left[-\psi\left(\frac{dq^*}{d\delta}\right)^2 + \phi'(q^*)\left(\frac{dq^*}{d\delta}\right)\right] + \frac{\alpha z}{\psi}\cdot\frac{Q''(q^*)}{Q'(q^*)}\cdot\frac{dx}{d\delta}\cdot\frac{dq^*}{d\delta} \\ &= \frac{\alpha z \mathcal{Q}'(q^*)}{(1-\theta)\psi^2}\left(\frac{\mathcal{Q}''(q^*)}{\mathcal{Q}'(q^*)}\left[\frac{\zeta-\psi q^*}{(1-\theta)\psi} + \phi(q^*)\right] + \left[\phi'(q^*) - \frac{1}{1-\theta}\right]\right). \end{aligned} \tag{B.26}$$

Given $\mathcal{Q}'(q) > 0$, the growth rate is concave if:

$$\frac{\mathcal{Q}''(q^*)}{\mathcal{Q}'(q^*)}\left[\frac{\zeta-\psi q^*}{(1-\theta)\psi} + \phi(q^*)\right] < \frac{1}{1-\theta} - \phi'(q^*). \tag{B.27}$$

B.4.3 Modified beta distribution

Consider the distribution $\mathcal{Q}(q) = 1-(1-q)^{1/\xi}$ with density $\mathcal{Q}'(q) = (1/\xi)(1-q)^{1/\xi-1}$, second derivative $\mathcal{Q}''(q) = \left(1-\xi^{-1}\right)\xi^{-1}(1-q)^{1/\xi-2}$ and scale parameter $1/\xi > 0$. We also note that $Q''(q)/Q'(q) = (\xi-1)/[\xi(1-q)]$. The expected value is:

$$E[q] = \int_0^1 \frac{q}{\xi}(1-q)^{\frac{1}{\xi}-1}\,dq = \frac{\xi}{1+\xi}. \tag{B.28}$$

Setting $\xi = 1$ should return us to the uniform distribution. The virtual valuation is:

$$\phi(q) = q - \frac{\xi(1-q)^{1/\xi}}{(1-q)^{1/\xi-1}} = q - \xi(1-q) \qquad \text{and} \qquad \phi'(q^*) = 1+\xi. \tag{B.29}$$

The creative destruction rate's first derivative with respect to δ is zero if:

$$\delta = (1-\theta)\frac{\zeta - \xi(1-\theta)\psi}{1-(1+\xi)(1-\theta)}. \tag{B.30}$$

Setting $\xi = 1$ returns us to the result in Proposition 1. Smaller values of ξ allow us to rescale the expected payoff for the creditor and ensure $\delta > 0$. The growth rate is concave if:

$$\frac{\xi-1}{\xi(1-q^*)}\left[\frac{\zeta-\psi q^*}{(1-\theta)\psi} + (1+\xi)q^* - \xi\right] < \frac{1}{1-\theta} - (1+\xi),$$

which is a linear equation in q^*. Solving:

$$\frac{1}{1-\theta}\left(\frac{(\xi-1)\zeta}{\psi}-1\right)+2\xi<\frac{1}{\xi}\left(1+\xi-\frac{1}{1-\theta}\right)q^*. \tag{B.31}$$

We can check that for $\xi=1$ the condition becomes $(1-q^*)(1-2\theta)/(1-\theta)<0$, which is the same as in Proposition 1.

C Decomposition

C.1 Direct versus indirect effects

C.1.1 Entry innovation

Firm entry is:

$$z^e=\frac{L^R}{\psi}-\frac{1}{\psi}\sum_{n=1}^{\infty}\Big(\zeta zn\mu(n,f)+[1-\mathcal{Q}(q^*)]\,(\zeta+\delta)\,zn\mu(n,s)\Big). \tag{C.32}$$

