Monetary Policy in Open Economies with Production Networks*

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Abstract

This paper studies the design of monetary policy in small open economies with domestic and cross-border production networks and nominal rigidities. The monetary policy that closes the domestic output gap is nearly optimal and is implemented by stabilizing the aggregate inflation index that weights sectoral inflation according to the sector's roles as a supplier of inputs and a net exporter of products within the international production networks. To close the output gap, monetary policy should assign large weights to inflation in sectors with small direct or indirect (i.e., via the downstream sectors) import shares and failing to account for the cross-border production networks *overemphasizes* inflation in sectors that export intensively directly and indirectly (i.e., via the downstream sectors). We validate our theoretical results using the World Input-Output Database and show that the monetary policy that closes the output gap outperforms alternative policies that abstract from the openness of the economy or the input-output linkages.

Keywords: Production networks, small open economy, monetary policy.

JEL Classification: C67, E52, F41.

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1 Introduction

Modern production revolves around intricate input-output relations within domestic firms and between domestic and foreign firms, and the position and import-export intensity of each domestic firm along the production networks are critical for an economy's response to shocks and the efficacy of stabilizing economic policies. Disruptions to the international input-output linkages during trade tensions between China and the US since the "China Section 301-Tariff Actions" in 2018 and the COVID-19 pandemic exemplify the primal role of international input-output linkages for the changes in output and prices and the stance of monetary policy.[1]

Yet, there is no systematic research focused on the design of monetary policy in open economies with both domestic and cross-border input-output relations—despite two separate strands of literature providing distinct insights on the issue. On the one hand, in a one-sector open economy model with nominal price rigidities and without input-output relations, the optimal monetary policy trades off the distortions from domestic inflation and from the terms of trade. On the other hand, in a multi-sector closed economy with domestic input-output linkages, the monetary policy should target a weighted average of sectoral inflation with the weights proportional to Domar weights (i.e., the sectoral-sales-to-GDP ratio) to account for the propagation of sectoral distortions along input-output linkages.

In light of these separate findings, it remains unknown what would be the policy prescription for a monetary authority that operates in an open, multi-sector economy with cross-border linkages between domestic and foreign firms in addition to domestic input-output linkages. Our paper sheds light on this outstanding issue by addressing the following two questions: (i) What is the role of domestic and cross-border input-output linkages in the design of monetary policy? (ii) What are the pitfalls of monetary policy that disregards cross-border input-output relations?

We study these issues by developing a multi-sector, small open economy model with production networks between domestic and foreign sectors that are subject to nominal rigidities. Our model combines the one-sector open economy framework in Galí and Monacelli (2005) with the production network framework similar to Ghassibe (2021b), La'O and Tahbaz-Salehi (2022), and Rubbo (2023).

In our multi-sector economy with nominal rigidities, inflation in the different sectors generates sectoral markup wedges that encapsulate the sectoral distortions that prevent the attainment of efficient allocations in the flexible-price equilibrium. The domestic and cross-border input-output linkages further propagate these sectoral distortions throughout the economy, resulting in aggregate distortions. Following the business cycle accounting approach in Chari et al. (2007), we use efficiency and labor wedges to characterize the aggregation of sectoral shocks and distortions in our economy. The efficiency wedge is a weighted average of exogenous sectoral shocks and is independent of sectoral markup wedges up to the first-order approximation. In contrast,

[1] See Auray et al. (2024) and Bai et al. (2024, 2025) for discussions on the impact of trade barriers and Covid-19 on output and monetary policy.

the labor wedge is a weighted average of sectoral markup wedges and, therefore, is relevant to monetary policy aimed at eliminating aggregate distortion. In particular, the labor wedge is proportional to the aggregate output gap (OG)—defined as the difference between the aggregate output in the sticky price and in the efficient, flexible price equilibria. Thus, the monetary policy that closes the aggregate output gap eliminates the first-order aggregate distortions in the open economy with production networks. We refer to this policy as the output gap (OG) monetary policy.

To close the aggregate output gap, the OG monetary policy stabilizes the aggregate inflation index that is proportional to the aggregate output gap by weighting the inflation of different sectors. The weight assigned to the inflation of each sector is the product of two components: (i) the sector's price rigidity that maps positive sectoral inflation into the negative sectoral markup wedge under nominal rigidities, similar to Rubbo (2023),[2] and (ii) the sector's OG weight that measures the contribution of the sectoral markup wedge to the aggregate output gap, which crucially depends on the interplay of domestic and cross-border input-output linkages. The size of the sectoral OG weight is determined by three channels that rely on distinct roles of the sector for the aggregate output in the network economy: (i) the *Consumer Price Index* (*CPI*), (ii) the *net export income*, and (iii) the *net profit income* channels. While the *CPI channel* is also present in closed economies, the *net-export* and *net-profit income channels are unique* to open economies.

In the *CPI channel*, a negative sectoral markup wedge leads to lower CPI than in the efficient, flexible-price equilibrium, which raises the real prices and, thereby, the supply of factors, hence generating a positive aggregate output gap. In the *net export income channel*, a negative sectoral markup wedge leads to lower domestic prices than in the efficient equilibrium, which increases net exports and, thus, the domestic labor income, generating a positive aggregate output gap. In the *net profit income channel*, a negative sectoral markup wedge has two opposite effects on the aggregate output gap: (i) one leading to lower domestic prices that increase the net profit income from increased exports, and (ii) the other increasing the costs of imported foreign inputs that reduce the net profit income.

The sizes of the three foregoing channels are determined by the different roles of the sector in the open-economy input-output network as a *supplier* of inputs and a *net exporter*. Because the CPI is the price of aggregate output, the size of the *CPI channel* is determined by the sector's direct and indirect (via the *downstream* sectors) contribution to domestic aggregate output as a supplier of inputs—which we measure using *domestic supplier centrality*. The size of the *net export income channel* is determined by the sector's direct and indirect (via the *downstream* sectors) contribution to the net exports and the resulting contribution to domestic labor income—which we measure using *net export centrality*. Finally, the *net profit income channel* is absent in closed economies, and its size is tiny in open economies, as will be shown in numerical simulations of our model.

[2]Under nominal rigidities, sticky-price firms cannot raise their prices in response to positive inflation in marginal costs, thereby generating a lower sectoral markup in the sticky price than in the efficient, flexible price equilibria.

Our model nests the closed economy framework with the production networks á la La'O and Tahbaz-Salehi (2022) and Rubbo (2023). Specifically, the OG weight is equivalent to the Domar weight, as the *net export and profit income channels* are absent, and the *domestic supplier centrality* encapsulating the *CPI channel* is equivalent to the Domar weight in the closed economy.

Using our small open economy model with production networks, we initially focus on our first research question regarding what is the role of domestic and cross-border input-output linkages in the weights of sectoral inflation the monetary policy should adopt to close the output gap. We find that the OG weight of a domestic sector decreases in the imports of foreign goods by the sector and its downstream sectors, because more imports reduce the sector's direct and indirect contribution to the domestic aggregate output, thereby reducing the size of the CPI channel and resulting in a smaller OG weight. As such, the monetary policy *should* assign large weights to the inflation in domestic sectors having small direct and indirect (via the downstream sectors) import shares.

We then focus on our second question and examine the pitfall in a monetary policy that adopts the Domar weights—which eliminate the output gap in the closed economy—disregarding the role of cross-border linkages. In open economies, sectoral products are sold to both domestic and foreign markets. Thus, the Domar weight in open economies is proportional to *total* sectoral sales that encapsulate the contribution of the sector to foreign demand in addition to *domestic* aggregate output. As a result, the monetary policy that aims at closing the *domestic* aggregate output gap—but yet adopts the Domar weight—over-emphasizes the inflation in sectors that export intensively to foreign countries directly and indirectly (via the downstream sectors). The difference between the Domar and OG weights is proportional to the degree of openness and is quantitatively significant in small open economies, as we show in our numerical simulations.

To study the quantitative relevance of our theoretical results and the welfare differences across alternative monetary policies, we calibrate the model to the World Input-Output Database (WIOD). The database comprises 43 countries with 56 major sectors for the year 2014. The variance decomposition of sectoral OG weights shows that the sizes of the CPI and net export income channels explain the bulk of the variation in the OG weight, with the importance of these two channels decreasing and increasing with the openness of the economy, respectively.

We use regression analysis to study the co-movements in the observed OG weights with the centrality measures for the countries in our sample. The results corroborate our two major theoretical findings for the OG policy: (i) the OG weight of a domestic sector increases with the *domestic supplier centrality*—which encapsulates the sector's contribution to domestic aggregate output as a supplier of inputs—and it decreases with the sector's *import intensity*—which encapsulates the imports of the sector and its downstream sectors, and (ii) the difference between the Domar and the OG weights for a domestic sector increases with the sector's *export intensity*—which encapsulates the direct and indirect (via downstream sectors) exports of the sector—and decreases in the *customer centrality*—which encapsulates the sector's contribution to domestic aggregate output as a customer of domestic labor. Moreover, compared to the import and export shares, the import and export intensities that account for the input-output linkages have higher explanatory power for the variation of sectoral OG weights. We use Mexico as a representative small open economy to illustrate key theoretical results on the difference between the OG and Domar weights. For instance, the export processing sector that manufactures machinery and equipment with 32% of its inputs from abroad and exports 99% of its output has an OG weight that is 87% lower than the Domar weight, corroborating our theoretical results on the relevance of international production networks for the stance of monetary policy.

Finally, we compare the welfare of alternative monetary policies, showing that the OG policy is close to the optimal monetary policy and outperforms three alternative monetary policies:[3] (i) the monetary policy that targets the Domar-weighted inflation index (and therefore abstracts from the openness of the economy); (ii) the monetary policy that targets the CPI-weighted inflation index (and thus abstracts from the openness and the input-output linkages); and (iii) the monetary policy that accounts for the openness but abstracts from the input-output linkages.[4] For instance, in Mexico, the OG policy improves over the Domar-weight and CPI-weight policies toward the optimal monetary policy by 67% and 99%, respectively. In the more open economy of Luxembourg, improvements from the OG policy are larger at 95% and 99%, respectively. In the more closed economy of the US, however, there is little welfare difference between the OG and Domar-weight policies, indicating that the imports and exports play a limited role in the design of monetary policy in countries with a low degree of openness. Accordingly, our numerical analysis further emphasizes the quantitative importance of considering both input-output linkages and the openness of the economy in designing monetary policies in small open economies.

In particular, we derive the analytical solutions of welfare loss and the optimal monetary policy to compute welfare loss. We find that the welfare-loss function in the open economy has similar components to the closed economy, as it comprises quadratic terms for the aggregate output gap and the within- and across-sector misallocations. However, compared to the closed economy, in an open economy, these common components differ in their contributions to welfare in three important ways: (i) the contributions of sectoral distortions to the aggregate output gap—encapsulated by the OG weights—are different from the Domar weight used in closed economies, (ii) the contributions of the aggregate output gap to sectoral inflation and thereby distortions—encapsulated by the slopes of the sectoral Phillips curves—include the nominal exchange rate channel that is unique to open economies, and (iii) the weights of sectoral distortions in the across-sector misallocation account for distortions in the use of home versus foreign products and in the nominal exchange rate, both of which are specific to open economies. These differences in welfare loss amount to substantial differences in the welfare-loss function and the optimal monetary policies between closed and open economies.

[3]See Online Appendix A for the analytical solutions of welfare loss, sectoral Phillips curves, and the optimal monetary policy.

[4]The Domar-weighted (vs. CPI-weighted) monetary policy targets the aggregate inflation index that weights each sector's inflation with the product of the sector's Domar (vs. CPI) weight and price rigidity.

Related literature. Our paper is related to four separate strands of literature. First, we relate to literature on the design of monetary policy in closed economies with production networks. La'O and Tahbaz-Salehi (2022) and Rubbo (2023) show that in closed economies, monetary policy that closes the output gap is nearly optimal, and weights inflation in the different sectors according to the sectoral Domar weights that account for the structure of the domestic production network. Compared to those two studies, we show that monetary policies in open economies need to account for the interplay between domestic and cross-border input-output linkages.

Second, we relate to literature that investigates the aggregation of sectoral distortions and shocks. Chari et al. (2007) use labor and efficiency wedges to characterize the aggregation of disaggregated shocks and distortions. Bigio and La'O (2020) extend that analysis to study a closed economy with production networks; they reveal that the efficiency wedge does not include first-order distortions and that only the labor wedge is critical to first-order economic efficiency. We generalize their results to an open economy with international production networks. Baqaee and Farhi (2024) study the distortions in a global economy with interconnected countries and sectors. Elliott and Jackson (2024) study the propagation of supply chain disruption in an international production network. Compared to their work, we examine the distortions in small open economies and focus our analysis on the design of monetary policy.

Third, we relate to literature on the transmission of monetary policy in production networks. Ghassibe (2021a,b) and Afrouzi and Bhattarai (2023) develop an analytical characterization of the transmission mechanism of monetary policy in closed economies with production networks. Nakamura and Steinsson (2010) and Pasten et al. (2020) provide a numerical characterization of the effect of monetary policy on aggregate output and inflation. Compared to those foregoing works, we focus on the design rather than the transmission of monetary policy in network economies.

Fourth, we link to the numerous studies on the design of monetary policies in small open economies without production networks. While earlier work focuses on one-sector small open economies (e.g., Galí and Monacelli, 2005; Soffritti and Zanetti, 2008; De Paoli, 2009), more recent studies by Matsumura (2022) and Wei and Xie (2020) explore small open economy models with multiple sectors. Compared to the foregoing studies, we derive closed-form solutions for the output gap and optimal monetary policies and provide a comprehensive analysis of the design of monetary policies in small open economies with fully-fledged domestic and cross-border input-output linkages.

Outline. The remainder of the paper is organized as follows. Section 2 describes our model of a small open economy with production networks. Section 3 studies the OG weights and characterizes the OG policy that eliminates the aggregate output gap. Section 4 quantifies the theoretical results using data and compares the welfare of alternative monetary policies. Section 5 concludes the paper.

2 Small open economy with production networks

2.1 Environment

The static, small open economy is populated by a representative household consuming domestic and imported sectoral products and supplying labor in exchange for wage income, a government that levies sector-specific taxes and manages the aggregate demand by controlling the money supply, and producers that operate in $N \in \mathbb{N}_+$ different sectors, indexed by $i \in \{1, 2, \cdots, N\}$.

Each sector i comprises two types of producers: (i) a unit mass of monopolistically competitive firms indexed by $f \in [0,1]$ that transform labor and intermediate inputs into differentiated goods, and (ii) a unit mass of perfectly competitive firms that pack the differentiated goods of each sector into a domestic sectoral product, which are both used domestically and exported to foreign consumers and producers. Each domestic sectoral product has an equivalent foreign sectoral product available for import. Consumption and intermediate inputs comprise domestic and foreign sectoral products.

2.2 Producers

Monopolistically competitive firms. Within each sector i, monopolistically competitive firms use a common constant-returns-to-scale production technology to transform labor and intermediate inputs into differentiated goods. The production technology of each firm f in sector i is

$$Y_{if} = A_i \cdot \left(\frac{L_{if}}{\alpha_i}\right)^{\alpha_i} \prod_{j=1}^{N} \left(\frac{X_{if,j}}{\omega_{i,j}}\right)^{\omega_{i,j}}, \tag{1}$$

where A_i is the sector-specific productivity shock, Y_{if} is the output of firm f in sector i, L_{if} is its labor input, and $X_{if,j}$ is the intermediate input acquired from sector j. Parameter α_i is the share of labor, and $\omega_{i,j}$ is the share of intermediate inputs from sector j. The collection of $\{\omega_{i,j}\}_{i,j}$ characterizes the input-output table. Constant returns-to-scale implies that $\alpha_i + \sum_{j=1}^{N} \omega_{i,j} = 1$.

The openness of the economy is reflected in the composition of $X_{if,j}$, which is aggregated from a domestic sectoral product $X_{Hif,Hj}$ and an imported foreign sectoral product $X_{Hif,Fj}$ according to the following constant-elasticity-of-substitution technology:

$$X_{if,j} = \left(v_{x,i,j}^{\frac{1}{\theta_j}} X_{Hif,Hj}^{\frac{\theta_j - 1}{\theta_j}} + (1 - v_{x,i,j})^{\frac{1}{\theta_j}} X_{Hif,Fj}^{\frac{\theta_j - 1}{\theta_j}} \right)^{\frac{\theta_j}{\theta_j - 1}}, \tag{2}$$

where θ_j is the elasticity of substitution between domestic and foreign sectoral products in intermediate input $X_{if,j}$. $v_{x,i,j}$ is the home bias parameter, which in equilibrium is equal to the steady-state expenditure share of $X_{Hif,Hj}$ in the composite intermediate input $X_{if,j}$.

The total cost of inputs used by the firm is

$$TC_{if} = WL_{if} + \sum_{j=1}^{N} (P_j X_{Hif,Hj} + S \cdot P^*_{IM,Fj} X_{Hif,Fj}), \tag{3}$$

where W is the nominal wage rate, P_j is the domestic sectoral price, $P^*_{IM,Fj}$ is the exogenous sectoral import price denominated in the foreign currency, and S is the nominal exchange rate. Given output Y_{if} and the production technology in equation (1), the firm optimally chooses labor and intermediate inputs to minimize TC_{if}, which yields the marginal cost of production that equals the average cost due to the constant-return-to-scale technology. Moreover, because all firms f in each sector i share the same production technology and face the same input prices, the marginal cost of production is identical across all firms in sector i, and we denote it by Φ_i. As a result, given the firm's price P_{if} and the sectoral tax (or subsidy if negative) rate τ_i on sales, the nominal profit of firm f in sector i equals

$$\Pi_{if} = (1 - \tau_i) P_{if} Y_{if} - \Phi_i \cdot Y_{if}. \tag{4}$$

Sectoral goods packers. In each sector i, the perfectly competitive and identical sectoral goods packers transform the differentiated goods produced by the monopolistically competitive firms into a sectoral product using the following constant-elasticity-of-substitution technology:

$$Y_i = \left(\int_0^1 Y_{if}^{\frac{\varepsilon_i - 1}{\varepsilon_i}} df \right)^{\frac{\varepsilon_i}{\varepsilon_i - 1}}, \tag{5}$$

where the within-sector elasticity of substitution between different firms' products is $\varepsilon_i > 1$. The cost minimization of the goods packers yields the following sectoral price index and demand function for the firms:

$$P_i = \left(\int_0^1 P_{if}^{1-\varepsilon_i} df \right)^{\frac{1}{1-\varepsilon_i}} \quad \text{and} \quad Y_{if} = \left(\frac{P_{if}}{P_i} \right)^{-\varepsilon_i} Y_i. \tag{6}$$

Nominal rigidity and sectoral markup wedges. Denote $P_i^{\#}$ the price that maximizes the firm's profit in equation (4) subject to the demand function in equation (6), which is equal to the following:

$$P_i^{\#} = \frac{1}{1 - \tau_i} \frac{\varepsilon_i}{\varepsilon_i - 1} \Phi_i \equiv \mu_i^{\#} \cdot \Phi_i, \tag{7}$$

where $\mu_i^{\#}$ denotes the desired sectoral (gross) markup. Nominal rigidity is modeled as static Calvo-pricing friction, in which only firms indexed by $f \leq \delta_i \in [0, 1]$ are allowed to choose their desired price $P_i^{\#}$ and the remaining firms maintain their price at the steady-state level. We refer to $(1-\delta_i)/\delta_i$ as the price rigidity of sector i. The sectoral markup $\mu_i \equiv P_i/\Phi_i$ differs from the desired markup $\mu_i^\#$ if the price rigidity of sector i is strictly positive, *viz*, $(1-\delta_i)/\delta_i > 0$. We define the *sectoral markup wedge* for domestic sector i as the log deviation of the sectoral markup from the desired markup, *viz*, $\ln(\mu_i) - \ln(\mu_i^\#)$.

2.3 Households

The preference of the representative household is described by the utility function defined over domestic aggregate consumption C and labor supply L:

$$u(C, L) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{L^{1+\varphi}}{1+\varphi}, \tag{8}$$

where σ is the degree of diminishing marginal utility of consumption and φ is the inverse of the Frisch elasticity of labor supply. In our static model without investment, domestic aggregate consumption is equivalent to the (domestic) aggregate output; thus, we refer to C as the aggregate output throughout the paper.

The (domestic) aggregate output C combines sectoral consumption $\{C_i\}_i$ that comprises domestic and imported components, C_{Hi} and C_{Fi}, respectively, for each sector i, represented by[5]

$$C = \prod_{i=1}^{N} \left(\frac{C_i}{\beta_i}\right)^{\beta_i}, \quad \text{where} \quad C_i = \left(v_i^{\frac{1}{\theta_i}} C_{Hi}^{\frac{\theta_i - 1}{\theta_i}} + (1-v_i)^{\frac{1}{\theta_i}} C_{Fi}^{\frac{\theta_i - 1}{\theta_i}}\right)^{\frac{\theta_i}{\theta_i - 1}}. \tag{9}$$

$\{\beta_i\}_i$ is the set of consumption shares satisfying $\sum_{i=1}^{N} \beta_i = 1$, and v_i is the home bias parameter for the consumption of sectoral products. Denote P_C as the price index of the aggregate output C, *viz*, the CPI. The budget constraint of the household is

$$P_C C = \sum_{i=1}^{N} \left(P_i C_{Hi} + S \cdot P_{IM,i}^{*} C_{Fi}\right) \leq WL + \sum_{i=1}^{N} \int_0^1 \Pi_{if} df + T, \tag{10}$$

where Π_{if} is the profit from firm f in sector i, and T is the lump-sum transfer of the tax revenues to the household. To purchase the consumption goods, households demand the following amount of money as the medium of exchange:

$$M_d = P_C C. \tag{11}$$

[5] As we show in equation (A.3) of Proposition 6 in Appendix A, the aggregate consumption gap ($\widetilde{C}$) drives sectoral inflation in the Phillips curves. For consistency with the terminology used in the optimal monetary policy literature, and with a slight abuse of notation, we will refer to $\widetilde{C}$ as the aggregate output gap, and to C as the aggregate output throughout the paper.

Cost minimization by the household yields the price index of aggregate output:

$$P_C = \prod_{i=1}^{N} \left(v_i P_i^{1-\theta_i} + (1 - v_i)(S \cdot P^*_{IM,Fi})^{1-\theta_i} \right)^{\frac{\beta_i}{1-\theta_i}}. \tag{12}$$

2.4 International trade

In addition to the sales subsidy $\{\tau_i\}_i$, the government also imposes sector-specific export tax $\{\tau_{EX,i}\}_i$ on the products exported to foreign countries. The no-arbitrage condition implies that there is no difference between the prices that producers receive from exporting (i.e., $(1 - \tau_{EX,i})P_{EX,i}$) and selling domestically (i.e., P_i):

$$(1 - \tau_{EX,i})P_{EX,i} = P_i, \quad \forall i \in \{1, 2, \cdots, N\}. \tag{13}$$

The export demand for sector i's product is modeled as the reduced-form demand function[6]

$$Y_{EX,i} = (P_{EX,i}/S)^{-\theta_{F,i}} D^*_{EX,Fi}, \tag{14}$$

where $D^*_{EX,Fi}$ is the exogenous component of foreign demand, $P_{EX,i}/S$ is the price of the exported domestic sector i goods in units of foreign currency, and the export demand is inversely related to domestic goods' export price, with $\theta_{F,i}$ as the price elasticity of export demand.

Trade is balanced in the static economy, which requires the value of exports to be exactly identical to the value of imports in the whole economy, resulting in the following:[7]

$$\sum_{i=1}^{N} P_{EX,i} Y_{EX,i} = S \sum_{i=1}^{N} P^*_{IM,Fi} \left(\sum_{j=1}^{N} \int_0^1 X_{Hjf,Fi} df + C_{Fi} \right). \tag{15}$$

This trade balance condition pins down the endogenous nominal exchange rate S in equilibrium. The trade balance condition is equivalent to the binding budget constraint of the households in the aggregate.

2.5 Aggregate states

There are three types of exogenous sector-level states in the economy: the productivity $\{A_i\}_i$, the foreign demand $\{D^*_{EX,Fi}\}_i$, and the import price $\{P^*_{IM,Fi}\}_i$. The aggregate state ξ collects the

[6]In general, the export demand in equation (14) can be written as $Y_{EX,i} = [P_{EX,i}/(S \cdot P^*_{EX,Fi})]^{-\theta_{F,i}} D^*_{EX,Fi}$, where $P^*_{EX,Fi}$ is the exogenous price for foreign-produced sector i's product in foreign markets, $D^*_{EX,Fi}$ is the exogenous foreign demand given the prices. Therefore, $D^*_{EX,i}$ in equation (14) captures the effects of both $P^*_{EX,Fi}$ and $D^*_{EX,Fi}$ on export demand.

[7]Engel (2016) advocates using a balanced trade assumption instead of the risk sharing condition in the complete market.

realized states:

$$\boldsymbol{\xi} \equiv \left\{A_i, D^*_{EX,Fi}, P^*_{IM,Fi}\right\}_{i\in\{1,2,\dots,N\}} \in \Xi = \mathbb{R}^{3N}_{\geq 0}. \tag{16}$$

2.6 Government: fiscal and monetary policies

The government sets fiscal and monetary policies. Fiscal policy includes a pair of non-contingent sectoral sales and export taxes $\{\tau_i, \tau_{EX,i}\}_i$ that do not respond to changes in exogenous states. The lump-sum transfer T to the households satisfies a fiscal budget balance:

$$T = \sum_{i=1}^{N} \left(\tau_i \int_0^1 P_{if} Y_{if} df + \tau_{EX,i} P_{EX,i} Y_{EX,i} \right). \tag{17}$$

The monetary policy is a one-dimensional state-contingent money supply $M(\boldsymbol{\xi})$ depending on the aggregate state $\boldsymbol{\xi}$. Our paper investigates the design of this monetary policy, with a particular focus on the monetary policy that eliminates the aggregate output gap.

2.7 Equilibrium definition

The market clearing conditions for product, labor, and money markets are:

$$Y_i(\boldsymbol{\xi}) = C_{Hi}(\boldsymbol{\xi}) + \sum_{j=1}^{N} \int_0^1 X_{Hjf,Hi}(\boldsymbol{\xi}) df + Y_{EX,i}(\boldsymbol{\xi}), \tag{18}$$

$$L(\boldsymbol{\xi}) = \sum_{i=1}^{N} \int_0^1 L_{if}(\boldsymbol{\xi}) df, \tag{19}$$

$$M(\boldsymbol{\xi}) = M_d(\boldsymbol{\xi}). \tag{20}$$

Definition 1. *A sticky-price equilibrium is a set of allocations, prices, and policies (i.e., $\{\tau_i, \tau_{EX,i}\}_i$ and $M(\boldsymbol{\xi})$) such that for any realized state $\boldsymbol{\xi} \in \Xi$,*

(i) producers optimally choose inputs to minimize the cost of production;

(ii) monopolistically competitive firms $f \in [0, \delta_i]$ set prices to maximize profits subject to their demand functions, and the remaining firms $f \in (\delta_i, 1]$ do not adjust prices;

(iii) representative household chooses consumption and labor to maximize utility subject to its budget constraint, and the total expenditure determines the money demand;

(iv) the government budget constraint is satisfied;

(v) all markets clear.

We define the *flexible-price equilibrium* as the special case of the *sticky-price equilibrium* in Definition 1 that involves no Calvo-pricing friction, as stated in the following definition:

Definition 2. *A flexible-price equilibrium is a set of allocations, prices, and policies satisfying all of the conditions stated in Definition 1, except that for any sector $i \in \{1, 2, \cdots, N\}$, $\delta_i = 1$,* viz, *all firms can adjust prices flexibly.*

While the *sticky-price equilibrium* is our focus, the allocation of the *flexible-price equilibrium* serves as a benchmark to obtain the distortions and welfare losses that nominal rigidities cause.

2.8 Efficient flexible-price equilibrium as reference equilibrium

As per Woodford (2003) and Galí (2015), we use non-contingent subsidies and taxes to eliminate distortion in the flexible-price equilibrium, as defined by the following assumption:[8]

Assumption 1. *The non-contingent tax rates for sales and exports are equal to*

$$\tau_i = -1/(\varepsilon_i - 1) \ \ \textit{and} \ \ \tau_{EX,i} = 1/\theta_{Fi}, \ \textit{respectively, for} \ \ \forall i \in \{1, ..., N\}. \tag{21}$$

Under Assumption 1, the *flexible-price equilibrium* is efficient for the home country, as stated in the following lemma:

Lemma 1. *Under Assumption 1, the flexible-price equilibrium implements the first-best allocation.*

Proof: See Section D.2 of the Supplementary Material.

Lemma 1 allows use of *flexible-price equilibrium* as the reference equilibrium for our further analyses of aggregate distortion and welfare loss.

2.9 Notations

This section summarizes the notation in the model to facilitate the tracking of vectors and matrices.

Deviations from the steady state and flexible-price equilibrium. We define the steady state of the static economy as the equilibrium in which all exogenous states A_i, $P^*_{IM,Fi}$, and $P^*_{EX,Fi}$ are at the steady state. We denote with x^{ss} and x^{flex} the values for the variable x in the steady state

[8] In one-sector closed economies, Woodford (2003) and Galí (2015) show that a sales subsidy eliminates the monopoly distortion and makes the flexible-price equilibrium efficient. La'O and Tahbaz-Salehi (2022) and Rubbo (2023) use sector-specific subsidies for the same purpose in a multi-sector closed economy. In small open economies, given that sales subsidies eliminate the monopoly distortion, the monopoly power of domestic producers on the international market needs to be retained to restore the efficiency of the flexible-price equilibrium. Therefore, we use sector-specific subsidies and export taxes to remove the monopoly distortion in the domestic market and the monopoly power in the international market, respectively, as in Matsumura (2022).

and in the flexible-price equilibrium, respectively. We express the log deviation of the variable x from the steady state x^{ss} and the *flexible-price equilibrium* x^{flex} as:

$$\widehat{x} \equiv \ln x - \ln x^{ss}, \quad \text{and} \quad \widehat{x}^{gap} \equiv \ln x - \ln x^{flex}, \tag{22}$$

respectively. We denote the vector that collects the sectoral inflation by $\widehat{\mathbf{P}} = (\widehat{P}_1, \widehat{P}_2, \cdots, \widehat{P}_N)^\top$.[9] We denote the aggregate output gap by $\widehat{C}^{gap}$. The sectoral markup wedge is $\ln(\mu_i) - \ln(\mu_i^{\#}) = \ln(\mu_i) - \ln(\mu_i^{ss}) \equiv \widehat{\mu}_i$ as the steady-state markup is equal to the desired markup.

Table 1: Notations of parameters and steady-state objects

Name	Expression
Consumption shares and home bias	$\boldsymbol{\beta} \equiv (\beta_1, \beta_2, \cdots, \beta_N)^\top$ & $\mathbf{v} \equiv (v_1, v_2, \cdots, v_N)^\top$
Labor shares	$\boldsymbol{\alpha} \equiv (\alpha_1, \alpha_2, \cdots, \alpha_N)^\top$
Intermediate input shares and home bias	$\boldsymbol{\Omega} \equiv \{\omega_{i,j}\}_{i,j\in\{1,2,\cdots,N\}}$ & $\mathbf{V}_x \equiv \{v_{x,i,j}\}_{i,j\in\{1,2,\cdots,N\}}$
Elasticity of home-foreign substitution	$\boldsymbol{\theta} \equiv (\theta_1, \theta_2, \cdots, \theta_N)^\top$ & $\boldsymbol{\theta}_F \equiv (\theta_{F,1}, \theta_{F,2}, \cdots, \theta_{F,N})^\top$
Frequency of price adjustment	$\boldsymbol{\Delta} = diag(\delta_1, \delta_2, \cdots, \delta_N)$
Steady-state sectoral Domar weight	$\boldsymbol{\lambda} \equiv (\lambda_1, \lambda_2, \cdots, \lambda_N)^\top \equiv \left(\frac{P_1^{ss}Y_1^{ss}}{P_C^{ss}C^{ss}}, \frac{P_2^{ss}Y_2^{ss}}{P_C^{ss}C^{ss}}, \cdots, \frac{P_N^{ss}Y_N^{ss}}{P_C^{ss}C^{ss}}\right)^\top$
Steady-state sectoral export-to-GDP ratio	$\boldsymbol{\lambda}_{EX} \equiv (\lambda_{EX,1}, \cdots, \lambda_{EX,N})^\top \equiv \left(\frac{P_1^{ss}Y_{EX,1}^{ss}}{P_C^{ss}C^{ss}}, \cdots, \frac{P_N^{ss}Y_{EX,N}^{ss}}{P_C^{ss}C^{ss}}\right)^\top$
Steady-state economy-wise labor share	$\Lambda_L \equiv W^{ss}L^{ss} / P_C^{ss}C^{ss}$

Parameters and steady-state objects. Summarized in Table 1 are the key parameters and steady-state variables, denoting vectors and matrices with bold fonts. For expositional simplicity, the superscript "ss" to denote the steady state is omitted when there is no obvious confusion.

3 Aggregate output gap and OG monetary policy

In Section 3, we first define the wedges—namely, the efficiency and labor wedges—that characterize the aggregation of shocks and distortions in the economy (subsection 3.1). We show that up to a first-order approximation, the labor wedge is a weighted average of sectoral distortions and proportional to the aggregate output gap and, thus, relevant for the aggregate distortion. However, the efficiency wedge involves exogenous disturbances and no structural distortions. Therefore, eliminating the labor wedge and closing the aggregate output gap are the primary objectives of monetary policy aimed at offsetting first-order distortions. As noted earlier, we refer to this policy as the output gap (OG) monetary policy.

To study the contributions of sectoral distortions to the aggregate distortions encapsulated by the labor wedge and the aggregate output gap, subsection 3.2 defines the centrality measures that describe the relative importance of each sector as a direct and indirect (via downstream or

[9] In our static model, inflation is identical to the log deviation of sectoral price from its steady-state level.

upstream sectors): (i) supplier of inputs to aggregate output, (ii) customer for domestic labor, and (iii) net exporter. Subsection 3.3 shows that the aggregate output gap is a weighted average of the sectoral distortions, and the weights—which we refer to as output gap (OG) weights—are composed of three distinct channels that are functions of the sector's centrality measures. Subsection 3.3 derives an analytical solution for the OG monetary policy that closes the aggregate output gap. Subsection 3.4 studies the interplay between import-export and network structures in determining the OG policy through the OG weights.

3.1 Aggregate wedges and aggregate output gap

We follow the spirit of Chari et al. (2007) to define the efficiency and labor wedges in the multi-sector, small open economy as follows.

Definition 3 (Aggregate wedges). *The two wedges $A_{agg} : \Xi \mapsto \mathbb{R}_+$ and $\Gamma_L : \Xi \mapsto \mathbb{R}_+$ allow equilibrium aggregate consumption and labor inputs to satisfy the following equations:*[10]

$$C(\boldsymbol{\xi}) = A_{agg}(\boldsymbol{\xi})L(\boldsymbol{\xi})^{\Lambda_L^{flex}(\boldsymbol{\xi})}, \quad \forall \boldsymbol{\xi} \in \Xi, \tag{23}$$

$$\frac{u_L(C(\boldsymbol{\xi}), L(\boldsymbol{\xi}))}{-u_C(C(\boldsymbol{\xi}), L(\boldsymbol{\xi}))} = \Gamma_L(\boldsymbol{\xi})\frac{\partial C}{\partial L}(\boldsymbol{\xi}), \quad \forall \boldsymbol{\xi} \in \Xi, \tag{24}$$

in the economy. We refer to $A_{agg}(\boldsymbol{\xi})$ as the efficiency wedge, or aggregate TFP, and $\Gamma_L(\boldsymbol{\xi})$ as the labor wedge, respectively, for any realized state $\boldsymbol{\xi} \in \Xi$.

The equilibrium of the economy is summarized by the aggregate production function in equation (23) and the intratemporal condition between aggregate consumption and labor supply in equation (24). The aggregate production function describes the transformation of labor inputs into aggregate consumption, where the transformation ratio equals the economy-wise share of domestic labor inputs in total inputs in the efficient, flexible-price equilibrium ($\Lambda_L^{flex}(\boldsymbol{\xi})$). The transformation ratio is less than one in our open economy, because domestic firms import foreign goods as inputs of production in addition to domestic labor. In contrast, the transformation ratio is equal to one in a closed economy—as per Bigio and La'O (2020)—because all domestic consumption is produced directly, or indirectly, using domestic labor. The efficiency wedge $A_{agg}(\boldsymbol{\xi})$ captures the shifts in the aggregate production function or the aggregate TFP.

The intratemporal condition in equation (24) relates the marginal product of labor for aggregate output (i.e., $\partial C/\partial L$) to the marginal rate of substitution between consumption and labor (i.e., $-u_L/u_C$), and the labor wedge $\Gamma_L(\boldsymbol{\xi})$ encapsulates the distortions that make the marginal product of labor different from the marginal rate of substitution.

[10]Section E.2 of the Supplementary Material shows that the marginal product of labor is $(\partial C/\partial L)(\boldsymbol{\xi}) = A_{agg}(\boldsymbol{\xi})\Lambda_L^{flex}L(\boldsymbol{\xi})^{\Lambda_L^{flex}(\boldsymbol{\xi})-1}$.

Based on the definition of the efficiency wedge in Definition 3, we establish the following open-economy version of Hulten's theorem.[11]

Lemma 2 (The open-economy version of Hulten's theorem)**.** *Up to the first-order approximation, the deviation of the efficiency wedge from the steady state is a weighted average of sectoral shocks as follows:*

$$\begin{aligned}\widehat{A}_{agg}(\boldsymbol{\xi}) &= \widehat{C}(\boldsymbol{\xi}) - \Lambda_L \widehat{L}(\boldsymbol{\xi}) \\ &= \boldsymbol{\lambda}^\top \widehat{\mathbf{A}} - \Big\{ \underbrace{[\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})]^\top}_{\text{imported consumption}} + \underbrace{\boldsymbol{\lambda}^\top (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})}_{\text{imported interm. inputs}} \Big\} \widehat{\mathbf{P}}^*_{IM,F} + \underbrace{[\boldsymbol{\lambda}_{EX} \oslash (\boldsymbol{\theta}_F - \mathbf{1})]^\top}_{\text{profits from exports}} \widehat{\mathbf{D}}^*_{EX,F}.\end{aligned} \tag{25}$$

Proof: See Section E.1 of the Supplementary Material.

Equation (25) shows that deviation of the efficiency wedge from the steady state is linked to the deviations of exogenous sectoral productivity ($\widehat{\mathbf{A}}$), import prices ($\widehat{\mathbf{P}}^*_{IM,F}$), and foreign demand ($\widehat{\mathbf{D}}^*_{EX,F}$), from the steady state. The elasticity of the efficiency wedge to the sectoral productivity is the Domar weight of the sector ($\boldsymbol{\lambda}$), as in the closed economy (Hulten, 1978; Bigio and La'O, 2020). In an open economy, however, the elasticities of the efficiency wedge to import prices and foreign demand depend on the linkages between the domestic and foreign economies. The elasticity of the efficiency wedge to a sector's import price shock equals the share of the sector's imports of consumption goods and intermediate inputs in aggregate output. Such elasticity is negative, as imported inflation materializes as a negative supply shock. The elasticity of the efficiency wedge to a shock to the sector's foreign demand equals the share of the sector's profits from exports in aggregate output. Such elasticity is positive because an increase in the foreign demand for domestic goods raises exports, which is the equivalent of a rise in the efficiency wedge for the small open economy.

Lemma 2 implies that—similar to the closed economy case—sectoral distortions have no first-order impact on the efficiency wedge in a small open economy with production networks. Therefore, the labor wedge encapsulates sectoral distortions entirely, as stated in the following proposition:

Proposition 1 (Sectoral distortion, efficiency and labor wedges, and the aggregate output gap)**.** *Up to the first-order approximation, the efficiency wedge in the sticky-price equilibrium is the same as the efficiency wedge in the efficient, flexible-price equilibrium:*

$$\widehat{A}_{agg}(\boldsymbol{\xi}) - \widehat{A}^{flex}_{agg}(\boldsymbol{\xi}) = \widehat{C}^{gap}(\boldsymbol{\xi}) - \Lambda_L \cdot \widehat{L}^{gap}(\boldsymbol{\xi}) = 0. \tag{26}$$

The labor wedge, though, deviates from the efficient, flexible-price level, and the deviation is proportional

[11] In closed economies with production networks, Bigio and La'O (2020) define a prototype economy and the corresponding efficiency and labor wedges. They also show that Hulten's theorem holds and that sectoral distortions have no first-order effect on the efficiency wedge.

to the aggregate output gap:[12]

$$\widehat{\Gamma}_L(\xi) - \widehat{\Gamma}_L^{flex}(\xi) = \widehat{\Gamma}_L(\xi) = [\sigma - 1 + (\varphi + 1)/\Lambda_L]\, \widehat{C}^{gap}(\xi). \tag{27}$$

Proof: See Section E.2 of the Supplementary Material.

Proposition 1 shows that up to the first-order approximation, the efficiency wedge is unaffected by sectoral distortions, but that the labor wedge is different from zero and it summarizes the distortions at the aggregate level. In particular, the deviation of the labor wedge from the efficient equilibrium is proportional to the aggregate output gap.

Comparison with Baqaee and Farhi (2024). Corollary 1 in Baqaee and Farhi (2024)—which studies an inter-connected global production network—decomposes the real GDP of an open economy into the efficiency wedge and the labor. This decomposition can be represented as the Hulten's theorem for the open economy. In that sense, our Lemma 2 is a corollary of their Corollary 1, but under the assumption of nominal rigidities and financial autarky. In particular, financial autarky in our set-up corresponds to zero lump-sum transfers across countries in Baqaee and Farhi (2024). Moreover, our Proposition 1 demonstrates the equivalence between the efficiency wedge in the flexible-price equilibrium (as in Baqaee and Farhi, 2024) and that in the sticky-price equilibrium, up to the first-order approximation. This equivalence implies that, to study the first-order inefficiencies and the monetary policy needed to eliminate them, we can focus on the impacts of nominal rigidities and inflation on the labor wedge instead of the efficiency wedge, which we pursue in Section 3.3.

3.2 Centrality measures in an open economy with networks

In a multi-sector open economy with production networks, the mapping of sectoral distortions into the aggregate distortions depends on the relevance of the sector within the network economy across three dimensions: (i) as a direct and indirect (via the downstream sectors) supplier of inputs to domestic aggregate output and to foreign countries, respectively, (ii) as a direct and indirect (via the upstream sectors) customer for domestic labor, and (iii) as a net exporter and the associated user of domestic labor. We refer to these different dimensions as the *supplier*, *customer*, and *net export centralities*, respectively. Before constructing our centrality measures for the open economy and comparing them with their counterparts in a closed economy, we define upstream and downstream relationships in the network economy.

[12] The deviation of the labor wedge from the efficient equilibrium equals the deviation of the labor wedge from the steady state. This is because the labor wedge equals one in the efficient, flexible-price equilibrium for any realized state ξ, including the steady state.

Definitions of upstream and downstream sectors in the open economy. We first introduce the open-economy version of the Leontief-inverse matrix

$$\mathbf{L}_{vx} \equiv (\mathbf{I} - \mathbf{\Omega} \odot \mathbf{V}_x)^{-1} = \{l_{vx,r,i}\}_{r,i}, \tag{28}$$

which affords defining the upstream and downstream relationships between sector pairs in an open economy with production networks.