Let tildes denote objects in the new steady state and Δ the difference of any two variables between steady state, for instance, $\Delta z^e\equiv\tilde{z}^e-z^e$ is the difference in entry rates between steady states and so on. For the entry rate we have:

$$\begin{aligned}\Delta z^e=&\frac{1}{\psi}\sum_{n=1}^{\infty}\zeta zn\mu(n,f)+\frac{1}{\psi}\sum_{n=1}^{\infty}[1-\mathcal{Q}(q^*)]\,(\zeta+\delta)\,zn\mu(n,s)\\&-\frac{1}{\psi}\sum_{n=1}^{\infty}\zeta zn\tilde{\mu}(n,f)-\frac{1}{\psi}\sum_{n=1}^{\infty}[1-\mathcal{Q}(\tilde{q}^*)]\,\big(\zeta+\tilde{\delta}\big)\,zn\tilde{\mu}(n,s).\end{aligned} \tag{C.33}$$

Using the definition $\Delta\mu(n,f)\equiv\tilde{\mu}(n,f)-\mu(n,f)$ from above and expanding the right-hand side of the equality in (C.33) by adding and subtracting $\frac{1}{\psi}\sum_{n=1}^{\infty}[1-\mathcal{Q}(\tilde{q}^*)](\zeta+\delta)zn\mu(n,s)$, we can write:

$$\begin{aligned}\Delta z^e=&-\frac{1}{\psi}\sum_{n=1}^{\infty}\zeta zn\Delta\mu(n,f)+\frac{1}{\psi}\sum_{n=1}^{\infty}[1-\mathcal{Q}(q^*)]\,(\zeta+\delta)\,zn\mu(n,s)-\frac{1}{\psi}\sum_{n=1}^{\infty}[1-\mathcal{Q}(\tilde{q}^*)]\,\big(\zeta+\tilde{\delta}\big)\,zn\tilde{\mu}(n,s)\\&+\frac{1}{\psi}\left(\sum_{n=1}^{\infty}[1-\mathcal{Q}(\tilde{q}^*)]\,(\zeta+\delta)\,zn\mu(n,s)-\sum_{n=1}^{\infty}[1-\mathcal{Q}(\tilde{q}^*)](\zeta+\delta)zn\mu(n,s)\right).\end{aligned} \tag{C.34}$$

Collecting terms again, then expanding by $\frac{1}{\psi}\sum_{n=1}^{\infty}[1-\mathcal{Q}(\tilde{q}^*)]\left(\zeta+\tilde{\delta}\right)zn\mu(n,s)$, yields:

$$\begin{aligned}\Delta z^e = &-\frac{1}{\psi}\sum_{n=1}^{\infty}\zeta zn\Delta\mu(n,f)+\frac{1}{\psi}\sum_{n=1}^{\infty}\Delta\mathcal{Q}(q^*)\,(\zeta+\delta)\,zn\mu(n,s)\\ &-\frac{1}{\psi}\sum_{n=1}^{\infty}[1-\mathcal{Q}(\tilde{q}^*)]\left(\zeta+\tilde{\delta}\right)zn\tilde{\mu}(n,s)+\frac{1}{\psi}\sum_{n=1}^{\infty}[1-\mathcal{Q}(\tilde{q}^*)]\,(\zeta+\delta)\,zn\mu(n,s)\\ &+\frac{1}{\psi}\left(\sum_{n=1}^{\infty}[1-\mathcal{Q}(\tilde{q}^*)]\left(\zeta+\tilde{\delta}\right)zn\mu(n,s)-\sum_{n=1}^{\infty}[1-\mathcal{Q}(\tilde{q}^*)]\left(\zeta+\tilde{\delta}\right)zn\mu(n,s)\right).\end{aligned} \tag{C.35}$$

Finally, collecting the remaining terms:

$$\begin{aligned}\Delta z^e = &-\frac{1}{\psi}\sum_{n=1}^{\infty}\zeta zn\Delta\mu(n,f)+\frac{1}{\psi}\sum_{n=1}^{\infty}\Delta\mathcal{Q}(q^*)\,(\zeta+\delta)\,zn\mu(n,s)\\ &-\frac{1}{\psi}\sum_{n=1}^{\infty}[1-\mathcal{Q}(\tilde{q}^*)]\Delta\delta zn\mu(n,s)-\frac{1}{\psi}\sum_{n=1}^{\infty}[1-\mathcal{Q}(\tilde{q}^*)]\left(\zeta+\tilde{\delta}\right)zn\Delta\mu(n,s),\end{aligned} \tag{C.36}$$

where $\Delta\mathcal{Q}(q^*)\equiv\mathcal{Q}(\tilde{q}^*)-\mathcal{Q}(q^*)$.