Definition 4. *For a pair of domestic sectors $r \neq i$, r is a downstream sector of i if $l_{vx,r,i} > 0$; r is an upstream sector of i if $l_{vx,i,r} > 0$. Accordingly, we define $l_{vx,r,i}$ and $l_{vx,i,r}$ as the downstream and upstream Leontief inverse of domestic sector i, respectively.*

We decompose the downstream and upstream relationships from the Leontief inverse into the direct and indirect components as follows

$$l_{vx,r,i} = \underbrace{\mathbf{1}(r=i)}_{\text{direct impact}} + \underbrace{\omega_{r,i} v_{x,r,i} l_{vx,i,i}}_{\text{direct downstream}} + \underbrace{\textstyle\sum_{s\neq i} \omega_{r,s} v_{x,r,s} l_{vx,s,i}}_{\text{indirect downstream}}, \tag{29}$$

$$l_{vx,i,r} = \underbrace{\mathbf{1}(r=i)}_{\text{direct impact}} + \underbrace{\omega_{i,r} v_{x,i,r} l_{vx,r,r}}_{\text{direct upstream}} + \underbrace{\textstyle\sum_{s\neq i} \omega_{i,s} v_{x,i,s} l_{vx,s,r}}_{\text{indirect upstream}}. \tag{30}$$

Equation (29) shows that if $r = i$, a shock to domestic sector i entails a direct impact on itself, captured by the first component on the right-hand side (RHS) of the equation. Otherwise, if $r \neq i$ is a downstream sector to i (i.e., $l_{vx,r,i}$), a shock to i propagates to the downstream sector r directly—as described by the second component on the RHS of the equation. Similarly, a shock to i can also propagate to the downstream sector r indirectly via an intermediate sector s that is downstream to i and upstream to r—as described by the third component on the RHS of the equation. In other words, the downstream Leontief inverse $l_{vx,r,i}$ captures the direct and indirect contribution of domestic sector i to downstream sectors r as an input supplier. Similarly, equation (30) shows that sector i uses the goods of its upstream sector r as inputs either directly (second component on the RHS of the equation) or indirectly via an intermediate sector s that is upstream to i and downstream to r (third component on the RHS of the equation)—as described by the different components of the upstream Leontief inverse $l_{vx,i,r}$.

Notably, equations (29) and (30) involve import shares (i.e., $v_{x,r,i}$) in the direct and indirect components. The two equations indicate that the import structure of an open economy interacts with the input-output linkages (i.e., $\omega_{r,i}$) to determine the upstream and downstream relationships between domestic sectors and, in turn, the centrality measures and the OG weights, which we discuss in detail in subsection 3.4.

Supplier and customer centralities in open economies. In open economies, producers in the domestic sector supply products to satisfy domestic and foreign demand. Thus, we partition the roles of a domestic sector as a supplier of inputs between the supply to domestic aggregate output and the foreign demand, and we refer to these different roles as *domestic supplier centrality* and *foreign supplier centrality*, respectively, and define them as follows:

Definition 5. *The* ***domestic supplier centrality*** $\widetilde{\lambda}_{D,i}$ *of the domestic sector i is defined as*

$$\widetilde{\lambda}_{D,i} \equiv \sum_r \beta_r v_r l_{vx,r,i}. \tag{31}$$

The ***foreign supplier centrality*** $\widetilde{\lambda}_{F,i}$ *of the domestic sector i is defined as*

$$\widetilde{\lambda}_{F,i} \equiv \sum_r \lambda_{EX,r} l_{vx,r,i}. \tag{32}$$

Equations (31) and (32) show that the domestic and foreign supplier centralities of sector i are functions of the sector's downstream Leontief inverse $l_{vx,r,i}$, thus evincing the direct and indirect contribution of domestic sector i as input suppliers to downstream sectors r. The goods of downstream sectors r are used for (domestic) aggregate output and for exports—captured by the consumption share $\beta_r v_r$ and the export-to-GDP ratio $\lambda_{EX,r}$, respectively. Therefore, the domestic supplier centrality $\widetilde{\lambda}_{D,i}$ (vs. foreign supplier centrality $\widetilde{\lambda}_{F,i}$) of a domestic sector i summarizes the importance of the sector in the network economy as both a direct and an indirect supplier (via downstream sectors) for the aggregate output (vs. foreign demand or exports).

In addition to the role as a supplier of inputs, a domestic sector also takes the role as a customer for domestic labor, which we summarize by the *customer centrality*, defined as follows:

Definition 6. *The* ***customer centrality*** $\widetilde{\alpha}_i$ *of the domestic sector i is given by*

$$\widetilde{\alpha}_i = \sum_r l_{vx,i,r} \alpha_r. \tag{33}$$

Equation (33) shows that the customer centrality of sector i is a function of the sector's upstream Leontief inverse $l_{vx,i,r}$, evincing sector i's direct and indirect use of upstream domestic sector r's goods, which further requires domestic labor—captured by the labor share α_r. Therefore, the *customer centrality* of a domestic sector i summarizes the sector's role in the network economy as both a direct and an indirect customer (via upstream sectors) of domestic labor.

Net export centrality: the role of sectoral distortions for the labor income from net exports. In open economies, domestic sectoral distortions are important for net exports. An increase in the price markup of domestic goods shifts consumption and demand for intermediate inputs away from domestic producers towards foreign producers, reducing the net exports of the economy and diminishing the use of domestic labor. The next definition characterizes the *net export centrality* that formalizes this channel.

Definition 7. *The **net export centrality** $\widetilde{\rho}_{NX,i}$ of domestic sector i is given by*

$$\widetilde{\rho}_{NX,i} = \sum_{r} \rho_{NX,r} \widetilde{\alpha}_r l_{vx,r,i}, \tag{34}$$

where

$$\rho_{NX,r} \equiv (\theta_{F,r} - 1) \lambda_{EX,r} + (\theta_r - 1) \left[\beta_r v_r (1 - v_r) + \sum_{s} \lambda_s \omega_{s,r} v_{x,s,r} (1 - v_{x,s,r}) \right] \tag{35}$$

*is the **elasticity of the net export** of the domestic sector r to the decline in the price markup wedge of the sector.*

The **net export centrality** and the **elasticity of net export** in equations (34) and (35), respectively, describe the mapping of sectoral distortions—encapsulated by the sectoral price markup wedges—into distortions in the net exports and the associated distortion in the use of domestic labor inputs.

More specifically, a decline in the price markup wedge of domestic sector i propagates downstream directly and indirectly to domestic sectors r and leads to lower prices in these sectors, as evinced by the downstream Leontief inverse $l_{vx,r,i}$. The resulting deflation in the domestic sector r leads to a shift in the demand for sector r goods from foreign to domestic products. It generates an increase in the exports of domestic sector r goods and a decrease in the imports of foreign sector r goods both as consumption goods and intermediate inputs—which are captured by the terms $(\theta_{F,r} - 1)\lambda_{EX,r}$, $(\theta_r - 1)\beta_r v_r(1 - v_r)$, and $(\theta_r - 1)\lambda_s \omega_{s,r} v_{x,s,r}(1 - v_{x,s,r})$ that comprise the elasticity of sector r's net export to its deflation (i.e., $\rho_{NX,r}$ in equation 35), respectively. The increase in the net exports of domestic sector r generates a rise in the direct and indirect use of domestic labor, as captured by the customer centrality $\widetilde{\alpha}_r$ in the net export centrality $\widetilde{\rho}_{NX,i}$ in equation (34).

Comparison of the centrality measures in open and closed economies. In open economies, the domestic output supplies both domestic and foreign demand, encapsulated by domestic and foreign supplier centralities, respectively. The domestic producers acquire intermediate inputs from foreign countries as well as from domestic sectors, thus making customer centrality different from the closed-economy counterpart, as shown by the open-economy version of Leontief inverse in equation (31).[13]

In closed economies, there is domestic demand only (i.e., $\lambda_{EX,i} = 0$, $\forall i$), and the *foreign supplier centrality* is equal to zero, as goods are entirely supplied to the domestic market. Thus, *supplier centrality* is equal to *domestic supplier centrality*, and it reduces to the Domar weight

[13] Subsection 3.4 discusses how imports and exports interplay with the network structure to determine centrality measures.

Table 2: Comparison of the centrality measures in open and closed economies

Measure of centrality	Economic meaning	(1) Open economy	(2) Closed economy
Domestic supplier centrality	**Supplier** for (domestic) aggregate output	$\widetilde{\lambda}_{D,i}$	λ_i (Domar weight)
Foreign supplier centrality	**Supplier** for foreign demand	$\widetilde{\lambda}_{F,i}$	0
Customer centrality	**Customer** for domestic labor	$\widetilde{\alpha}_i$	1
Net export centrality	**Net-export**-driven changes in use of domestic labor	$\widetilde{\rho}_{NX,i}$	0

(Baqaee, 2018), as we show in Lemma 5.[14] Similarly, *customer centrality* equals one (i.e. $\widetilde{\alpha}_i = 1$) in closed economies, because the inputs of a sector directly or indirectly are produced by domestic labor. The *net export centrality* equals zero in closed economies, as there are no imports or exports. Shown in Table 2 is a comparison between our centrality measures in open economies (Column 1) and those in closed economies (Column 2).

3.3 Aggregate output gap and OG monetary policy

In this subsection, we show that the aggregate output gap originates from sectoral distortions, and can be expressed as a weighted average of sectoral markup wedges. The weight assigned to each sector—which we refer to as the sectoral OG weight—measures the contribution of the sector's markup wedge to the aggregate output gap. It is composed of three distinct channels: the *CPI*, the *net export income*, and the *net profit income channels*. The size of each of these channels in the OG weight is determined by the centrality measures defined in the previous subsection.

We further define the monetary policy that achieves the zero aggregate output gap (referring to it as the OG monetary policy). We show that the OG policy is implemented by setting the money supply to stabilize the aggregate inflation index that appropriately weights the sectoral inflation. Specifically, the weight of sectoral inflation in the OG monetary policy is the product of two components: (i) the sectoral price-rigidity that maps sectoral inflation into the sectoral markup wedge and (ii) the OG weight that maps the sectoral markup wedge into the aggregate output gap.

Sectoral distortions and the aggregate output gap. Under nominal rigidities, sectoral inflation generates negative sectoral markup wedges, because the fraction $(1 - \delta_i)$ of sector i's firms cannot adjust prices in response to changes in marginal costs. As a result, sectoral markup wedges—encapsulating sectoral distortions—are linked to sectoral inflation through sectoral price rigidities as follows:[15]

$$\widehat{\mu}_i(\boldsymbol{\xi}) = -(1 - \delta_i)/\delta_i \cdot \widehat{P}_i(\boldsymbol{\xi}). \tag{36}$$

[14] Our standard assumption of Cobb-Douglas production function is crucial for the equivalence between the *supplier centrality* and the Domar weight, as Baqaee (2018) avers.

[15] Exogenous shocks to sectoral productivity, import prices, and export demand drive sectoral inflation in the sticky-price equilibrium. Section D.6 of the Supplementary Material provides the proof of equation (36).

Because the aggregate output is manufactured using goods from the different sectors, sectoral markup wedges ($\widehat{\mu}_i$) influence the aggregate output gap *by* distorting the price of the aggregate output (i.e., the CPI), as outlined in the following lemma:

Lemma 3 (Impact of sectoral distortions on CPI). *Up to the first-order approximation, the deviation of the CPI from the efficient, flexible-price level (i.e., the CPI gap $\widehat{P}_C^{gap}$) depends on the nominal wage gap ($\widehat{W}^{gap}$), the nominal exchange rate gap ($\widehat{S}^{gap}$), and the sectoral markup wedges ($\widehat{\mu}_i$) as follows:*

$$\widehat{P}_C^{gap}(\boldsymbol{\xi}) = \Big(\sum_{i=1}^{N} \widetilde{\lambda}_{D,i}\alpha_i\Big)\widehat{W}^{gap}(\boldsymbol{\xi}) + \Big(1 - \sum_{i=1}^{N} \widetilde{\lambda}_{D,i}\alpha_i\Big)\widehat{S}^{gap}(\boldsymbol{\xi}) + \sum_{i=1}^{N} \widetilde{\lambda}_{D,i} \cdot \widehat{\mu}_i(\boldsymbol{\xi}). \tag{37}$$

Proof: See Section E.3 of the Supplementary Material.

The aggregate output is produced using inputs from either domestic labor or imported foreign factors, directly or indirectly (via downstream sectors). Accordingly, equation (37) in Lemma 3 shows that the distortion in the price of the aggregate output ($\widehat{P}_C^{gap}$) arises from three different components: (i) the nominal wage gap ($\widehat{W}^{gap}$) that represents the distortion in the price of domestic labor; (ii) the nominal exchange rate gap ($\widehat{S}^{gap}$) that represents the distortion in the price of imported foreign factors; and (iii) the sectoral markup wedges ($\widehat{\mu}_i$) that encapsulate the distortion in sectoral markups arising from price rigidities. On the RHS of equation (37), the weight of the wage gap equals the cost share of domestic labor inputs in the production of aggregate output: $\sum_{i=1}^{N} \widetilde{\lambda}_{D,i}\alpha_i$. Similarly, the weight of the exchange rate gap equals the cost share of imported foreign inputs in the production of aggregate output: $(1 - \sum_{i=1}^{N} \widetilde{\lambda}_{D,i}\alpha_i)$.[16] Finally, the weight of the markup wedge of each sector i equals the sector's direct and indirect (via downstream sectors) supply of inputs to the production of aggregate output, encapsulated by the *domestic supplier centrality* ($\widetilde{\lambda}_{D,i}$) in the third term on the RHS.

Equation (37) implies that the negative sectoral markup wedges result in a lower CPI in the sticky-price relative to the efficient, flexible-price equilibrium. In response to the lower CPI, the real wage (W/P_C) increases and the real exchange rate depreciates (i.e., S/P_C increases), fostering a higher supply of domestic labor (Lemma 12 in Section E.4 of the Supplementary Material) and an increase in imported foreign factors due to current account improvement (Lemma 13 in Section E.5 of the Supplementary Material), respectively. The increased supply of domestic and foreign factors results in a positive aggregate output gap in response to negative sectoral markup wedges, as outlined in the following theorem:

[16]The direct and indirect use of domestic labor in the production of aggregate output gap *via* domestic sector i equals the product of two sectoral measures: first, the domestic supplier centrality $\widetilde{\lambda}_{D,i}$ that encapsulates the direct and indirect contribution of domestic sector i to the production of aggregate output; and second, the labor cost share α_i that encapsulates sector i's use of domestic labor. The cost share of domestic labor in the production of aggregate output is the sum of the above product (i.e., $\widetilde{\lambda}_{D,i}\alpha_i$) across all domestic sectors i. The remaining part $(1 - \sum_{i=1}^{N} \widetilde{\lambda}_{D,i}\alpha_i)$ is the cost share of imported foreign factors in the production of aggregate output.

Theorem 1 (Aggregate output gap and sectoral distortions)**.** *In a sticky-price equilibrium, negative sectoral markup wedges $\{\widehat{\mu}_i(\boldsymbol{\xi})\}_i$ contribute to a positive aggregate output gap $\widehat{C}^{gap}(\boldsymbol{\xi})$ as follows:*

$$\kappa_C \cdot \widehat{C}^{gap}(\boldsymbol{\xi}) = -\sum_{i=1}^{N} \mathcal{M}_{OG,i} \cdot \widehat{\mu}_i(\boldsymbol{\xi}), \tag{38}$$

where the sectoral OG weight ($\mathcal{M}_{OG,i}$) is equal to:

$$\mathcal{M}_{OG,i} \equiv \underbrace{\widetilde{\lambda}_{D,i}}_{\text{CPI channel}} + \underbrace{\kappa_S \cdot \widetilde{\rho}_{NX,i}}_{\text{net export income channel}} + \underbrace{\kappa_S \cdot [\widetilde{\lambda}_{F,i} - \lambda_i(1-\widetilde{\alpha}_i)]}_{\text{net profit income channel}}, \tag{39}$$

$$\kappa_S \equiv \frac{1 - \sum_{i=1}^{N} \widetilde{\lambda}_{D,i}\alpha_i}{1 - \sum_{i=1}^{N} \widetilde{\lambda}_{D,i}\alpha_i + \sum_{i=1}^{N}(\rho_{NX,i}\widetilde{\alpha}_i + \lambda_{EX,i})\widetilde{\alpha}_i},$$

$$\kappa_C \equiv \kappa_S\Big(1 - \sum_{i=1}^{N} \widetilde{\lambda}_{D,i}\alpha_i\Big) + \Big[1 - \kappa_S\Big(1 - \sum_{i=1}^{N} \widetilde{\lambda}_{D,i}\alpha_i\Big)\Big](\sigma + \varphi/\Lambda_L).$$

Proof: See Section E.6 of the Supplementary Material.

Equation (38) shows that negative sectoral markup wedges contribute to a positive aggregate output gap. The OG weight ($\mathcal{M}_{OG,i}$) in equation (39) measures the contribution of the markup wedge of each sector to the aggregate output gap, and its size is determined by three distinct channels: (i) the *CPI*, (ii) the *net export income*, and (iii) the *net profit income* channels. The *CPI channel* is a direct implication from Lemma 3, and is also present in the closed economy. As shown in equation (37), negative markup wedges generate a negative CPI gap, thus increasing the real prices of factor inputs. The increase in the real prices of factors generates a higher supply of factors, fostering production and generating a positive aggregate output gap.[17]

The *net export income channel*—unique to the open economy—contributes to the increase in the aggregate output gap. Specifically, negative sectoral markup wedges lower domestic sectoral prices relative to the efficient equilibrium, increasing the net exports and, thereby, the domestic labor income, and resulting in a positive aggregate output gap.[18] Finally, the *net profit income channel*—also unique to the open economy—contributes to the aggregate output gap in two countervailing ways: while negative sectoral markup wedges reduce domestic sectoral prices that increase the export taxes and improve the current account, they also increase the use of foreign inputs and worsen the current account, making the impact of the *net profit income channel* on the aggregate output gap undetermined.[19]

The size of each different channel determining the sector's OG weight ($\mathcal{M}_{OG,i}$) depends

[17]The negative sign on the RHS of equation (38) indicates that negative sectoral markup wedges resulting from the positive sectoral inflation lead to a positive aggregate output gap.

[18]Lemma 13 in Section E.5 of the Supplementary Material provides a formal proof of the effects of sectoral markup wedges on net exports and profits.

[19]As we will show in Section 4 (e.g., see Figure 1), the *CPI* and *net profit income channels* (the first and second elements in equation 39) are the most quantitatively relevant channels while the net profit income channel (the third element in equation 39) is tiny and the least relevant. Therefore, in general, negative markup wedges lead to a positive aggregate output gap.

on the contribution of the sector to the aggregate output as a supplier of inputs, a customer of domestic labor, and a net exporter, encapsulated by the centrality measures on the RHS of equation (39), as we discuss below.

The size of the *CPI channel* is determined by the sector's direct and indirect (via downstream sectors) contribution to the aggregate output as a supplier of inputs to different sectors, encapsulated by the *domestic supplier centrality* ($\widetilde{\lambda}_{D,i}$). The size of the *net export income channel* is determined by the contribution of the sector to the net exports and the associated use of domestic labor, encapsulated by the *net export centrality* ($\widetilde{\rho}_{NX,i}$). Finally, the size of the *net profit income channel* is determined by the profits of the sector from international trade. The net profit income from sector i equals the export tax (encapsulated by the foreign supplier centrality $\widetilde{\lambda}_{F,i}$) net of the direct and indirect use of foreign factors by the sector (encapsulated by the term $\lambda_i(1-\widetilde{\alpha}_i)$).[20]

As shown in Proposition 1 in Section 3.1, the labor wedge is proportional to the aggregate output gap. Substituting equation (38) into equation (27) shows how the sectoral markup wedges contribute to the labor wedge, which determines the aggregate distortion in the economy. Therefore, the monetary policy that sets the weighted average of sectoral markup wedges to zero eliminates the first-order labor wedge and closes the output gap, as discussed in the next paragraph.

Definition and implementation of OG monetary policy. The next definition states the monetary policy that closes the aggregate output gap.

Definition 8. *The output gap monetary policy (OG policy for short) eliminates the aggregate output gap,* viz, $\widehat{C}^{gap}(\boldsymbol{\xi}) = 0$, *for any realized state* $\boldsymbol{\xi} \in \Xi$.[21]

To implement the OG policy, the monetary authority chooses the money supply to stabilize the aggregate inflation index that appropriately weights the domestic sectoral inflation. The aggregate inflation index accounts for (i) the mapping from sectoral inflation into sectoral markup wedges, as shown in equation (36); and (ii) the contribution of sectoral markup wedges to the aggregate output gap, as shown in Theorem 1. The next proposition formally characterizes the implementation of the OG monetary policy.

Proposition 2. *The OG monetary policy is implemented by setting the following aggregate inflation index to zero:*

$$\sum_{i=1}^{N} \mathcal{M}_{OG,i} \cdot (1-\delta_i)/\delta_i \cdot \widehat{P}_i(\boldsymbol{\xi}) = 0, \tag{40}$$

[20] Lemma 13 in Section E.5 of the Supplementary Material provides details on how the centrality measures determine the size of the *net export income* and *net profit income channels*. The parameter κ_S on the RHS of equation (39) determines the contribution of the net export income and net profit income channels relative to the CPI channel. It increases in the degree of openness of the economy, evinced by the economy-wise share of imported foreign inputs in total inputs $(1-\sum_{i=1}^{N}\widetilde{\lambda}_{D,i}\alpha_i)$ in its numerator.

[21] Lemma 14 in Section E.10 of the Supplementary Material shows that the monetary policy that controls the supply of money $M(\boldsymbol{\xi})$ uniquely determines the aggregate output gap. Therefore, our OG monetary policy is well-defined.

for any realized state $\boldsymbol{\xi} \in \Xi$. The OG monetary policy achieves zero labor wedge and aggregate output gap up to the first-order approximation, viz,

$$[\sigma - 1 + (\varphi + 1)/\Lambda_L]^{-1} \kappa_C \widehat{\Gamma}_L(\boldsymbol{\xi}) = \kappa_C \cdot \widehat{C}^{gap}(\boldsymbol{\xi}) = 0.$$

Proof: Straightforward substitution of equation (36) in equation (38) from Theorem 1.

Proposition 2 shows that the monetary authority implements the OG policy by choosing the money supply that makes the aggregate inflation index in equation (40) equal to zero.[22] Equation (40) reveals that the weight assigned to sector i in the aggregate inflation index is proportional to the sectoral price rigidity $(1 - \delta_i)/\delta_i$. The OG policy assigns higher weights to sectors with high nominal rigidities, which is consistent with the results in closed economies (La'O and Tahbaz-Salehi, 2022; Rubbo, 2023). Important in an open economy, however, equation (40) indicates that the weight assigned to sector i is proportional to the OG weight $\mathcal{M}_{OG,i}$ defined in equation (39), which internalizes the structure of the domestic and cross-border input-output linkages as stated in Theorem 1. The next section studies the role of the structure of import-export and the production network for the OG monetary policy.

3.4 Role of domestic and cross-border input-output linkages

In this section, we study how the shares of sectoral import and export interplay with the input-output linkages to determine the OG policy through the sectoral OG weights. We focus on the following two questions: (i) What are the roles of the domestic and cross-border input-output linkages for the weights of sectoral inflation in the OG policy? (ii) What is the pitfall in the monetary policy that adopts the Domar weights—which close the output gap in the closed economy, disregarding the roles of cross-border linkages?

Import shares and OG weights. Our definitions of centralities (i.e., domestic supplier, customer, and net export centralities in equations (31), (33), and (34)) include the Leontief inverse that depends on the import shares and input-output matrix. Therefore, by combining the equations of centralities and the decomposition equations of the Leontief inverse (29) and (30), respectively, we determine how the import structure of the economy influences our centrality measures and the sectoral OG weights, as summarized by the following propositions.

Proposition 3. *Domestic supplier centrality of the domestic sector i (i.e., $\widetilde{\lambda}_{D,i}$) strictly decreases in its import share of consumption $(1 - v_i)$ if and only if $\beta_i > 0$; $\widetilde{\lambda}_{D,i}$ strictly decreases in its direct downstream sector r's import share of sector i goods (i.e., $\omega_{r,i} v_{x,r,i} > 0$) if and only if $\widetilde{\lambda}_{D,r} > 0$; $\widetilde{\lambda}_{D,i}$ strictly decreases*

[22] The OG monetary policy can be achieved because of two reasons: first, the aggregate output gap strictly increases in the amount of money supply (see Lemma 14 in Section E.10 of the Supplementary Material); and second, inflation in each sector strictly increases in the aggregate output gap as a result of the positive slopes of the sectoral Phillips curves (see equation A.3 in Online Appendix A).

in its indirect downstream sector s′ import share of sector r goods if and only if $\widetilde{\lambda}_{D,s} > 0$, $\omega_{s,r} > 0$, *and* $\ell_{vx,r,i} > 0$.

Proof: See Section E.7 of the Supplementary Material.

Proposition 3 shows that the domestic supplier centrality of a domestic sector i decreases in sector i's import share of foreign goods as consumption, as well as sector i's direct and indirect downstream sectors' import shares (of intermediate inputs). Intuitively, more direct and indirect imports reduce the sector's contribution to the domestic aggregate output, thereby reducing the size of the CPI channel and resulting in a smaller OG weight. This implies that monetary policy *should* assign higher weights to inflation in domestic sectors with small direct and indirect (via downstream sectors) import shares.[23]

To summarize the relevance of the different import shares for a sector's domestic supplier centrality and thus the OG weight, we construct a single measure—the *import intensity*—defined as follows:[24]

Definition 9. *The* ***import intensity*** *of a domestic sector i is defined as*

$$\textit{Import Intensity} = 1 - \widetilde{\lambda}_{D,i}/\widetilde{\lambda}_{All,D,i} \geq 0, \tag{41}$$

where $\widetilde{\lambda}_{All,D,i}$ *is the i-th entry of the vector* $\boldsymbol{\beta}^{\top}(\mathbf{I}-\boldsymbol{\Omega})^{-1}$.

The term $\widetilde{\lambda}_{All,D,i}$ in equation (41) captures the domestic demand that reaches the domestic sector i directly and indirectly via downstream sectors *if* the entire economy—including sector i and its downstream sectors—does not import from abroad (i.e., $v_r = 1$ for all r and $v_{x,r,s} = 1$ for all r and s). Accordingly, $\widetilde{\lambda}_{D,i}/\widetilde{\lambda}_{All,D,i}$ captures the ratio of the domestic demand for sector i's goods in the baseline economy with imports to the domestic demand in the counterfactual economy without imports. Therefore, the single metric of *import intensity* measures the impact of the import shares of the economy on the demand for sector i's goods by domestic consumers and producers.[25]

Pitfalls in the monetary policy that disregards cross-border linkages. To investigate the relevance of accounting for the degree of openness for the monetary policy, we study the pitfalls of adopting the sectoral weights that close the output gap in the closed economy instead of the OG weights, disregarding the role of cross-border linkages.

[23] Our model with a fully-fledged production network and analytical solutions allows us to identify three channels determining the sectoral weights in the monetary policy. The net export centrality in our analysis encompasses the export share of upstream sector that Wei and Xie (2020) outline by numerical simulations in the special case of a vertical network.

[24] Using the single measure of the relevance of the different import shares for a sector's domestic supplier centrality, we quantitatively examine the importance of import intensity for sectoral OG weights in Section 4.1.

[25] In our quantitative analysis in Section 4.1, we show that domestic supplier centrality is the most relevant element for the level of OG weight in the data. Therefore, the OG weight of a domestic sector—which increases with the domestic supplier centrality as in equation (39)—decreases with the *import intensity*.

As a first step, we derive the OG weight in closed economies. In a closed economy, as we discussed in Section 3.2 (Table 2), the domestic supplier centrality in the OG weight in equation (38) reduces to the Domar weight, and the *net export income* and *net profit income channels* are equal to zero. Therefore, the OG weights are equal to the Domar weights in closed economies, which is consistent with the results in La'O and Tahbaz-Salehi (2022) and Rubbo (2023), as summarized by the next lemma:

Lemma 4. *In a closed economy, the OG weight of any sector reduces to the Domar weight, i.e., $\mathcal{M}_{OG,i} = \lambda_i$ for each sector $i \in \{1, 2, \cdots, N\}$.*

Proof: See Section E.8 of the Supplementary Material.

Lemma 4 implies that the monetary policy that aims at closing the domestic output gap but yet disregards the cross-border linkages will adopt the Domar weight in place of the OG weight. Unlike in closed economies, the Domar weight in an open economy comprises not only the *domestic supplier centrality* ($\widetilde{\lambda}_{D,i}$), but also the *foreign supplier centrality* ($\widetilde{\lambda}_{F,i}$), as stated by the following lemma:

Lemma 5. *In the open economy, the Domar weight of the sector i equals the sum of the sectoral domestic and foreign supplier centralities, i.e.,*

$$\lambda_i = \widetilde{\lambda}_{D,i} + \widetilde{\lambda}_{F,i}. \tag{42}$$

Proof: See Section E.9 of Supplementary Material.

Lemma 5 shows that in the open economy, the sales of each sector—which is proportional to the sector's Domar weight—comprises the sales to both domestic and foreign customers, captured by the terms $\widetilde{\lambda}_{D,i}$ and $\widetilde{\lambda}_{F,i}$, respectively, in equation (42) for the Domar weight.

Combining Lemmas 4 and 5 yields the percentage deviation of the closed-economy OG weight (i.e., the Domar weight) from the open-economy OG weight, as outlined in the following proposition:

Proposition 4. *The percentage deviation of the Domar weight from the OG weight is equal to*

$$\frac{\lambda_i - \mathcal{M}_{OG,i}}{\lambda_i} = \frac{\widetilde{\lambda}_{F,i}}{\lambda_i} - \kappa_S \cdot \frac{\sum_r \rho_{NX,r} \widetilde{\alpha}_r l_{vx,r,i}}{\lambda_i} - \kappa_S \cdot \frac{\widetilde{\lambda}_{F,i} - \lambda_i (1 - \widetilde{\alpha}_i)}{\lambda_i}. \tag{43}$$

Proof: This a straightforward result from Lemma 5 and Theorem 1.

Proposition 4 shows that the Domar weight is different from the OG weight, especially in the domestic sectors with large $\widetilde{\lambda}_{F,i}/\lambda_i$, which we define as the *export intensity* in the next definition.

Definition 10. *The **export intensity** of a domestic sector i is defined as*

$$\textit{Export Intensity} = \widetilde{\lambda}_{F,i}/\lambda_i. \tag{44}$$

The *export intensity* in Definition 10 measures the share of a sector's direct and indirect exports (via downstream sectors) to foreign countries in its total sales.

Proposition 4 demonstrates that the difference between the Domar weight and the OG weight is larger for domestic sectors that are important direct and indirect suppliers to foreign demand, as evinced by the *export intensity* (i.e., the first term on the RHS of equation 44). Intuitively, sectors with large export intensity chiefly supply inputs to foreign countries rather than to the domestic aggregate output. Therefore, the monetary policy that aims at closing the *domestic* aggregate output gap, but yet disregards the cross-border linkages and uses the Domar weight, will over-emphasize the inflation of these sectors.

Proposition 4 also shows that the difference between the Domar and the OG weights is larger for domestic sectors that use *less* domestic labor directly or indirectly (via upstream sectors), as evinced by the customer centrality $\widetilde{\alpha}_r$ in the second term on the RHS of equation (43). Intuitively, sectors with small customer centrality use limited domestic inputs and contribute less to domestic labor income and aggregate output. Therefore, treating the economy as closed and assuming that all sectors contribute only to domestic-labor income (i.e., $\widetilde{\alpha}_r = 1$ for all r) over-emphasizes the importance of inflation in sectors with small customer centrality for the aggregate output gap. The difference between the Domar and the OG weights increases with the degree of openness of the economy, as evinced by the parameter κ_S in the RHS of equation (43).

Typical sectors with large export intensity and small customer centrality are the *export processing* sectors, which primarily supply inputs to foreign instead of domestic demand and use mostly foreign inputs rather than domestic labor.

4 Quantitative analysis

In this section, we quantify our theoretical results by calibrating the model to the input-output matrices of the major OECD economies. Subsection 4.1 studies the relevance of different channels in explaining the differences in the OG weights across sectors for the open economies in our sample, showing that the CPI and the net export income channels explain the bulk of the OG weight. Subsection 4.2 studies the role of different centrality measures for the OG weight and for the difference between Domar weights and OG weights. Subsection 4.3 compares the welfare of alternative monetary policies, showing that the OG policy is welfare-enhancing in open economies. However, adopting the OG weights in place of the Domar weights generates limited welfare improvement in economies with a low degree of openness like the US.

4.1 Variance decomposition of sectoral OG weights

We study the relevance of the three channels for the sectoral OG weights using data for 43 economies (28 EU and 15 OECD countries, each of them comprising 56 sectors) from the WIOD for the year 2014.[26] We calibrate the input-output matrix and import and export shares using the WIOD sector-level data for each country. Shown in Table 3 is the calibration of the key parameters in our model. Online Appendix C.1 presents the calibration for all parameters and provides relevant details on the WIOD.

Table 3: Model calibration

Parameters	Data variables/moments used
Common across all countries	
Risk aversion, $\sigma = 2$	Business cycle literature (e.g., Corsetti et al., 2010; Arellano et al., 2019)
Labor supply elasticity, $\varphi = 1$	Business cycle literature (e.g., Corsetti et al., 2010; Arellano et al., 2019)
Elasticity of substitution (EOS) across varieties, $\varepsilon_i = 8$	Atkeson and Burstein (2008)
EOS. btw. domestic and foreign goods, $\theta_i = \theta_{Fi} = 5$	Head and Mayer (2014)
Sector-level frequency of price adjustment, δ_i	Pasten et al. (2024)
Frequency of wage adjustment, δ_0	Beraja et al. (2019) and Barattieri et al. (2014)
Country specific	
Input-output matrix, $\mathbf{\Omega}$	Sectoral gross output, intermediate goods from both domestic and foreign
Home bias for firms' import $\mathbf{V}_x$	Intermediate goods from both domestic and foreign
Labor share, $\boldsymbol{\alpha}$	Sectoral gross output, labor compensation
Export to foreign countries in steady state, $\mathbf{D}_H^{*,ss}$	Sectoral exports to foreign countries
Consumer consumption share, $\boldsymbol{\beta}$	Sectoral consumption from both domestic and foreign, GDP
Consumer consumption home bias, $\boldsymbol{v}$	Sectoral consumption from both domestic and foreign

For each country, we compute the percentage contributions of each of the three components in the OG weights shown in equation (39)—namely, the *CPI channel* $\widetilde{\lambda}_{D,i}$, the *net export income channel* $\kappa_S \widetilde{\rho}_{NX,i}$, and the *net profit income channel* $\kappa_S[\widetilde{\lambda}_{F,i} - \lambda_i(1-\widetilde{\alpha}_i)]$, respectively—to the variance of the OG weight using the following variance decomposition:

$$100\% = \frac{cov(\widetilde{\lambda}_{D,i}, \mathcal{M}_{OG,i})}{var(\mathcal{M}_{OG,i})} + \frac{cov(\kappa_S \widetilde{\rho}_{NX,i}, \mathcal{M}_{OG,i})}{var(\mathcal{M}_{OG,i})} + \frac{cov(\kappa_S[\widetilde{\lambda}_{F,i} - \lambda_i(1-\widetilde{\alpha}_i)], \mathcal{M}_{OG,i})}{var(\mathcal{M}_{OG,i})}. \tag{45}$$

Plotted in Figure 1 are the percentage contributions of each of the three channels to the total variation of OG weights for each country in the sample. Each set of the vertically aligned markers in blue circles, red dots, and green stars represents the contributions of the *CPI channel*, the *net export income channel*, and the *net profit income channel*, respectively, for a specific country. The vertical dashed lines show the cases for the USA, Mexico, and Luxembourg, as representative economies with different degrees of openness (from relatively closed to fully open). The dashed-blue, solid-red, and dash-dotted-green lines show the fitted curves for each of the three channels across countries.

As depicted in the figure, the *CPI channel* (blue circle) and *net export income channel* (red dot) explain the bulk of the variation in the sectoral OG weights across sectors for all countries. In contrast, the contribution of the *net profit income channel* (green star) is negligible, as evinced by the near zero dashed-dotted green line. Moreover, the percentage contribution of the *net export income channel* (*CPI channel*) increases (declines) with the openness of the country measured by the economy-wise export-to-GDP ratio, as shown by the rising solid-red line (the declining dashed-blue line).[27] For example, in Luxembourg—the most open economy in our sample with

[26]Data source: https://www.rug.nl/ggdc/valuechain/wiod/?lang=en. The release of WIOD in 2016 provides information for the period 2000-2014. In our analysis, we use the latest available year of 2014.

[27]The patterns are robust to the alternative measurement of the degree of openness using the economy-wise import-to-GDP ratio and the ratio of total trade volume to GDP.

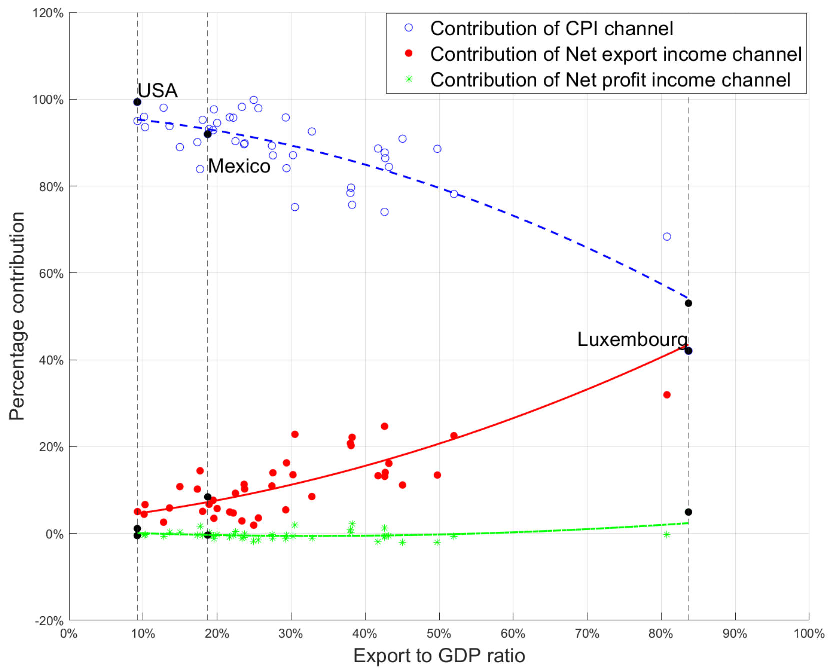

Figure 1: Variance decomposition of OG weights

Notes: Shown in the scatter plot are the percentage contributions of each of the three channels to the OG weight for each country (y-axis) against the country's economy-wise export-to-GDP ratio (x-axis). The CPI, the net export income, and the net profit income channels are marked as blue circles, red dots, and green stars, respectively. The dashed-blue, solid-red, and dash-dotted-green lines are the fitted curves for the CPI, the net export income, and the net profit income channels across countries, respectively.

an economy-wise export-to-GDP ratio of 83%—the contribution of the *CPI channel* is inferior to the *net export income channel* (42% vs. 53%). In contrast, in Mexico—a moderately open economy with an export-to-GDP ratio of 19%—the contribution of the *CPI channel* to the OG weight is large compared to the *net export income channel* (92% vs. 8%). Finally, for the US—a nearly closed economy with an export-to-GDP ratio of 9%—the *CPI channel* contributes to almost the entire variation in OG weights (99%) while the contribution of the *net export income channel* is minimal (1%).

4.2 Determinants of OG weights and Domar-OG difference

In this section, we use panel regressions to study the relevance of our centrality measures for the variation in the sectoral OG weight. We quantify the theoretical results (shown in Section 3.4) by showing that sectors with large *domestic supplier centrality* and small *import intensity* have large OG weights, and sectors with large *export intensity* and small *customer centrality* have a large difference between the Domar and OG weights—the former correspond to the OG weights in closed economies.

Panel regression across sectors and countries. We validate our theoretical results using the following regressions:[28]

$$y_{c,i} = \mathbf{X}_{c,i}^{\top}\boldsymbol{\beta} + \eta_c + \epsilon_{c,i}, \quad \text{with} \quad y_{c,i} \in \{\mathcal{M}_{OG,c,i}, (\lambda_{c,i} - \mathcal{M}_{OG,c,i})/\lambda_{c,i}\}, \tag{46}$$

where the dependent variable $y_{c,i}$ is either the level of the OG weight ($\mathcal{M}_{OG,c,i}$), or the percentage difference between the Domar and OG weights ($(\lambda_{c,i} - \mathcal{M}_{OG,c,i})/\lambda_{c,i}$) for sector i and country c. The variable $\boldsymbol{X}_{c,i}$ includes our centrality measures for the regressions with the OG weight as the dependent variable (see Table 4); it also includes interaction terms between centralities for the regressions with the Domar-OG difference as the dependent variable (see Table 5). The variable η_c is the country fixed effect.

Equation (46) allows testing the two main theoretical results (Section 3.4): (i) the sectoral OG weight increases in the domestic supplier centrality and decreases in the import intensity, and (ii) sectors with large export intensity or small customer centrality entail large differences between the Domar and OG weights.

Table 4: Centrality measures and the OG weight in the data

	(1)	(2)	(3)	(4)	(5)
Domar weight	0.620***				0.544***
	(0.089)				(0.080)
Domestic supplier centrality		1.022***			
		(0.013)			
Import share			-0.000154*		
			(8.20e-05)		
Import intensity				-0.063***	-0.034***
				(0.005)	(0.005)
Observations	601	601	601	601	601
Adjusted R-square	0.739	0.927	0.010	0.322	0.821
Country FE	Yes	Yes	Yes	Yes	Yes

Notes: Shown in the table are the regression results based on equation (46), which regresses the level of the sectoral OG weight over the centrality measures defined in Section 3.2. The analysis includes the subsample of 11 relatively open economies—in terms of the economy-wise export-to-GDP ratio—out of all 43 economies. Country fixed effects are controlled. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Shown in Table 4 are the estimates of the equation with the level of sectoral OG weight ($\mathcal{M}_{OG,c,i}$) as the dependent variable. Column (2) shows that the domestic supplier centrality is positively related to the OG weight of the sector with a coefficient equal to 1.02 and an R-square equal to 0.93.

In contrast, Column (1) shows that the Domar weight—which is the nearly optimal OG weight

[28]We focus on the subsample of 11 relatively open economies—in terms of the economy-wise export-to-GDP ratio—out of all 43 economies. Results are robust, albeit less strong, for less open economies. We do not include sectoral fixed effects in the regression, as our main purpose is to explore the variations in OG weights across different sectors.

in closed economies á la Rubbo (2023) and La'O and Tahbaz-Salehi (2022)—has a small explanatory power compared to domestic supplier centrality, as evinced by the smaller R-square of 0.74.

Columns (4) and (5) examine how the import intensity is related to the sectoral OG weight. As shown in Column (4), a sector's OG weight significantly decreases with the import intensity, as evinced by the negative coefficients of import intensity.[29] Column (5) shows that the import intensity significantly increases the explanatory power of the Domar weight for the closed-economy policy to the variation in the sector's OG weight, as evinced by the larger R-square of 0.82 than 0.74 in Column (1).