We can split the above expression into three components: A direct cost effect through changes in δ, a selection effect through changes in the cutoff q^*, and an indirect effect through the distribution:

$$\begin{aligned}\Delta z^e = &-\frac{1}{\psi}\sum_{n=1}^{\infty}[1-\mathcal{Q}(\tilde{q}^*)]\Delta\delta zn\mu(n,s)+\frac{1}{\psi}\sum_{n=1}^{\infty}\Delta\mathcal{Q}(q^*)\,(\zeta+\delta)\,zn\mu(n,s)\\ &-\frac{1}{\psi}\sum_{n=1}^{\infty}\left(\zeta zn\Delta\mu(n,f)+[1-\mathcal{Q}(\tilde{q}^*)]\left(\zeta+\tilde{\delta}\right)zn\Delta\mu(n,s)\right).\end{aligned} \tag{C.37}$$

C.1.2 Incumbent innovation

The aggregate creative destruction rate is $x=z^e+z^i$ as defined in (27), where z^i denotes incumbent innovation $z^i=\sum_{n=1}^{\infty}\left(\kappa zn\mu(n,f)+\underline{q}(q^*)zn\mu(n,s)\right)$ such that $\Delta z^i\equiv\tilde{z}^i-z^i$ is:

$$\Delta z^i=\sum_{n=1}^{\infty}\kappa zn\tilde{\mu}(n,f)+\sum_{n=1}^{\infty}\underline{q}(\tilde{q}^*)zn\tilde{\mu}(n,s)-\sum_{n=1}^{\infty}\kappa zn\mu(n,f)-\sum_{n=1}^{\infty}\underline{q}(q^*)zn\mu(n,s). \tag{C.38}$$

Expanding by $\sum_{n=1}^{\infty} \underline{q}(\tilde{q}^*) zn\mu(n,s)$ and collecting terms yields:

$$\begin{aligned} \Delta z^i = & \sum_{n=1}^{\infty} \kappa zn\Delta\mu(n,f) + \sum_{n=1}^{\infty} \underline{q}(\tilde{q}^*) zn\tilde{\mu}(n,s) - \sum_{n=1}^{\infty} \underline{q}(q^*) zn\mu(n,s) \\ & + \left(\sum_{n=1}^{\infty} \underline{q}(\tilde{q}^*) zn\mu(n,s) - \sum_{n=1}^{\infty} \underline{q}(\tilde{q}^*) zn\mu(n,s) \right). \end{aligned} \tag{C.39}$$

Finally, we have:

$$\Delta z^i = \sum_{n=1}^{\infty} \Delta \underline{q}(q^*) zn\mu(n,s) + \sum_{n=1}^{\infty} \Big(\kappa zn\Delta\mu(n,f) + \underline{q}(\tilde{q}^*) zn\Delta\mu(n,s) \Big). \tag{C.40}$$

C.1.3 Creative destruction

Combining (C.37) and (C.40) yields an expression for the steady-state-on-steady-state change in the creative destruction rate $\Delta x = \Delta z^e + \Delta z^i$:

$$\begin{aligned} \Delta x = & \sum_{n=1}^{\infty} \kappa zn\Delta\mu(n,f) + \sum_{n=1}^{\infty} \Delta \underline{q}(q^*) zn\mu(n,s) + \sum_{n=1}^{\infty} \underline{q}(\tilde{q}^*) zn\Delta\mu(n,s) \\ & - \frac{1}{\psi} \sum_{n=1}^{\infty} [1 - \mathcal{Q}(\tilde{q}^*)] \Delta\delta zn\mu(n,s) + \frac{1}{\psi} \sum_{n=1}^{\infty} \Delta \mathcal{Q}(q^*) \left(\zeta + \delta\right) zn\mu(n,s) \\ & - \frac{1}{\psi} \sum_{n=1}^{\infty} \left(\zeta zn\Delta\mu(n,f) + [1 - \mathcal{Q}(\tilde{q}^*)] \left(\zeta + \tilde{\delta}\right) zn\Delta\mu(n,s) \right). \end{aligned} \tag{C.41}$$