Overall, the estimates in Table 4 corroborate our result that the sectoral OG weight increases in the domestic supplier centrality and, therefore, decreases with the import intensity of the sector that measures its direct and indirect (via downstream sectors) imports of foreign products (Proposition 3).

Table 5: Centrality measures and the difference between Domar and OG weights

	(1)	(2)	(3)	(4)	(5)
Export intensity	0.722***				1.107***
	(0.012)				(0.040)
Export share		0.569***		0.617***	
		(0.009)		(0.040)	
Customer centrality			-1.070***	-0.249***	0.065
			(0.041)	(0.048)	(0.041)
Export share × Customer centrality				-0.249***	
				(0.053)	
Export intensity × Customer centrality					-0.756***
					(0.050)
Observations	601	601	601	601	601
Adjusted R-square	0.891	0.868	0.591	0.908	0.948
Country FE	Yes	Yes	Yes	Yes	Yes

Notes: Shown in the table are the regression results based on equation (46), which regresses the sectoral Domar-OG percentage difference $(\lambda_i - \mathcal{M}_{OG,i})/\lambda_i$ over the centrality measures defined in Section 3.2 and the interaction terms between the centrality measures. The analysis includes the subsample of 11 relatively open economies—in terms of the economy-wise export-to-GDP ratio—out of all 43 economies. Country fixed effects are controlled. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Shown in Table 5 are the results for the version of the regression in equation (46) with the percentage difference between the Domar and OG weights ($(\lambda_{c,i} - \mathcal{M}_{OG,c,i})/\lambda_{c,i}$) as the dependent variable. Columns (1) to (3) show that the Domar-OG percentage difference increases with the export share and intensity of the sector, and it decreases with customer centrality. The export intensity is positively related to the Domar-OG difference with a coefficient of 0.72 (Column 1), which is consistent with our theoretical results in Proposition 4. Furthermore, it has the largest explanatory power among all single explanatory variables of the Domar-OG difference,

[29] Additionally, we find that a sector's OG weight also decreases in its export share and export intensity, but with small R-squares of 0.156 and 0.207, respectively.

as evinced by the largest R-square of 0.89.

Column (5) shows that conditional on the export intensity, customer centrality is negatively related to the Domar-OG difference with a coefficient of -0.76, and the customer centrality and export intensity explain most of the variations in the Domar-OG difference, as evinced by the large R-square of 0.95.

Overall, results in Table 5 support Proposition 4, demonstrating that monetary policy adopting Domar weights over-emphasizes inflation in sectors with either large export intensity or limited customer centrality.

The role of production networks in small open economies. In a small open economy without production networks, each sector contributes only directly to domestic and foreign demand as an input supplier, leading the import and export intensities in equations (41) and (44) to reduce to the import and export shares (i.e., $1 - v_i$ and $\lambda_{EX,i}/\lambda_i$), respectively. We determine the role of production networks for monetary policies in small open economies by comparing the explanatory powers of import intensities over import shares for the OG weights in Tables 4, as well as by comparing the export intensities over export shares for the Domar-OG difference in Table 5.

Comparing Columns (3) and (4) of Table 4 reveals that import intensity explains more variation in OG weights than the direct import share, as evinced by the larger R-square for import intensity (0.32) and the almost negligible R-square for import share (0.01). Thus, indirect imports via upstream sectors—captured by import intensity rather than import shares—are important for the sectoral OG weights, hence supporting the relevance of input-output linkages for monetary policy. Moreover, comparing Columns (5) and (4) of Table 5 indicates that export intensity explains a larger variation in the sectoral Domar-OG difference than the direct export share, as evinced by the higher R-square in the regression with export intensity (0.95) than in the regression with export share (0.91). Thus, indirect exports via downstream sectors—captured by export intensity rather than export shares—are important for the sectoral Domar-OG difference, again underlying the relevance of input-output linkages for monetary policy.

Therefore, we conclude that the structure of input-output linkages interplays with the imports and exports of the small open economy to determine the weights of the OG policy that closes the aggregate output gap.

The case of Mexico. To provide concrete examples of sectors that illustrate the relationship between the sectoral OG weights and our centrality measures outlined in Section 3.4, we examine the case of Mexico, a prototypical small open economy.

Shown in Table 6 is the role of direct and indirect downstream sectors' import shares in determining the OG weight for two representative sectors. The sector *manufacture of computer, electronic, and optical products* (sector 17) imports heavily (directly and indirectly), with an import intensity of 96.1%, leading to a tiny OG weight of 0.003. In contrast, the sector *manufacture of*

Table 6: Import intensity and sectoral OG weights for typical sectors in Mexico

Sector name	Sector ID	Import intensity	OG weight
Manufacture of computer, electronic, and optical products	17	96.1%	0.003
Manufacture of motor vehicles, trailers, and semi-trailers	20	33.0%	0.05

motor vehicles, trailers, and semi-trailers (sector 20) has limited imports with an import intensity of 33%, leading to a significantly larger OG weight (0.05). Thus, we conclude that sectors with small import intensities tend to have large OG weights, so the monetary policy should assign them larger weights, consistent with Columns (4) and (5) in Table 4 and Proposition 3 in Section 3.4.

Table 7: Export intensity and Domar-OG difference for typical sectors in Mexico

Sector name	Sector ID	Export shares	Export intensity	Customer centrality	Domar-OG difference
Accommodation and food service activities	36	0%	1.3%	0.922	1.2%
Manufacture of machinery and equipment	19	99%	99%	0.681	87%
Manufacture of basic metals	15	38%	82%	0.785	65%

Displayed in Table 7 are three representative sectors in Mexico to illustrate the role of the direct and indirect exports via downstream sectors in determining the Domar-OG difference of a domestic sector. First, the *manufacture of machinery and equipment* (sector 19) has a large export share and export intensity of about 99%, leading to a Domar-OG difference of 87%. In contrast, sector *accommodation and food service activities* (sector 36)—which exports little with a small export intensity of 1.3%—has a tiny Domar-OG difference of 1.2%. Moreover, although the sector *manufacture of basic metals* (sector 15) does not directly export (i.e., with a small export share of 38%), it is mainly a supplier of inputs for the exporting sector *manufacture of electrical equipment* (sector 18) that has a large export share of 99%. As a result, the sector *manufacture of basic metals* (sector 15) has a large export intensity of 82%, leading to a higher Domar weight than OG weight by as much as 65%, despite not being a direct exporter.

4.3 Welfare comparison of alternative monetary policies

In this section, we compare the welfare losses of the economy under alternative monetary policies. Specifically, welfare loss is defined as the difference between the welfare under the sticky-price equilibrium (with specific monetary policies) and the efficient flexible-price equilibrium up to the second-order approximation, as shown in equation (A.1) of Proposition 5 in Online Appendix A. In Online Appendix A, we derive the analytical solutions of welfare loss, sectoral Phillips curves, and the optimal monetary policy that minimizes welfare loss, and discuss how they differ in open economies from closed economies.

Simulation results using the above analytical solutions show that the OG policy performs closely to the optimal monetary policy, and outperforms the policies that: (i) weight sectoral inflation with the Domar weights and sectoral price rigidities—which we refer to as the Domar-weight policy—and (ii) weight sectoral inflation with consumption weights (i.e., β_i) and sectoral price rigidities—which we refer to as the CPI-weight policy. We focus on these alternative policies because they are widely used policies that target PPI and CPI inflation, respectively—the former ignores the openness of the economy, and the latter ignores both the openness and the input-output linkages.[30]

Specifically, we compare the welfare loss under the following five alternative monetary policies: the optimal policy, the OG policy, the Domar-weight policy, the CPI-weight policy, and the OG policy that ignores the network structure of the economy. The Domar-weight (CPI-weight) policy targets an aggregate inflation index where the Domar weight λ_i (consumption share β_i), after adjusting for sectoral price rigidities (i.e., multiply by $(1-\delta_i)/\delta_i$), is used as the weight for each sector i's inflation. Intuitively, we can interpret that the Domar-weight policy is the OG monetary policy that treats the economy as closed, and the CPI-weight policy is the OG monetary policy that *both* treats the economy as closed *and* ignores the input-output linkages across sectors.

Shown in Table 8 is the total welfare loss expressed as a percentage of the steady-state consumption under the alternative monetary policies. We consider the welfare loss for Mexico, Luxembourg, and the US that represent countries with medium, large, and small degrees of openness, respectively—as measured by the economy-wise export-to-GDP ratio (19%, 83%, and 9%). Using equation (A.1) in Online Appendix A, we decompose the welfare loss into the *output gap misallocation* and the *within- and across-sector misallocation*. We also use equation (A.5) to decompose the welfare loss arising from the *within- and across-sector misallocation* into two sub-components: (i) the output-gap-related term, and (ii) a policy-irrelevant term, respectively.

As shown in Table 8, the OG monetary policy yields a welfare loss that is close to the optimal policy and significantly outperforms the Domar-weight and CPI-weight policies, which ignore cross-border linkages and *both* domestic *and* cross-border input-output linkages, respectively. For Mexico, the difference in the welfare loss between the optimal and the OG policies is tiny and equal to 0.020 percent of the steady-state consumption (-1.859 vs. -1.879), establishing that the OG policy is nearly optimal. Important to our analysis, the OG policy improves the welfare loss over the Domar-weight policy by 0.043 percent of the steady-state consumption, and it generates an even larger improvement over CPI-weight policies (-1.879 vs. -1.922 vs. -4.968). The welfare improvement of the OG policy over the Domar-weight (CPI-weight) policy corresponds to 67.1% (99.3%) of the welfare difference between the optimal and the Domar-weight (CPI-weight) policy. The welfare improvement of the OG policy over the Domar-weight and CPI-weight policies exhibits welfare enhancement if the design of monetary policy accounts for openness and the

[30]For each economy, we compute welfare losses under different monetary policies using the same simulations of log-normal shocks to the sectoral import prices. For simplicity, we assume that the shocks to different sectors share the same mean. We set the mean of sectoral shocks to generate an average CPI inflation of 2% for each economy to compare—under the same aggregate level of inflation—the welfare losses across different economies with different openness and structures of input-output linkages. The variance-covariance matrix of these shocks is calibrated to that of Mexico. We simulate the shocks 100,000 times to compute the expected welfare loss under each of the alternative monetary policies.

Table 8: Welfare loss under different monetary policies

	(1) Optimal	(2) OG	(3) Domar	(4) CPI	(5) OG w/o network
Mexico Export-to-GDP ratio: 19%					
Total welfare loss	-1.859	-1.879	-1.922	-4.968	-4.948
Improvement by OG policy towards optimal			67.1%	99.3%	99.3%
Output gap misallocation	-0.003	0.000	-0.002	-0.388	-0.385
Within- and across-sector misallocation					
— output-gap-related	0.024	0.000	-0.041	-2.701	-2.684
— policy-irrelevant	-1.879	-1.879	-1.879	-1.879	-1.879
Luxembourg Export-to-GDP ratio: 83%					
Total welfare loss	-7.742	-7.777	-8.504	-10.675	-11.551
Improvement by OG policy towards optimal			95.4%	98.8%	99.1%
Output gap misallocation	-0.006	0.000	-0.089	-0.427	-0.569
Within- and across-sector misallocation					
— output-gap-related	0.041	0.000	-0.638	-2.471	-3.205
— policy-irrelevant	-7.777	-7.777	-7.777	-7.777	-7.777
US Export-to-GDP ratio: 9%					
Total welfare loss	-1.400	-1.472	-1.476	-6.757	-6.546
Improvement by OG policy towards optimal			5.4%	98.6%	98.6%
Output gap misallocation	-0.011	0.000	0.000	-0.623	-0.596
Within- and across-sector misallocation					
— output-gap-related	0.083	0.000	-0.004	-4.662	-4.478
— policy-irrelevant	-1.472	-1.472	-1.472	-1.472	-1.472

Notes: Reported in the table is the welfare loss—expressed in units of percent of steady-state consumption—under different monetary policy designs. Columns (1) to (5) show the welfare losses under the optimal policy, the OG policy, the Domar-weight policy, the CPI-weight policy, and the OG policy that ignores the production network, respectively. The sectoral weights in all five policies adjust for sectoral price rigidities. Online Appendix C.2 outlines the sectoral weights adopted by the alternative monetary policies.

input-output linkages of the economy.[31]

The role of input-output linkages for the welfare loss in open economies can be assessed by comparing Column (5) to Column (2) in Table 8. The OG monetary policy in Column (5) ignores the production network entirely (i.e., $\boldsymbol{\Omega} = \mathbf{0}_{N\times N}$, $\boldsymbol{\alpha} = \mathbf{1}$), generating a much larger welfare loss than the OG monetary policy that accounts for the production network (-4.948 vs. -1.879).

Decomposing the total welfare loss into different components illustrates why the OG policy is closer to the optimal policy and improves over Domar-weight and CPI-weight policies. The OG policy closes the aggregate output gap, therefore, setting the welfare loss arising from the output gap misallocation and the output-gap-related component in the cross- and within-sector misallocation to zero. Quantitatively, Table 8 shows that these two components related to the

[31] In Online Appendix C.3, we show our results are robust to alternative shocks, including shocks to the import prices of only manufacturing sectors (with sector IDs from 6 to 24 in Table C.1) and shocks to sectoral productivity.

aggregate output gap generate large welfare losses in Mexico for the Domar-weight policy (-0.002 and -0.041), and even larger losses for the CPI-weight policy (-0.388 and -2.701). This result shows that the OG policy improves over the policies that target price-rigidity-adjusted PPI and CPI inflation (i.e., Domar-weight and CPI-weighted policies) by eliminating the aggregate output gap, which supports the adoption of the OG policy to enhance welfare in small open economies.

Finally, we examine the welfare loss under alternative monetary policies for two *additional* economies: namely, Luxembourg and the US, which represent the polar cases of open and closed economies, respectively. In the most open economy of Luxembourg (the middle panel of Table 8), the OG policy improves over the Domar-weight policy by a large 95.4%, compared to a more limited 67.1% for Mexico. The same qualitative results outlined for Mexico hold for Luxembourg and are stronger quantitatively. The bottom panel of Table 8 presents the welfare loss for the nearly closed economy of the US, showing that the OG and Domar-weight policies yield similar welfare loss and they are equally close to the optimal policy, echoing the results of La'O and Tahbaz-Salehi (2022) and Rubbo (2023) in closed economies. Therefore, we conclude that the difference between the OG and the Domar-weight policies is significant for open economies while its importance diminishes in relatively closed economies like the US.

5 Conclusion

This paper investigates the design of monetary policy in small open economies with domestic and cross-border input-output linkages and nominal rigidities. Aggregate distortions are proportional to the aggregate output gap, which can be expressed as a weighted average of sectoral markup wedges that encapsulate the inefficiency in each sector. Monetary policy can close the output gap and offset the sectoral distortions by stabilizing the aggregate index of inflation that weights inflation in each sector based on the degree of nominal rigidities and the centrality of the sector as a supplier of inputs and a net exporter of products within the international production networks. To close the output gap, monetary policy should assign larger weights to inflation in sectors with small direct or indirect (via the downstream sectors) import shares, and failing to account for the cross-border production networks overemphasizes the inflation in sectors that export intensively directly and indirectly (via the downstream sectors), generating quantitatively significant welfare losses that rise with the degree of openness of the economy. We derive the closed-form solution for the optimal monetary policy that minimizes the welfare losses up to the second-order approximation and show that the OG policy generates welfare losses quantitatively close to the optimal policy and, therefore, is nearly optimal.

We calibrate our model to the WIOD and validate our theoretical results. The OG policy outperforms alternative monetary policies that abstract from the openness of the economy by using Domar weights or the input-output linkages by targeting the open-economy CPI inflation index. Overall, our analysis demonstrates that openness and domestic input-output linkages are jointly important for the conduct of monetary policy in small open economies with international production networks.

Our study suggests several interesting avenues for future research. First, the analysis could be extended by relaxing the assumption of financial autarky and studying the interplay between the incompleteness of the financial market and the production networks for the design of monetary policy. Second, the analysis could be extended to cases in which fiscal policy fails to offset the first-order distortions with non-contingent subsidies, leading to an inefficient flexible-price equilibrium as in Baqaee and Farhi (2024), such that the monetary policy needs to account for the interaction between the supply-side effect of monetary policy and the openness of the economy to improve efficiency. Finally, the analysis could be extended to consider large open economies where monetary policy would need to account for feedback effects from the responses of foreign economies to the domestic policy—which may interplay with international product networks to determine the impact of the domestic monetary policy. We plan to investigate some of these issues in future work.

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Online Appendix

A Welfare loss and optimal monetary policy

In this section, we study optimal monetary policy. As per Woodford (2003) and Galí (2015), we derive the closed-form solution of the optimal monetary policy that minimizes welfare losses (or, equivalently, the aggregate wedges) up to the second-order approximation.

Welfare loss and sectoral Phillips curves. Based on the assumption of non-contingent subsidy and tax rates that offset monopolistic distortions (Lemma 1), the *flexible-price equilibrium* is efficient. We define *welfare loss* as the difference in utility of the representative household between the *sticky* and *flexible-price equilibria*: $u(\boldsymbol{\xi}) - u^{flex}(\boldsymbol{\xi})$. We approximate the welfare-loss function up to the second-order approximation and show that the loss originates from the labor *and* efficiency wedges, as stated in the following proposition:

Proposition 5 (Welfare loss). *Given the realized state $\boldsymbol{\xi} \in \Xi$, the welfare loss is a function of the aggregate output gap $\widehat{C}^{gap}(\boldsymbol{\xi})$ and the sectoral inflation $\widehat{\mathbf{P}}(\boldsymbol{\xi})$, defined as follows:*[32]

$$u(\boldsymbol{\xi}) - u^{flex}(\boldsymbol{\xi}) = \underbrace{-\frac{1}{2}\Big(\sigma - 1 + \frac{\varphi+1}{\Lambda_L}\Big)\widehat{C}^{gap}(\boldsymbol{\xi})^2}_{\text{output gap misallocation}} \underbrace{-\frac{1}{2}\widehat{\mathbf{P}}(\boldsymbol{\xi})^\top \boldsymbol{\mathcal{L}} \widehat{\mathbf{P}}(\boldsymbol{\xi})}_{\text{within-and-across misallocation}} + o(\|\widehat{\boldsymbol{\xi}}\|^2) \tag{A.1}$$

$$= \underbrace{-\frac{1}{2}\Big(\sigma - 1 + \frac{\varphi+1}{\Lambda_L}\Big)^{-1}\widehat{\Gamma}_L(\boldsymbol{\xi})^2}_{\text{from labor wedge}} \underbrace{-\Big(\widehat{A}_{agg}(\boldsymbol{\xi}) - \widehat{A}^{flex}_{agg}(\boldsymbol{\xi})\Big)}_{\text{from efficiency wedge}} + o(\|\widehat{\boldsymbol{\xi}}\|^2), \tag{A.2}$$

where Λ_L is the steady-state economy-wise labor share in GDP, and $\boldsymbol{\mathcal{L}} \equiv \boldsymbol{\mathcal{L}}^{within} + \boldsymbol{\mathcal{L}}^{across}$ describes the mapping of sectoral inflation into misallocation within *and* across *sectors ($\boldsymbol{\mathcal{L}}^{within}$ and $\boldsymbol{\mathcal{L}}^{across}$, respectively). The within-sector misallocation matrix $\boldsymbol{\mathcal{L}}^{within}$ is diagonal with $\mathcal{L}^{within}_{i,i} \equiv \lambda_i \varepsilon_i (1-\delta_i)/\delta_i$. The across-sector misallocation matrix $\boldsymbol{\mathcal{L}}^{across}$ is presented in Appendix B.1.*

Proof: See Appendix B.1.

Equation (A.1) shows that up to the second-order approximation, *welfare loss* is the sum of the losses from the output gap misallocation and the within- and across-sector misallocation in closed economies, as in La'O and Tahbaz-Salehi (2022) and Rubbo (2023). Equation (A.2) re-writes the welfare loss by substituting equation (27) into equation (A.1), showing that the output gap and the within- and across-sector misallocation correspond to the aggregate labor and efficiency wedges—up to the second-order approximation—defined in equations (24) and (23), respectively.[33] In other words, the second-order welfare loss includes distortions from the

[32] Throughout the paper, minimization of welfare loss refers to maximization of equation (A.1).

[33] The second-order approximation of the term representing the efficiency wedge in equation (A.2) equals the term representing the within-and-across misallocation in equation (A.1).

labor *and* the efficiency wedges. As a result, the OG policy that closes the aggregate output gap and eliminates the labor wedge (obtained in Proposition 2) does not simultaneously balance the distortions arising from the efficiency wedge, such that the OG policy results in a higher welfare loss compared to the optimal policy.

In an open economy, the aggregate output gap, sectoral inflation, and the weights in the welfare loss function depend on the interplay between the openness and structure of the networks. The aggregate output gap in the open economy entails sectoral OG weights different from the Domar weights of the closed economy, as shown in Theorem 1. In particular, Proposition 4 shows that sectors with large export intensity and small customer centrality have smaller OG weights than the Domar weights of a closed economy.

The openness of the economy is important for the mapping from sectoral inflation into the within- and across-sector misallocation (i.e., $\boldsymbol{\mathcal{L}}$). While the matrix representing the within-sector misallocation (i.e., $\boldsymbol{\mathcal{L}}^{within}$) is the same as in a closed economy (La'O and Tahbaz-Salehi, 2022; Rubbo, 2023), the matrix representing the across-sector misallocation (i.e., $\boldsymbol{\mathcal{L}}^{across}$) internalizes the openness of the economy. More specifically, the analytical solution of the across-sector misallocation matrix in Appendix B.1 reveals that the across-sector misallocation in the multi-sector open economy accounts for the distortions in: (i) the allocation of domestic labor into the different sectors; (ii) the allocation of domestic sectoral products into consumption, intermediate inputs, and exports; (iii) the division of the use of consumption goods and intermediate inputs between home and foreign products by domestic households and firms, and (iv) the nominal exchange rate due to distortions in the current account. While the across-sector distortions (i) and (ii) reduce to their closed-economy versions in closed economies (as in La'O and Tahbaz-Salehi, 2022 and Rubbo, 2023), distortions (iii) and (iv) are unique to open economies.

To interpret the welfare loss function in Proposition 5 and attain the optimal monetary policy analytically, we derive the sectoral Phillips curves that link the aggregate output gap and the exogenous sectoral shocks to sectoral inflation, as stated in the next Proposition:

Proposition 6 (Sectoral Phillips curves). *In the* sticky-price equilibrium*, the following multi-sector Phillips curves hold:*

$$\widehat{\mathbf{P}}(\boldsymbol{\xi}) = \underbrace{\boldsymbol{\mathcal{B}}\widehat{C}^{gap}(\boldsymbol{\xi})}_{\text{output-gap-driven inflation}} + \underbrace{\boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}}}_{\text{cost-push inflation}} + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{A.3}$$

where $\widehat{\mathbf{P}}(\boldsymbol{\xi})$ is an N-by-1 vector with sectoral inflation, and parameters $\boldsymbol{\mathcal{B}}$ (an N-by-1 vector) and $\boldsymbol{\mathcal{V}}$ (an N-by-3N matrix) are the slopes of Phillips curves and the coefficients of exogenous shocks, respectively.[34]

Proof: See Appendix B.2.

[34]The definitions and the intuitions of parameter $\boldsymbol{\mathcal{V}}$ and parameters $\boldsymbol{\Delta}_{\Phi}, \Gamma_C, \boldsymbol{\mathcal{M}}_{EX}$, and $\boldsymbol{\mathcal{M}}_{IM}$ that compose $\boldsymbol{\mathcal{B}}$ as in equation (A.4) are presented in Appendix B.2. Specifically, Γ_C is shown in equation (B.39); $\boldsymbol{\mathcal{M}}_{EX}$ and $\boldsymbol{\mathcal{M}}_{IM}$ in equations (B.35) and (B.36) capture the impacts of export demand shocks and import price shocks on the current account, respectively.

In Proposition 6, the slopes of the sectoral Phillips curves are equal to

$$\boldsymbol{\mathcal{B}} \equiv \boldsymbol{\Delta}_{\Phi}\Big\{ \underbrace{\boldsymbol{\alpha}\Gamma_{W,C}}_{\text{nominal wage channel}} + \underbrace{\left(\boldsymbol{\Omega} \odot \mathbf{V}_{\mathbf{1}-\boldsymbol{x}}\right)\mathbf{1}\Gamma_{S,C}}_{\text{nominal exchange rate channel}} \Big\}, \tag{A.4}$$

where $\Gamma_{W,C} \equiv (\Gamma_C + \sigma + \varphi/\Lambda_L)$ and $\Gamma_{S,C} \equiv [(\boldsymbol{\mathcal{M}}_{EX} + \boldsymbol{\mathcal{M}}_{IM})^\top \mathbf{1}]^{-1}(\Gamma_C + 1)$ capture the impacts of the aggregate output gap on sectoral marginal costs and inflation *via* the nominal wage and nominal exchange rate, respectively.

The nominal wage channel in equation (A.4) comprises two sub-effects: first, a positive aggregate output gap increases CPI via the current account and nominal exchange rate, captured by the term Γ_C in $\Gamma_{W,C}$; second, a positive aggregate output gap increases the real wage via the labor supply, captured by the term $(\sigma + \varphi/\Lambda_L)$. In closed economies, the first sub-effect of CPI adjustments via the current account is absent, and the second sub-effect is present but smaller, as the elasticity of labor supply rises to unitary (i.e., $\Lambda_L = 1$).

The nominal exchange rate channel in equation (A.4) works as now described. A positive aggregate output gap raises the nominal expenditure and worsens the current account, leading to a depreciation of the domestic currency and an increase in the nominal exchange rate, represented by the term $\Gamma_{S,C}$ in the equation. The increase in the nominal exchange rate propagates into the costs of imported inputs and thus sectoral inflation, encapsulated by the term $\left(\boldsymbol{\Omega} \odot \mathbf{V}_{\mathbf{1}-\boldsymbol{x}}\right)\mathbf{1}$. The nominal exchange rate channel is unique to open economies and is absent in closed economies (i.e., $\left(\boldsymbol{\Omega} \odot \mathbf{V}_{\mathbf{1}-\boldsymbol{x}}\right)\mathbf{1} = \mathbf{0}$). This channel increases with the openness of the economy, evinced by the matrix of the import shares $\mathbf{V}_{1-x}$. Because both the nominal wage and exchange rate channels are positive, the slopes of the sectoral Phillips curves are positive for all sectors.

In Appendix B.2, we show that the coefficient matrix of the exogenous shocks $\boldsymbol{\mathcal{V}}$ also depends on the openness of the economy. Overall, our analysis demonstrates that the open economy dimension plays an important role in the influence of the aggregate output gap and the exogenous shocks on sectoral inflation.

Welfare loss as a function of the aggregate output gap. We substitute the sectoral Phillips curves (equation A.3) in Proposition 6 into the welfare loss (equation A.1) in Proposition 5 to rewrite the welfare loss as a function of the aggregate output gap and exogenous shocks, yielding the following:

$$u(\boldsymbol{\xi}) - u^{flex}(\boldsymbol{\xi}) = \underbrace{-\frac{1}{2}\left[\sigma - 1 + (\varphi+1)/\Lambda_L\right]\widehat{C}^{gap}(\boldsymbol{\xi})^2}_{\textit{output gap misallocation}}$$

$$\underbrace{\underbrace{-\frac{1}{2}\boldsymbol{\mathcal{B}}^\top\boldsymbol{\mathcal{L}}\boldsymbol{\mathcal{B}}\cdot\widehat{C}^{gap}(\boldsymbol{\xi})^2 - (\boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}})^\top\boldsymbol{\mathcal{L}}\boldsymbol{\mathcal{B}}\cdot\widehat{C}^{gap}(\boldsymbol{\xi})}_{\text{output-gap-related}}\underbrace{-\frac{1}{2}(\boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}})^\top\boldsymbol{\mathcal{L}}(\boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}})}_{\text{policy-irrelevant}}}_{\textit{within- and across-sector misallocation}} + o(\|\widehat{\boldsymbol{\xi}}\|^2). \tag{A.5}$$

Equation (A.5) shows that the welfare loss depends on the *output gap misallocation* (the first line on the RHS of equation A.5, as already shown in equation A.1), as well as the *within- and across-sector misallocation* (the second line of equation A.5). This second component is further decomposed into two sub-components: (i) the output-gap-related component, and (ii) the policy-irrelevant component of exogenous shocks that cannot be influenced by monetary policy.

Equation (A.5) shows that closing the output gap (i.e., $\widehat{C}^{gap}(\boldsymbol{\xi}) = 0$) eliminates the output gap misallocation and the output-gap-related component of the within- and across-sector misallocation, but it is unable to eliminate the misallocation arising from the policy-irrelevant sectoral shocks.

Optimal monetary policy. We analytically derive the optimal monetary policy, and compare it to the OG policy stated in Definition 8. The optimal monetary policy chooses the money supply—which is equivalent to choosing the aggregate output gap $\widehat{C}^{gap}(\boldsymbol{\xi})$ according to Lemma 14 in Section E.10 of the Supplementary Material—that minimizes the welfare loss in equation (A.5), defined as follows.

Definition 11 (Optimal monetary policy)**.** *For any realized aggregate state $\boldsymbol{\xi} \in \Xi$, the optimal monetary policy chooses the money supply $M(\boldsymbol{\xi})$—which is equivalent to choosing the aggregate output gap $\widehat{C}^{gap}(\boldsymbol{\xi})$ in equilibrium—to minimize the welfare loss in equation (A.1)* subject to *the sectoral Phillips curves (A.3).*

Consistent with Definition 11, we derive the aggregate inflation index that the monetary authority should target to implement the optimal monetary policy. Substituting the sectoral Phillips curves (A.3) into the welfare loss function (A.1) yields the implementation rule for the optimal policy, as stated in the next proposition.

Proposition 7 (Implementation of the optimal monetary policy)**.** *The optimal monetary policy is implemented by setting the following aggregate inflation index to zero:*

$$\left\{ \left[\sigma - 1 + (\varphi + 1)/\Lambda_L\right] \kappa_C^{-1} \boldsymbol{\mathcal{M}}_{OG}^{\top} (\boldsymbol{\Delta}^{-1} - \mathbf{I}) + \boldsymbol{\mathcal{B}}^{\top} \boldsymbol{\mathcal{L}} \right\} \widehat{\mathbf{P}} = 0, \tag{A.6}$$

for any realized state $\boldsymbol{\xi} \in \Xi$.

Proof: See Appendix B.3.

Equation (A.6) shows that the optimal monetary policy in open economies differs from the optimal monetary policy in closed economies in three important aspects: (i) the mapping from sectoral inflation into the aggregate output gap (i.e., the OG weights $\boldsymbol{\mathcal{M}}_{OG}$) is such that the Domar weight for the closed-economy policy fails to close the output gap in the open economy, as discussed in Section 3.4; (ii) the effects of movements in the aggregate output gap for sectoral inflation depend on the openness of the economy—evinced by the dependence of the slopes of the sectoral Phillips curve (i.e., $\boldsymbol{\mathcal{B}}$) on the openness of the economy, as shown in equation (A.4);

and (iii) the matrix representing the across-sector misallocation (i.e., $\boldsymbol{\mathcal{L}}^{across}$) in the welfare loss in equation (A.2) depends on the openness of the economy, as discussed in Proposition 5.

Comparing the optimal monetary policy in equation (A.6) with the OG policy in equation (40) shows that the optimal policy accounts for *both* the output gap misallocation—evinced by the OG weights $\boldsymbol{\mathcal{M}}_{OG}^{\top}$ as the first term in the brackets—*and* the within- and across-sector misallocation generated by sectoral distortions—evinced by the second term $\boldsymbol{\mathcal{B}}^{\top}\boldsymbol{\mathcal{L}}$ in the brackets.

To further study the difference between the optimal and the OG monetary policies, we relate the optimal monetary policy to the aggregate output gap by noticing that the optimal monetary policy is equivalent to choosing the aggregate output gap $\widehat{C}^{gap}(\boldsymbol{\xi})$ that minimizes welfare loss in equation (A.5).

Proposition 8 (Aggregate output gap in the optimal monetary policy)**.** *The optimal monetary policy satisfies the first-order condition of equation (A.5) with respect to the aggregate output gap $\widehat{C}^{gap}(\boldsymbol{\xi})$, i.e.,*

$$\left[\sigma - 1 + (\varphi + 1)/\Lambda_L + \boldsymbol{\mathcal{B}}^{\top}\boldsymbol{\mathcal{L}}\boldsymbol{\mathcal{B}}\right]\widehat{C}^{gap}(\boldsymbol{\xi}) + \boldsymbol{\mathcal{B}}^{\top}\boldsymbol{\mathcal{L}}\boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}} = 0. \tag{A.7}$$

Proof: See Appendix B.3.

Proposition 8 highlights that the OG policy—which closes the aggregate output gap (i.e., $\widehat{C}^{gap}(\boldsymbol{\xi}) = 0$)—does not satisfy condition (A.7) for the optimal monetary policy. In multi-sector economies, those sector-specific cost-push components in sectoral Phillips curves do not comove with the one-dimensional aggregate output gap (i.e., $\boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}} \neq \mathbf{0}$ in equation A.3), thus making the OG policy unable to simultaneously minimize the within- and across-sector misallocation (captured by $\boldsymbol{\mathcal{B}}^{\top}\boldsymbol{\mathcal{L}}\boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}}$ in equation A.7). Proposition 8 shows that the "divine coincidence" in multi-sector open economies breaks down as in multi-sector closed economies: the OG policy that closes the output gap does not simultaneously minimize the within- and across-sector misallocation, and is therefore suboptimal.

B Proofs of the theoretical results in Appendix A

This appendix derives the welfare loss up to the second-order approximation and the sectoral Phillips curves, from which we derive the analytical solution of the optimal monetary policy.

B.1 Proof of Proposition 5: welfare loss up to the second-order approximation

Approximating the utility function around the *flexible-price equilibrium* up to the second order yields

$$u - u^{flex} = u_C^{flex} C^{flex}\left[\widehat{C}^{gap} - \frac{\sigma - 1}{2}(\widehat{C}^{gap})^2\right] + u_L^{flex} L^{flex}\left[\widehat{L}^{gap} + \frac{\varphi + 1}{2}(\widehat{L}^{gap})^2\right] + o(\|\widehat{\boldsymbol{\xi}}\|^2). \tag{B.8}$$

Substituting in equation (B.8) the optimality condition of labor supply $-u_L^{flex}/u_C^{flex} = W^{flex}/P_C^{flex}$, the approximation of labor share $\Lambda_L^{flex} \equiv (W^{flex}L^{flex})/(P_C^{flex}C^{flex}) = \Lambda_L + O(\|\widehat{\xi}\|)$, and the approximation of the coefficient $u_C^{flex}C^{flex} = \left(C^{flex}\right)^{1-\sigma} = 1 + O(\|\widehat{\xi}\|)$ under normalized $C^{ss} = 1$, yields

$$u(\xi) - u^{flex}(\xi) = \widehat{C}^{gap}(\xi) - \Lambda_L^{flex}(\xi)\widehat{L}^{gap}(\xi) \\ - \frac{1}{2}\left[(\sigma-1)\widehat{C}^{gap}(\xi)^2 + \Lambda_L(\varphi+1)\widehat{L}^{gap}(\xi)^2\right] + o(\|\widehat{\xi}\|^2). \quad \text{(B.9)}$$

Taking the difference between the log deviation of the efficiency wedge (equation (23) in Definition 3) from the steady state in the *sticky-price equilibrium* and in the *flexible-price equilibrium* and substituting it into equation (B.9), yields the following:

$$\begin{aligned} u(\xi) - u^{flex}(\xi) &= \widehat{A}_{agg}(\xi) - \widehat{A}_{agg}^{flex}(\xi) - \frac{1}{2}\left[(\sigma-1)\widehat{C}^{gap}(\xi)^2 + \Lambda_L(\varphi+1)\widehat{L}^{gap}(\xi)^2\right] + o(\|\widehat{\xi}\|^2) \\ &= \widehat{A}_{agg}(\xi) - \widehat{A}_{agg}^{flex}(\xi) - \frac{1}{2}\left[\sigma - 1 + (\varphi+1)/\Lambda_L\right]\widehat{C}^{gap}(\xi)^2 + o(\|\widehat{\xi}\|^2) \\ &= \widehat{A}_{agg}(\xi) - \widehat{A}_{agg}^{flex}(\xi) - \frac{1}{2}\left[\sigma - 1 + (\varphi+1)/\Lambda_L\right]^{-1}\widehat{\Gamma}_L(\xi)^2 + o(\|\widehat{\xi}\|^2), \end{aligned}$$

where $\widehat{A}_{agg}(\xi)$ and $\widehat{A}_{agg}^{flex}(\xi)$ are the exact log deviations of the efficiency wedge from the steady state in the *sticky-price equilibrium* and in the *flexible-price equilibrium*, respectively. The last two equalities are obtained by substituting in equations (26) and (27) from Proposition 1, respectively.

Next, we derive the second-order approximation of the labor and efficiency wedges separately.

Labor wedge up to the second-order approximation. Combining equation (27) in Proposition 1 and equation (38) in Theorem 1 yields a quadratic form of $\widehat{\mu}(\xi)$:

$$\left[\sigma - 1 + (\varphi+1)/\Lambda_L\right]\widehat{C}^{gap}(\xi)^2 \\ = \kappa_C^{-2}\left[\sigma - 1 + (\varphi+1)/\Lambda_L\right]\widehat{\mu}(\xi)^\top \mathcal{M}_{OG}^\top \mathcal{M}_{OG}\widehat{\mu}(\xi) + o(\|\widehat{\xi}\|^2). \quad \text{(B.10)}$$

An equivalent economy with sectoral markup wedges. To facilitate the derivation of the efficiency wedge, we construct an equivalent economy with sectoral markup wedges. Specifically, for the *sticky-price equilibrium* under any realized shocks $\widehat{\xi}$, the equivalent economy satisfies all of the equilibrium conditions in Definition 1 except that in condition (ii), the markups of sticky-price firms, μ_{if}, are derived from $1 - \delta_i + \delta_i\mu_{if}^{1-\theta_i} = \mu_i(\xi)^{1-\theta_i}$ for all sector i, where the sectoral markup wedge $\widehat{\mu}_i(\xi)$ is identical to that in the *sticky-price equilibrium*. Therefore, the constructed economy has *exactly the same allocations, prices, and welfare loss* as in the *sticky-price equilibirum* for any realized shock $\widehat{\xi}$, and thus we refer to it as the *equivalent economy*. With a slight abuse of notation, in the remainder of this subsection, we express the utility and other sector-level allocations and prices in the equivalent economy as functions of $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$ and $\widehat{\boldsymbol{\xi}}$, using the same function names as in the *sticky-price equilibirum* (e.g., $u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \widehat{\boldsymbol{\xi}})$ and $C(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \widehat{\boldsymbol{\xi}})$).

Then, we show that under any realized shock $\widehat{\boldsymbol{\xi}}$ and the corresponding sectoral markup wedges $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$ in the *sticky-price equilibrium*, up to the second-order approximation, the welfare loss in the *equivalent economy* is equal to the welfare loss in the economy with the identical sectoral markup wedges $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$ *but* shutting down all realized shocks (e.g., $u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \mathbf{0})$), as stated in the following lemma.[35]

Lemma 6. *Let $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$ be the sectoral markup wedges in the* sticky-price equilibrium *under realized shocks $\widehat{\boldsymbol{\xi}}$. Up to the second-order approximation, the welfare loss in the* sticky-price equilibrium *under any shock $\widehat{\boldsymbol{\xi}}$ is equivalent to the welfare loss in an economy with the identical sectoral markup wedges $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$ but shutting down all shocks*, viz,

$$u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \widehat{\boldsymbol{\xi}}) - u(\mathbf{0}, \widehat{\boldsymbol{\xi}}) \;=\; u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \mathbf{0}) - u(\mathbf{0}, \mathbf{0}) + o(||\widehat{\boldsymbol{\xi}}||^2) \;=\; \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})^\top \boldsymbol{\mathcal{L}}^u_{\mu\mu} \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(||\widehat{\boldsymbol{\xi}}||^2). \quad \text{(B.11)}$$

The first equality in equation (B.11) illustrates the equivalence of welfare loss in economies with and without the realized shocks. This equivalence is supported by the second equality in equation (B.11), showing that up to the second-order approximation, the welfare loss is a function of only sectoral markup wedges and involves no shocks.

To prove Lemma 6, we first write down the complete form of the second-order welfare loss in the *sticky-price equilibrium* under any realized shock $\widehat{\boldsymbol{\xi}}$ as follows:

$$u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \widehat{\boldsymbol{\xi}}) - u(\mathbf{0}, \widehat{\boldsymbol{\xi}}) = \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})^\top \boldsymbol{\mathcal{L}}^u_{\mu\mu} \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + \widehat{\boldsymbol{\xi}}^\top \boldsymbol{\mathcal{L}}^u_{\xi\mu} \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + \widehat{\boldsymbol{\xi}}^\top \boldsymbol{\mathcal{L}}^u_{\xi\xi} \widehat{\boldsymbol{\xi}} + o(||\widehat{\boldsymbol{\xi}}||^2). \quad \text{(B.12)}$$

The efficiency of the *flexible-price equilibrium* indicates that the welfare is maximized at $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) = \mathbf{0}$ and $u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \widehat{\boldsymbol{\xi}}) \leq u(\mathbf{0}, \widehat{\boldsymbol{\xi}})$ for any realized shocks $\widehat{\boldsymbol{\xi}}$, based on which we prove Lemma 6. First, since the welfare is maximized at $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) = \mathbf{0}$, the derivative of the RHS of equation (B.12) with respect to $\widehat{\boldsymbol{\mu}}$ equals $\mathbf{0}$ at $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) = 0$ for any realized shocks $\widehat{\boldsymbol{\xi}}$, requiring $\boldsymbol{\mathcal{L}}^u_{\xi\mu} = \mathbf{0}$. Second, if $\boldsymbol{\mathcal{L}}^u_{\xi\xi} \neq \mathbf{0}$, there exists some realized shocks $\widehat{\boldsymbol{\xi}}$ such that the RHS of equation (B.12) is strictly positive at $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) = \mathbf{0}$ (i.e., $\widehat{\boldsymbol{\xi}}^\top \boldsymbol{\mathcal{L}}^u_{\xi\xi} \widehat{\boldsymbol{\xi}} > 0$), which contradicts $u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \widehat{\boldsymbol{\xi}}) \leq u(\mathbf{0}, \widehat{\boldsymbol{\xi}})$.

Therefore, we conclude that $\boldsymbol{\mathcal{L}}^u_{\xi\mu} = \mathbf{0}$, $\boldsymbol{\mathcal{L}}^u_{\xi\xi} = \mathbf{0}$, and the RHS of equation (B.12) degenerates to $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})^\top \boldsymbol{\mathcal{L}}^u_{\mu\mu} \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$, which proves the second equality in equation (B.12) of Lemma 6. Since up to the second-order approximation, $u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \widehat{\boldsymbol{\xi}}) - u(\mathbf{0}, \widehat{\boldsymbol{\xi}})$ is a function of only $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$ and involves no $\widehat{\boldsymbol{\xi}}$, we have $u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \widehat{\boldsymbol{\xi}}) - u(\mathbf{0}, \widehat{\boldsymbol{\xi}}) = u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \mathbf{0}) - u(\mathbf{0}, \mathbf{0}) + o(||\widehat{\boldsymbol{\xi}}||^2)$, which proves the first equality in equation (B.12) of Lemma 6.