Grouping terms along direct and indirect effects, as well as changes in the distribution, we obtain:

$$\begin{aligned} \Delta x = & \sum_{n=1}^{\infty} \left[\Delta \underline{q}(\tilde{q}^*) + \Delta \mathcal{Q}(q^*) \frac{\zeta + \delta}{\psi} \right] zn\mu(n,s) - \frac{1}{\psi} \sum_{n=1}^{\infty} [1 - \mathcal{Q}(\tilde{q}^*)] \Delta\delta zn\mu(n,s) \\ & + \sum_{n=1}^{\infty} \left[\left(\kappa - \frac{\zeta}{\psi} \right) zn\Delta\mu(n,f) + [1 - \mathcal{Q}(\tilde{q}^*)] \left(\frac{\underline{q}(\tilde{q}^*)}{1 - \mathcal{Q}(\tilde{q}^*)} - \frac{\zeta + \tilde{\delta}}{\psi} \right) zn\Delta\mu(n,s) \right]. \end{aligned} \tag{C.42}$$

The first line contains the direct effect of changes in the threshold and the indirect effect on entry due to a higher cost of refinancing δ. The second line contains the indirect, compositional effect on creative destruction through the firm size distribution.

C.2 Within versus between effects

Denoting the new distribution by $\tilde{\mu}(n,j)$, and the change in the distribution as $\Delta\mu(n,j) \equiv \tilde{\mu}(n,j) - \mu(n,j)$, we have:

$$\begin{aligned}\Delta\mu(n,j) = & \; [\mu(n)\tilde{\mu}(j|n) - \mu(n,j)] \\ & + [\tilde{\mu}(n)\mu(j|n) - \mu(n)\tilde{\mu}(j|n)] \\ & + [\tilde{\mu}(n,j) - \tilde{\mu}(n)\mu(j|n)] \,.\end{aligned} \tag{C.43}$$

The first term captures the effect of changes in types j given fixed firm size n. The second term captures the effect of changes in firm size n given fixed types j. The last term captures interaction effect.

The conditional distributions $\mu(j|n)$ and $\tilde{\mu}(j|n)$ are given by:

$$\mu(j|n) = \frac{\mu(n,j)}{\mu(n)} \qquad \text{and} \qquad \tilde{\mu}(j|n) = \frac{\tilde{\mu}(n,j)}{\tilde{\mu}(n)}. \tag{C.44}$$

Finally, to account for different size bins, we define weighted versions of the within, between and interaction effects as follows:

$$\begin{aligned} WE^{\Omega}(j) &= \frac{\sum_{n=1}^{\infty} \Omega_n \cdot WE(n,j)}{\sum_{n=1}^{\infty} \Omega_n \cdot TE(n,j)} = \frac{\sum_{n=1}^{\infty} \Omega_n \, [\mu(n)\tilde{\mu}(j|n) - \mu(n,j)]}{\sum_{n=1}^{\infty} \Omega_n \, [\tilde{\mu}(n,j) - \mu(n,j)]}, \\ BE^{\Omega}(j) &= \frac{\sum_{n=1}^{\infty} \Omega_n \cdot BE(n,j)}{\sum_{n=1}^{\infty} \Omega_n \cdot TE(n,j)} = \frac{\sum_{n=1}^{\infty} \Omega_n \, [\tilde{\mu}(n)\mu(j|n) - \mu(n)\tilde{\mu}(j|n)]}{\sum_{n=1}^{\infty} \Omega_n \, [\tilde{\mu}(n,j) - \mu(n,j)]}, \\ IE^{\Omega}(j) &= \frac{\sum_{n=1}^{\infty} \Omega_n \cdot BE(n,j)}{\sum_{n=1}^{\infty} \Omega_n \cdot TE(n,j)} = \frac{\sum_{n=1}^{\infty} \Omega_n \, [\tilde{\mu}(n,j) - \tilde{\mu}(n)\mu(j|n)]}{\sum_{n=1}^{\infty} \Omega_n \, [\tilde{\mu}(n,j) - \mu(n,j)]}, \end{aligned} \tag{C.45}$$

where we use $\Omega_n = \{1, n\}$ as weights to obtain equally and size-weighted averages.