Based on Lemma 6, we derive the welfare loss up to the second-order approximation by deriving the equivalent $u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \mathbf{0}) - u(\mathbf{0}, \mathbf{0})$ with the sectoral markup wedges $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$ resulting from

[35] The economy with the identical sectoral markup wedges $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$ *but* shutting down all realized shocks satisfies all of the equilibrium conditions in Definition 1 except that the markups of sticky-price firms are derived from $1 - \delta_i + \delta_i \mu_{if}^{1-\theta_i} = \mu_i(\boldsymbol{\xi})^{1-\theta_i}$ for all sector i in condition (ii), and $\{\widehat{\mathbf{A}}, \widehat{\mathbf{D}}^*_{EX}, \widehat{\mathbf{P}}^*_{IM}\}$ are set to zero.

shocks $\widehat{\boldsymbol{\xi}}$ in the *sticky-price equilibrium*. Particularly, because equation (B.11) shows that up to the second-order approximation, the labor wedge is a quadratic function of sectoral markup wedges, the efficiency wedge—as the remaining component of the welfare loss—is also a quadratic function of only sectoral markup wedges. Therefore, we arrive at the following:

$$\widehat{A}_{agg}(\boldsymbol{\xi}) - \widehat{A}^{flex}_{agg}(\boldsymbol{\xi}) = \widehat{A}_{agg}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \widehat{\boldsymbol{\xi}}) - \widehat{A}_{agg}(\mathbf{0}, \widehat{\boldsymbol{\xi}}) = \widehat{A}_{agg}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \mathbf{0}) - \widehat{A}_{agg}(\mathbf{0}, \mathbf{0}) + o(||\widehat{\boldsymbol{\xi}}||^2)$$
$$= \widehat{C}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \mathbf{0}) - \Lambda^{flex}_L(\mathbf{0})\widehat{L}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \mathbf{0}) + o(||\widehat{\boldsymbol{\xi}}||^2) = \widehat{C}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \mathbf{0}) - \Lambda_L \widehat{L}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \mathbf{0}) + o(||\widehat{\boldsymbol{\xi}}||^2).$$

For simplicity of notation, in the remainder of this subsection, we denote $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$ by $\widehat{\boldsymbol{\mu}}$ and ignore the entry of $\mathbf{0}$ for any function in the equivalent economy with sectoral markup wedges $\widehat{\boldsymbol{\mu}}$ but no realized shocks, e.g., $\widehat{C}(\widehat{\boldsymbol{\mu}}) - \Lambda_L \widehat{L}(\widehat{\boldsymbol{\mu}}) \equiv \widehat{C}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \mathbf{0}) - \Lambda_L \widehat{L}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}), \mathbf{0})$.

In the remaining proof of this subsection, for any variable x, we use $\widehat{x}(\widehat{\boldsymbol{\mu}})$ to replace $\widehat{x}(\widehat{\boldsymbol{\mu}}) - \widehat{x}(\mathbf{0})$, because $\widehat{x}(\mathbf{0}) = \mathbf{0}$ when all sectoral markup wedges are set to zero to represent both the *flexible-price equilibrium* and the steady state.

Efficiency wedge up to the second-order approximation. Using the equivalent economy without realized shocks, we derive the efficiency wedge up to the second-order approximation. The equilibrium of such an economy under sectoral markup wedges $\widehat{\boldsymbol{\mu}}$ satisfies conditions (D.50)-(D.56) of the *feasible allocation* in Definition 12, which has the following approximation around the steady state up to the second order:

$$L^{ss}\widehat{L}(\widehat{\boldsymbol{\mu}}) = \sum_i L^{ss}_i \widehat{L}_i(\widehat{\boldsymbol{\mu}}) + \widehat{L}^{O2}(\widehat{\boldsymbol{\mu}}), \tag{B.13}$$
$$Y^{ss}_i \widehat{Y}_i(\widehat{\boldsymbol{\mu}}) = C^{ss}_{Hi}\widehat{C}_{Hi}(\widehat{\boldsymbol{\mu}}) + \sum_j X^{ss}_{Hj,Hi}\widehat{X}_{Hj,Hi}(\widehat{\boldsymbol{\mu}}) + Y^{ss}_{EX,i}\widehat{Y}_{EX,i}(\widehat{\boldsymbol{\mu}}) + \widehat{Y}^{O2}_i(\widehat{\boldsymbol{\mu}}), \tag{B.14}$$
$$C^{ss}\widehat{C}(\widehat{\boldsymbol{\mu}}) = \sum_i C^{ss}_i \widehat{C}_i(\widehat{\boldsymbol{\mu}}) + \widehat{C}^{O2}(\widehat{\boldsymbol{\mu}}), \tag{B.15}$$
$$Y^{ss}_i \widehat{Y}_i(\widehat{\boldsymbol{\mu}}) = W^{ss} L^{ss}_i \widehat{L}_i(\widehat{\boldsymbol{\mu}}) + \sum_j X^{ss}_{i,j}\widehat{X}_{i,j}(\widehat{\boldsymbol{\mu}}) + \widehat{F}^{O2}_i(\widehat{\boldsymbol{\mu}}), \tag{B.16}$$
$$C^{ss}_i \widehat{C}_i(\widehat{\boldsymbol{\mu}}) = C^{ss}_{Hi}\widehat{C}_{Hi}(\widehat{\boldsymbol{\mu}}) + C^{ss}_{Fi}\widehat{C}_{Fi}(\widehat{\boldsymbol{\mu}}) + \widehat{C}^{O2}_i(\widehat{\boldsymbol{\mu}}), \tag{B.17}$$
$$X^{ss}_{i,j}\widehat{X}_{i,j}(\widehat{\boldsymbol{\mu}}) = X^{ss}_{Hi,Hj}\widehat{X}_{Hi,Hj}(\widehat{\boldsymbol{\mu}}) + X^{ss}_{Hi,Fj}\widehat{X}_{Hi,Fj}(\widehat{\boldsymbol{\mu}}) + \widehat{X}^{O2}_{i,j}(\widehat{\boldsymbol{\mu}}), \tag{B.18}$$
$$EX^{ss}\widehat{EX}(\widehat{\boldsymbol{\mu}}) = \sum_i Y^{ss}_{EX,i}\widehat{Y}_{EX,i}(\widehat{\boldsymbol{\mu}}) = \sum_i C^{ss}_{Fi}\widehat{C}_{Fi}(\widehat{\boldsymbol{\mu}}) + \sum_i\sum_j X^{ss}_{Hi,Fj}\widehat{X}_{Hi,Fj}(\widehat{\boldsymbol{\mu}}) + \widehat{S}^{O2}(\widehat{\boldsymbol{\mu}}), \tag{B.19}$$

where $\widehat{L}^{O2}, \{\widehat{Y}^{O2}_i\}_i, \widehat{C}^{O2}, \{\widehat{F}^{O2}_i\}_i, \{\widehat{C}^{O2}_i\}_i, \{\widehat{X}^{O2}_{i,j}\}_{i,j}, \widehat{S}^{O2}$ denote second- or higher-order terms of each equation. Comparing equations (B.13)-(B.19) to the first-order approximation equations (E.91)-(E.97) in the proof of Lemma 2 (Section E.1 of the Supplementary Material), yields the following expression of efficiency wedge:

$$\widehat{C}(\widehat{\boldsymbol{\mu}}) - \Lambda_L \widehat{L}(\widehat{\boldsymbol{\mu}}) = -\widehat{L}^{O2}(\widehat{\boldsymbol{\mu}}) - \sum_i \widehat{Y}^{O2}_i(\widehat{\boldsymbol{\mu}})$$

$$+ \widehat{C}^{O2}(\widehat{\boldsymbol{\mu}}) + \sum_i \left[\widehat{F}_i^{O2}(\widehat{\boldsymbol{\mu}}) + \widehat{C}_i^{O2}(\widehat{\boldsymbol{\mu}}) + \sum_j \widehat{X}_{i,j}^{O2}(\widehat{\boldsymbol{\mu}}) \right] - \widehat{S}^{O2}(\widehat{\boldsymbol{\mu}}), \quad \text{(B.20)}$$

where only second- or higher-order terms appear because up to the first-order approximation, the efficiency wedge is unaffected by sectoral markup wedges, as shown in Proposition 1.

Next, we derive each term on the RHS of equation (B.20). In equations (B.13) and (B.14),

$$\widehat{L}^{O2}(\widehat{\boldsymbol{\mu}}) = \frac{1}{2} \sum_i \lambda_i \alpha_i \big[\widehat{L}_i(\widehat{\boldsymbol{\mu}}) - \widehat{L}(\widehat{\boldsymbol{\mu}})\big]^2 + o(\|\widehat{\boldsymbol{\mu}}\|^2), \quad \text{(B.21)}$$

$$\widehat{Y}_i^{O2}(\widehat{\boldsymbol{\mu}}) = \frac{1}{2}\beta_i v_i \big[\widehat{C}_{Hi}(\widehat{\boldsymbol{\mu}}) - \widehat{Y}_i(\widehat{\boldsymbol{\mu}})\big]^2 + \frac{1}{2}\sum_j \lambda_j \omega_{j,i} v_{x,j,i} \big[\widehat{X}_{Hj,Hi}(\widehat{\boldsymbol{\mu}}) - \widehat{Y}_i(\widehat{\boldsymbol{\mu}})\big]^2 \quad \text{(B.22)}$$
$$+ \frac{1}{2}\lambda_{EX,i} \big[\widehat{Y}_{EX,i}(\widehat{\boldsymbol{\mu}}) - \widehat{Y}_i(\widehat{\boldsymbol{\mu}})\big]^2 + o(\|\widehat{\boldsymbol{\mu}}\|^2).$$

Equation (B.21) shows that $\widehat{L}^{O2}(\widehat{\boldsymbol{\mu}})$ captures the distortion in the efficiency wedge due to the disproportionate allocation of labor across different sectors. Equation (B.22) shows that $\widehat{Y}_i^{O2}(\widehat{\boldsymbol{\mu}})$ captures the distortion in the efficiency wedge due to the allocation of sectoral production ($\widehat{Y}_i$) that is disproportionate to sectoral consumption, use of intermediate inputs, and exports.

In equations (B.15) and (B.16), under the Cobb-Douglas aggregate consumption and production function $\mathcal{C}$ and $\{F_i\}_i$, we have

$$\widehat{C}^{O2}(\widehat{\boldsymbol{\mu}}) = 0, \quad \text{(B.23)}$$
$$\widehat{F}_i^{O2}(\widehat{\boldsymbol{\mu}}) = \lambda_i \widehat{\iota}_i(\widehat{\boldsymbol{\mu}}). \quad \text{(B.24)}$$

Equation (B.23) shows that sectoral markup wedges lead to no distortion in the efficiency wedge from cross-sector allocation of consumption. Equation (B.24) shows that $\widehat{F}_i^{O2}(\widehat{\boldsymbol{\mu}})$ captures the distortion in the efficiency wedge due to within-sector misallocation.

To further derive the second-order approximation of ι_i, we first approximate the CES aggregator of differentiated goods up to the second order as follows:

$$1 = \int_0^1 \left(\frac{Y_{if}}{Y_i}\right)^{\frac{\varepsilon_i - 1}{\varepsilon_i}} df = \int_0^1 \exp\left(\widehat{Y}_{if} - \widehat{Y}_i\right)^{\frac{\varepsilon_i - 1}{\varepsilon_i}} df$$
$$= \int_0^1 \left[1 - \frac{\varepsilon_i - 1}{\varepsilon_i}(\widehat{Y}_{if} - \widehat{Y}_i) + \frac{1}{2}\left(\frac{\varepsilon_i - 1}{\varepsilon_i}\right)^2 (\widehat{Y}_{if} - \widehat{Y}_i)^2\right] df + o\left(\sup_f \|\widehat{Y}_{if} - \widehat{Y}_i\|^2\right),$$

which yields the following equation up to the second-order approximation:

$$\int_0^1 (\widehat{Y}_{if} - \widehat{Y}_i) df = \frac{\varepsilon_i - 1}{2\varepsilon_i} \int_0^1 (\widehat{Y}_{if} - \widehat{Y}_i)^2 df + o\left(\sup_f \|\widehat{Y}_{if} - \widehat{Y}_i\|^2\right).$$

As a result, the second-order approximation of $\widehat{\iota}_i$ is

$$\widehat{\iota}_i = -\ln\Big[\int_0^1 \Big(\frac{Y_{if}}{Y_i}\Big)df\Big] = -\ln\Big\{\int_0^1 \Big[1-(\widehat{Y}_{if}-\widehat{Y}_i)+\frac{1}{2}(\widehat{Y}_{if}-\widehat{Y}_i)^2\Big]df + o\big(\sup_f\|\widehat{Y}_{if}-\widehat{Y}_i\|^2\big)\Big\}$$
$$= -\frac{1}{2\varepsilon_i}\int_0^1 (\widehat{Y}_{if}-\widehat{Y}_i)^2 df + o\big(\sup_f\|\widehat{Y}_{if}-\widehat{Y}_i\|^2\big).$$

Using the demand function in equation (6) and the definition of firm-level markup $\mu_{if} \equiv P_{if}/\Phi_i$, we have $\widehat{Y}_{if}-\widehat{Y}_i = -\varepsilon_i(\widehat{P}_{if}-\widehat{P}_i) = -\varepsilon_i(\widehat{\mu}_{if}-\widehat{\mu}_i)$, which yields

$$\widehat{\iota}_i = -\frac{\varepsilon_i}{2}\int_0^1 (\widehat{\mu}_{if}-\widehat{\mu}_i)^2 df + o\big(\sup_f\|\widehat{\mu}_{if}-\widehat{\mu}_i\|^2\big).$$

Under Calvo-pricing, $\widehat{\mu}_{if}=0$ for $f\in[0,\delta_i]$ and $\widehat{\mu}_{if}=(1-\delta_i)^{-1}\widehat{\mu}_i + o(||\widehat{\mu}_i||)$ for $f\in(\delta_i,1]$, which yields

$$\widehat{\iota}_i(\widehat{\boldsymbol{\mu}}) = -\frac{1}{2}\frac{\varepsilon_i\delta_i}{1-\delta_i}\widehat{\mu}_i^2 + o(\|\widehat{\boldsymbol{\mu}}\|^2).$$

In equations (B.17) and (B.18),

$$\widehat{C}_i^{O2}(\widehat{\boldsymbol{\mu}}) = \frac{1}{2}(1-\theta_i^{-1})\beta_i v_i(1-v_i)\big[\widehat{C}_{Hi}(\widehat{\boldsymbol{\mu}})-\widehat{C}_{Fi}(\widehat{\boldsymbol{\mu}})\big]^2 + o(\|\widehat{\boldsymbol{\mu}}\|^2), \tag{B.25}$$
$$\widehat{X}_{i,j}^{O2}(\widehat{\boldsymbol{\mu}}) = \frac{1}{2}(1-\theta_j^{-1})\lambda_i\omega_{i,j}v_{x,i,j}(1-v_{x,i,j})\big[\widehat{X}_{Hi,Hj}(\widehat{\boldsymbol{\mu}})-\widehat{X}_{Hi,Fj}(\widehat{\boldsymbol{\mu}})\big]^2 + o(\|\widehat{\boldsymbol{\mu}}\|^2). \tag{B.26}$$

Equation (B.25) shows that $\widehat{C}_i^{O2}(\widehat{\boldsymbol{\mu}})$ captures the distortion in the efficiency wedge due to disproportionate consumption of domestic and foreign sectoral goods. Equation (B.26) shows that $\widehat{X}_{i,j}^{O2}(\widehat{\boldsymbol{\mu}})$ captures the distortion in the efficiency wedge due to disproportionate use of intermediate inputs from domestic and foreign sectoral products. Finally, in equation (B.19),

$$\widehat{S}^{O2}(\widehat{\boldsymbol{\mu}}) = \frac{1}{2}\sum_i \frac{\theta_{F,i}}{\theta_{F,i}-1}\lambda_{EX,i}^{ss}\big[(1-\theta_{F,i}^{-1})\widehat{Y}_{EX,i}(\widehat{\boldsymbol{\mu}})-\widehat{EX}(\widehat{\boldsymbol{\mu}})\big]^2 \tag{B.27}$$
$$-\frac{1}{2}\sum_i \beta_i(1-v_i)\big[\widehat{C}_{Fi}(\widehat{\boldsymbol{\mu}})-\widehat{EX}(\widehat{\boldsymbol{\mu}})\big]^2 - \frac{1}{2}\sum_i\sum_j \omega_{i,j}(1-v_{x,i,j})\big[\widehat{X}_{Hi,Fj}(\widehat{\boldsymbol{\mu}})-\widehat{EX}(\widehat{\boldsymbol{\mu}})\big]^2 + o(\|\widehat{\boldsymbol{\mu}}\|^2).$$

Equation (B.27) shows that $\widehat{S}^{O2}(\widehat{\boldsymbol{\mu}})$ captures the distortion in the efficiency wedge due to the distortion in the nominal exchange rate, which is driven by the changes in the sectoral consumption of foreign goods, use of intermediate inputs from foreign sectors, and sectoral exports that are disproportionate to the change in the economy-wise exports in terms of foreign currencies ($\widehat{EX}(\widehat{\boldsymbol{\mu}})$).

Combining equations (B.21)-(B.27), yields the RHS of equation (B.20) as:

$$\widehat{C}(\widehat{\boldsymbol{\mu}}) - \Lambda_L\widehat{L}(\widehat{\boldsymbol{\mu}}) =$$

$$
\begin{aligned}
&-\frac{1}{2}\sum_i \lambda_i \varepsilon_i \frac{\delta_i}{1-\delta_i}\widehat{\mu}_i^2 && \Big\} \text{ within-sector misallocation} \\
&-\frac{1}{2}\sum_i \lambda_i \alpha_i \big[\widehat{L}_i(\widehat{\boldsymbol{\mu}}) - \widehat{L}(\widehat{\boldsymbol{\mu}})\big]^2 && \Big\} \text{ cross-sector allocation of labor} \\
&-\frac{1}{2}\sum_i \beta_i v_i \big[\widehat{C}_{Hi}(\widehat{\boldsymbol{\mu}}) - \widehat{Y}_i(\widehat{\boldsymbol{\mu}})\big]^2 \\
&-\frac{1}{2}\sum_i\sum_j \lambda_j \omega_{j,i} v_{x,j,i} \big[\widehat{X}_{Hj,Hi}(\widehat{\boldsymbol{\mu}}) - \widehat{Y}_i(\widehat{\boldsymbol{\mu}})\big]^2 && \Big\} \text{ allocation of sectoral products into consumption, intermediate inputs, and exports} \\
&-\frac{1}{2}\sum_i \lambda_{EX,i} \big[\widehat{Y}_{EX,i}(\widehat{\boldsymbol{\mu}}) - \widehat{Y}_i(\widehat{\boldsymbol{\mu}})\big]^2 \\
&+\frac{1}{2}\sum_i \frac{\theta_i - 1}{\theta_i}\beta_i v_i (1-v_i) \big[\widehat{C}_{Hi}(\widehat{\boldsymbol{\mu}}) - \widehat{C}_{Fi}(\widehat{\boldsymbol{\mu}})\big]^2 \\
&+\frac{1}{2}\sum_i\sum_j \frac{\theta_j - 1}{\theta_j}\lambda_i \omega_{i,j} v_{x,i,j}(1-v_{x,i,j}) \big[\widehat{X}_{Hi,Hj}(\widehat{\boldsymbol{\mu}}) - \widehat{X}_{Hi,Fj}(\widehat{\boldsymbol{\mu}})\big]^2 && \Big\} \text{ division of consumption and intermediate inputs between home and foreign products} \\
&+\frac{1}{2}\sum_i \frac{\theta_{F,i}}{\theta_{F,i}-1}\lambda_{EX,i}^{ss} \Big[\frac{\theta_{F,i}-1}{\theta_{F,i}}\widehat{Y}_{EX,i}(\widehat{\boldsymbol{\mu}}) - \widehat{EX}(\widehat{\boldsymbol{\mu}})\Big]^2 \\
&-\frac{1}{2}\sum_i \beta_i (1-v_i) \big[\widehat{C}_{Fi}(\widehat{\boldsymbol{\mu}}) - \widehat{EX}(\widehat{\boldsymbol{\mu}})\big]^2 && \Big\} \text{ distortion in nominal exchange rate} \\
&-\frac{1}{2}\sum_i\sum_j \omega_{i,j}(1-v_{x,i,j}) \big[\widehat{X}_{Hi,Fj}(\widehat{\boldsymbol{\mu}}) - \widehat{EX}(\widehat{\boldsymbol{\mu}})\big]^2 \\
&+ o(\|\widehat{\boldsymbol{\mu}}\|^2). \qquad \text{(B.28)}
\end{aligned}
$$

Next, we derive the RHS of equation (B.28) as a function of sectoral market wedges $\widehat{\boldsymbol{\mu}}$. Because the RHS includes only squared terms of the changes in the allocations, to derive the RHS up to the second-order approximation, we approximate changes in the allocations up to the first order.

To facilitate the proof, we define the following real prices and quantity in units of domestic and foreign currencies:

$$
\begin{aligned}
w &\equiv W/P_C, \qquad s \equiv S/P_C, \qquad p_i \equiv P_i/P_C, \\
c_S &\equiv C/s, \qquad p_{S,i} \equiv p_i/s.
\end{aligned}
$$

We first derive three variables that are iteratively used for the remainder of the proof, namely, $\widehat{c}_S(\widehat{\boldsymbol{\mu}})$, $\widehat{p}_{S,i}(\widehat{\boldsymbol{\mu}})$, and $\widehat{\lambda}_i(\widehat{\boldsymbol{\mu}})$. As a preparation, equation (38) in Theorem 1 indicates

$$
\widehat{C}(\widehat{\boldsymbol{\mu}}) = \kappa_C^{-1}\sum_k \mathcal{M}_{OG,k}\widehat{\mu}_k + o(\|\widehat{\boldsymbol{\mu}}\|).
$$

Equation (E.111) in Lemma E.4 of the Supplementary Material indicates

$$
\widehat{w}(\widehat{\boldsymbol{\mu}}) = (\sigma + \varphi/\Lambda_L)\widehat{C}(\widehat{\boldsymbol{\mu}}) + o(\|\widehat{\boldsymbol{\mu}}\|).
$$

Rearranging equation (E.110) yields

$$\widehat{s}(\widehat{\boldsymbol{\mu}}) = -\frac{\sum_k \widetilde{\lambda}_{D,k}\widehat{\mu}_k + \sum_k \widetilde{\lambda}_{D,k}\alpha_k \cdot \widehat{w}(\widehat{\boldsymbol{\mu}})}{1 - \sum_k \widetilde{\lambda}_{D,k}\alpha_k} + o(\|\widehat{\boldsymbol{\mu}}\|).$$

Subtracting the log deviation of CPI from both sides of equation (E.106) yields

$$\widehat{p}_i(\widehat{\boldsymbol{\mu}}) = \sum_k \ell_{vx,i,k}\left[\alpha_k \cdot \widehat{w}(\widehat{\boldsymbol{\mu}}) + \sum_h \omega_{k,h}v_{x,k,h} \cdot \widehat{s}(\widehat{\boldsymbol{\mu}}) + \widehat{\mu}_k\right] + o(\|\widehat{\boldsymbol{\mu}}\|).$$

Based on the derived $\widehat{C}(\widehat{\boldsymbol{\mu}})$, $\widehat{w}(\widehat{\boldsymbol{\mu}})$, $\widehat{s}(\widehat{\boldsymbol{\mu}})$, and $\{\widehat{p}_i(\widehat{\boldsymbol{\mu}})\}_i$, the log deviations of the real aggregate output and sectoral prices in units of foreign currency are equal to

$$\widehat{c}_S(\widehat{\boldsymbol{\mu}}) = \widehat{C}(\widehat{\boldsymbol{\mu}}) - \widehat{s}(\widehat{\boldsymbol{\mu}}), \tag{B.29}$$

$$\widehat{p}_{S,i}(\widehat{\boldsymbol{\mu}}) = \widehat{p}_i(\widehat{\boldsymbol{\mu}}) - \widehat{s}(\widehat{\boldsymbol{\mu}}). \tag{B.30}$$

Rearranging equation (D.78) in Lemma 9 yields

$$\widehat{\lambda}_i(\widehat{\boldsymbol{\mu}}) = -\lambda_i^{-1}\sum_k \ell_{vx,k,i}\left[\lambda_{EX,k}\widehat{c}_S(\widehat{\boldsymbol{\mu}}) + \rho_{NX,k}\widehat{p}_{S,k}(\widehat{\boldsymbol{\mu}}) + \lambda_k\widehat{\mu}_k\right] + \widehat{\mu}_i + o(\|\widehat{\boldsymbol{\mu}}\|). \tag{B.31}$$

Next, we derive the allocations on the RHS of equation (B.28) as functions of $\widehat{c}_S(\widehat{\boldsymbol{\mu}})$, $\widehat{p}_{S,i}(\widehat{\boldsymbol{\mu}})$, and $\widehat{\lambda}_i(\widehat{\boldsymbol{\mu}})$ in equations (B.29), (B.30), and (B.31), which yields

$$\begin{aligned}
&\widehat{C}(\widehat{\boldsymbol{\mu}}) - \Lambda_L\widehat{L}(\widehat{\boldsymbol{\mu}}) = \\
&-\frac{1}{2}\sum_i \lambda_i\varepsilon_i\frac{\delta_i}{1-\delta_i}\widehat{\mu}_i^2 \qquad \Big\} \text{within-sector misallocation} \\
&-\frac{1}{2}\sum_i \lambda_i\alpha_i\left[\widehat{\lambda}_i(\widehat{\boldsymbol{\mu}}) - \widehat{\mu}_i - \frac{\sum_k \lambda_k\alpha_k\left(\widehat{\lambda}_k(\widehat{\boldsymbol{\mu}}) - \widehat{\mu}_k\right)}{\sum_k \lambda_k\alpha_k}\right]^2 \qquad \Big\} \text{cross-sector allocation of labor} \\
&\left.\begin{aligned}
&-\frac{1}{2}\sum_i \beta_i v_i\left[(\theta_i - 1)(1 - v_i)\widehat{p}_{S,i}(\widehat{\boldsymbol{\mu}}) + \widehat{\lambda}_i(\widehat{\boldsymbol{\mu}})\right]^2 \\
&-\frac{1}{2}\sum_i\sum_j \lambda_j\omega_{j,i}v_{x,j,i}\left[(\theta_i - 1)(1 - v_{x,j,i})\widehat{p}_{S,i}(\widehat{\boldsymbol{\mu}}) + \widehat{\lambda}_i(\widehat{\boldsymbol{\mu}}) - \widehat{\lambda}_j(\widehat{\boldsymbol{\mu}}) + \widehat{\mu}_j\right]^2 \\
&-\frac{1}{2}\sum_i \lambda_{EX,i}\left[\widehat{c}_S(\widehat{\boldsymbol{\mu}}) + (\theta_{F,i} - 1)\widehat{p}_{S,i}(\widehat{\boldsymbol{\mu}}) + \widehat{\lambda}_i(\widehat{\boldsymbol{\mu}})\right]^2
\end{aligned}\right\} \text{allocation of sectoral products into consumption, intermediate inputs, and exports} \\
&+\frac{1}{2}\sum_i \theta_i(\theta_i - 1)\left[\beta_i v_i(1 - v_i) + \sum_j \lambda_{j,i}v_{x,j,i}(1 - v_{x,j,i})\right]\widehat{p}_{S,i}(\widehat{\boldsymbol{\mu}})^2 \qquad \Big\} \text{division of consumption and intermediate inputs between home and foreign products}
\end{aligned}$$

$$
\left.\begin{aligned}
&+\frac{1}{2}\sum_i \frac{\theta_{F,i}}{\theta_{F,i}-1}\lambda_{EX,i}\left[-(\theta_{F,i}-1)\widehat{p}_{S,i}(\widehat{\boldsymbol{\mu}})+\frac{\sum_k \lambda_{EX,k}\theta_{F,k}\widehat{p}_{S,k}(\widehat{\boldsymbol{\mu}})}{\sum_{k'}\lambda_{EX,k'}}\right]^2 \\
&-\frac{1}{2}\sum_i \beta_i(1-v_i)\left[\widehat{c}_S(\widehat{\boldsymbol{\mu}})+(\theta_i-1)v_i\widehat{p}_{S,i}(\widehat{\boldsymbol{\mu}})+\frac{\sum_k \lambda_{EX,k}\theta_{F,k}\widehat{p}_{S,k}(\widehat{\boldsymbol{\mu}})}{\sum_{k'}\lambda_{EX,k'}}\right]^2 \\
&-\frac{1}{2}\sum_i\sum_j \omega_{i,j}(1-v_{x,i,j})\left[\widehat{c}_S(\widehat{\boldsymbol{\mu}})+(\theta_j-1)v_{x,i,j}\widehat{p}_{S,j}(\widehat{\boldsymbol{\mu}})+\frac{\sum_k \lambda_{EX,k}\theta_{F,k}\widehat{p}_{S,k}(\widehat{\boldsymbol{\mu}})}{\sum_{k'}\lambda_{EX,k'}}+\widehat{\lambda}_i(\widehat{\boldsymbol{\mu}})-\widehat{\mu}_i\right]^2
\end{aligned}\right\}\text{distortion in nominal exchange rate}
$$

$$
+o(\|\widehat{\boldsymbol{\mu}}\|^2). \tag{B.32}
$$

Rewriting equation (B.32) back into the form of the gaps of aggregate consumption and labor and substituting $\widehat{\mu}_i$ by $\widehat{P}_i$ according to equation (B.32) yield

$$
\widehat{C}^{gap}(\boldsymbol{\xi})-\Lambda_L\widehat{L}^{gap}(\boldsymbol{\xi}) = \underbrace{-\frac{1}{2}\widehat{\mathbf{P}}(\boldsymbol{\xi})^\top\boldsymbol{\mathcal{L}}^{within}\widehat{\mathbf{P}}(\boldsymbol{\xi})}_{\text{with-sector misallocation}}\underbrace{-\frac{1}{2}\widehat{\mathbf{P}}(\boldsymbol{\xi})^\top\boldsymbol{\mathcal{L}}^{across}\widehat{\mathbf{P}}(\boldsymbol{\xi})}_{\text{across-sector misallocation}}+o(\|\widehat{\boldsymbol{\xi}}\|^2),
$$

where $-\frac{1}{2}\widehat{\mathbf{P}}^\top\boldsymbol{\mathcal{L}}^{within}\widehat{\mathbf{P}}$ and the rest as $-\frac{1}{2}\widehat{\mathbf{P}}^\top\boldsymbol{\mathcal{L}}^{across}\widehat{\mathbf{P}}$ are equal to the first line and the rest lines on the RHS of equation (B.32), respectively. The non-negative welfare losses due to the within-sector and across-sector misallocations imply that $\boldsymbol{\mathcal{L}}^{within}$ and $\boldsymbol{\mathcal{L}}^{across}$ are both positive semi-definite. □

Efficiency wedge in closed economies. In closed economies á la La'O and Tahbaz-Salehi (2022) and Rubbo (2023), $v_i = v_{x,i,j} = \Lambda_L = 1$, $\lambda_{EX,i} = \rho_{NX,i} = 0$, $\mathcal{M}_{OG,i} = \widetilde{\lambda}_{D,i} = \lambda_i$, and $\ell_{vx,i,j}$ degenerates to $\ell_{i,j}$.[36] Therefore, equation (B.32) becomes the following:

$$
\begin{aligned}
\widehat{C}(\widehat{\boldsymbol{\mu}})-\Lambda_L\widehat{L}(\widehat{\boldsymbol{\mu}}) =& -\frac{1}{2}\sum_i\lambda_i\varepsilon_i\frac{\delta_i}{1-\delta_i}\widehat{\mu}_i^2-\frac{1}{2}\sum_i\lambda_i\alpha_i\Big\{\big[\widehat{\lambda}_i(\widehat{\boldsymbol{\mu}})-\widehat{\mu}_i\big]-\sum_j\lambda_j\alpha_j\big[\widehat{\lambda}_j(\widehat{\boldsymbol{\mu}})-\widehat{\mu}_j\big]\Big\}^2 \\
&-\frac{1}{2}\sum_i\beta_i v_i\widehat{\lambda}_i(\widehat{\boldsymbol{\mu}})^2-\frac{1}{2}\sum_i\sum_j\lambda_j\omega_{j,i}v_{x,j,i}\left[\widehat{\lambda}_i(\widehat{\boldsymbol{\mu}})-\widehat{\lambda}_j(\widehat{\boldsymbol{\mu}})+\widehat{\mu}_j\right]^2+o(\|\widehat{\boldsymbol{\mu}}\|^2) \\
=& -\frac{1}{2}\sum_i\lambda_i\varepsilon_i\frac{\delta_i}{1-\delta_i}\widehat{\mu}_i^2-\frac{1}{2}\sum_i\lambda_i\widehat{\mu}_i^2-\sum_i\sum_j\lambda_i\ell_{i,j}\widehat{\mu}_i\widehat{\mu}_j+\frac{1}{2}\Big(\sum_i\lambda_i\widehat{\mu}_i\Big)^2+o(\|\widehat{\boldsymbol{\mu}}\|^2),
\end{aligned}
$$

which is consistent with Rubbo (2023).

On the other hand, sectoral markup wedges $\{\widehat{\mu}_i\}_i$ are linked to the pricing errors $\{\bar{e}_i\}_i$ in La'O and Tahbaz-Salehi (2022) as follows:

$$
\bar{e}_i=\sum_j\ell_{i,j}\widehat{\mu}_j, \qquad \widehat{\mu}_i=\bar{e}_i-\sum_j\omega_{i,j}\bar{e}_j, \qquad \text{and} \qquad \sum_j\beta_j\bar{e}_j=\sum_i\lambda_i^{ss}\widehat{\mu}_i,
$$

[36] $\ell_{i,j}$ is the (i,j)-th element of matrix $(\mathbf{I}-\boldsymbol{\Omega})^{-1}$.

which implies that

$$\sum_i \sum_j \lambda_i^{ss} \ell_{i,j} \widehat{\mu}_i \widehat{\mu}_j = \sum_i \lambda_i^{ss} \Big(\sum_j \ell_{i,j} \widehat{\mu}_j \Big) \widehat{\mu}_i = \sum_i \lambda_i^{ss} \overline{e}_i \widehat{\mu}_i.$$

Hence, the across-sector misallocation in the efficiency wedge can be rewritten as:

$$\begin{aligned}
&\widehat{C}(\widehat{\boldsymbol{\mu}}) - \Lambda_L \widehat{L}(\widehat{\boldsymbol{\mu}}) + \frac{1}{2} \sum_i \lambda_i \varepsilon_i \frac{\delta_i}{1-\delta_i} \widehat{\mu}_i^2 \\
&\qquad = - \Big(\sum_i \lambda_i^{ss} \widehat{\mu}_i \Big)^2 - \sum_i \lambda_i^{ss} \widehat{\mu}_i^2 + 2 \sum_i \lambda_i^{ss} \overline{e}_i \widehat{\mu}_i + o(\|\widehat{\boldsymbol{\mu}}\|^2) \\
&\qquad = - \Big(\sum_i \lambda_i^{ss} \widehat{\mu}_i \Big)^2 + \sum_i \lambda_i^{ss} \overline{e}_i^2 - \sum_i \lambda_i^{ss} \overline{e}_i^2 - \sum_i \lambda_i^{ss} \widehat{\mu}_i^2 + 2 \sum_i \lambda_i^{ss} \overline{e}_i \widehat{\mu}_i + o(\|\widehat{\boldsymbol{\mu}}\|^2) \\
&\qquad = - \Big(\sum_i \beta_i \overline{e}_i \Big)^2 + \sum_j \beta_j \overline{e}_j^2 + \sum_i \sum_j \lambda_i^{ss} \omega_{i,j} \overline{e}_j^2 - \sum_i \lambda_i^{ss} \Big(\sum_j \omega_{i,j} \overline{e}_j \Big)^2 + o(\|\widehat{\boldsymbol{\mu}}\|^2) \\
&\qquad \equiv xvar_0(\overline{\mathbf{e}}) + \sum_i \lambda_i^{ss} xvar_i(\overline{\mathbf{e}}) + o(\|\widehat{\boldsymbol{\mu}}\|^2),
\end{aligned}$$

where notation $xvar_i$ for all $i \in \{0, 1, 2, \cdots, N\}$ follows La'O and Tahbaz-Salehi (2022). Our welfare loss function in the special case of closed economies is consistent with La'O and Tahbaz-Salehi (2022).

B.2 Proof of Proposition 6: sectoral Phillips curves

Step 1: derive $\widehat{S}$ and $\widehat{P}_C$ as functions of $\{\widehat{C}, \widehat{\mathbf{P}}, \widehat{\boldsymbol{\xi}}\}$. Following every step in the proof of Lemma 13 in Section E.5 of the Supplementary Material—except for the *sticky-price equilibrium* instead of for the difference between the *sticky-price* and *flexible-price equilibria*—yields

$$\begin{aligned}
&\big[1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha} + (\boldsymbol{\rho}_{NX} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \mathbf{1}\big] \widehat{S}(\boldsymbol{\xi}) \\
&\quad = (\boldsymbol{\rho}_{NX} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \widehat{\mathbf{P}}(\boldsymbol{\xi}) + [\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})]^\top \boldsymbol{\Delta}^{-1} (\mathbf{I} - \boldsymbol{\Delta}) \widehat{\mathbf{P}}(\boldsymbol{\xi}) + (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})(\widehat{P}_C(\boldsymbol{\xi}) + \widehat{C}(\boldsymbol{\xi})) \\
&\qquad - [\widetilde{\boldsymbol{\lambda}}_F \odot \boldsymbol{\alpha} + \boldsymbol{\lambda}_{EX} \oslash (\boldsymbol{\theta}_F - \mathbf{1})]^\top \widehat{\mathbf{D}}^*_{EX,F} - (\boldsymbol{\rho}_{IM} \odot \widetilde{\boldsymbol{\alpha}})^\top \widehat{\mathbf{P}}^*_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|), \quad \text{(B.33)}
\end{aligned}$$

where $\boldsymbol{\rho}_{IM} \equiv \boldsymbol{\rho}_{NX} - (\boldsymbol{\theta}_F - \mathbf{1}) \odot \boldsymbol{\lambda}_{EX}$ is the elasticity of sectoral imports to import price shocks, which is equal to the net export elasticity $\boldsymbol{\rho}_{EX}$ diminished by the export component $(\boldsymbol{\theta}_F - \mathbf{1}) \odot \boldsymbol{\lambda}_{EX}$.

Rearranging equation (B.33) and introducing shorthand notations, yield

$$\begin{aligned}
(\boldsymbol{\mathcal{M}}_{EX} + \boldsymbol{\mathcal{M}}_{IM})^\top \mathbf{1} \widehat{S}(\boldsymbol{\xi}) &= \widehat{P}_C(\boldsymbol{\xi}) + \widehat{C}(\boldsymbol{\xi}) + (\boldsymbol{\mathcal{M}}_p + \boldsymbol{\mathcal{M}}_\mu)^\top \widehat{\mathbf{P}}(\boldsymbol{\xi}) \\
&\quad - (\boldsymbol{\mathcal{M}}_{EX} \oslash \boldsymbol{\theta}_F)^\top \widehat{\mathbf{D}}^*_{EX,F} - \boldsymbol{\mathcal{M}}_{IM}^\top \widehat{\mathbf{P}}^*_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|), \quad \text{(B.34)}
\end{aligned}$$

where the shorthand notations are as follows:

$$\begin{aligned}
\boldsymbol{\mathcal{M}}_{EX} &\equiv (1-\widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})^{-1}[\widetilde{\boldsymbol{\lambda}}_F \odot \boldsymbol{\alpha} + \boldsymbol{\lambda}_{EX} \oslash (\boldsymbol{\theta}_F - \mathbf{1})] \odot \boldsymbol{\theta}_F, && \text{(B.35)}\\
\boldsymbol{\mathcal{M}}_{IM} &\equiv (1-\widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})^{-1}(\boldsymbol{\rho}_{IM} \odot \widetilde{\boldsymbol{\alpha}}), && \text{(B.36)}\\
\boldsymbol{\mathcal{M}}_{p} &\equiv (1-\widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})^{-1}(\boldsymbol{\rho}_{NX} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})\\
\boldsymbol{\mathcal{M}}_{\mu} &\equiv (1-\widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})^{-1}(\boldsymbol{\Delta}^{-1} - \mathbf{I})[\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})].
\end{aligned}$$

According to equation (D.87) in Section D.5 of the Supplementary Material, $1 + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1} = (\boldsymbol{\mathcal{M}}_{EX} + \boldsymbol{\mathcal{M}}_{IM})^\top \mathbf{1}$.

Substituting equation (E.108) in Section E.3 of the Supplementary Material into equation (B.34), yield the following $\widehat{S}$ and $\widehat{P}_C$ as functions of $\{\widehat{C}, \widehat{\mathbf{P}}, \widehat{\boldsymbol{\xi}}\}$:

$$\begin{aligned}
\widehat{S}(\boldsymbol{\xi}) &= \Gamma_{S,C}\widehat{C}(\boldsymbol{\xi}) + \boldsymbol{\Gamma}_{S,P}^\top \widehat{\mathbf{P}}(\boldsymbol{\xi}) + \boldsymbol{\Gamma}_{S,EX}^\top \widehat{\mathbf{D}}_{EX,F}^* + \boldsymbol{\Gamma}_{S,IM}^\top \widehat{\mathbf{P}}_{IM,F}^* + o(\|\widehat{\boldsymbol{\xi}}\|), && \text{(B.37)}\\
\widehat{P}_C(\boldsymbol{\xi}) &= \Gamma_{C}\widehat{C}(\boldsymbol{\xi}) + \boldsymbol{\Gamma}_{P}^\top \widehat{\mathbf{P}}(\boldsymbol{\xi}) + \boldsymbol{\Gamma}_{EX}^\top \widehat{\mathbf{D}}_{EX}^* + \boldsymbol{\Gamma}_{IM}^\top \widehat{\mathbf{P}}_{IM}^* + o(\|\widehat{\boldsymbol{\xi}}\|). && \text{(B.38)}
\end{aligned}$$

where the shorthand notations are

$$\begin{aligned}
\Gamma_{S,C} &\equiv (\boldsymbol{\beta}^\top \mathbf{v} + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1})^{-1},\\
\boldsymbol{\Gamma}_{S,P} &\equiv \Gamma_{S,C} \cdot (\boldsymbol{\mathcal{M}}_p + \boldsymbol{\mathcal{M}}_\mu) + \Gamma_{S,C} \cdot (\boldsymbol{\beta} \odot \mathbf{v}),\\
\boldsymbol{\Gamma}_{S,EX} &\equiv -\Gamma_{S,C} \cdot (\boldsymbol{\mathcal{M}}_{EX} \oslash \boldsymbol{\theta}_F),\\
\boldsymbol{\Gamma}_{S,IM} &\equiv -\Gamma_{S,C} \cdot \boldsymbol{\mathcal{M}}_{IM} + \Gamma_{S,C} \cdot [\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})],\\
\Gamma_{C} &\equiv \Gamma_{S,C} \cdot (1 - \boldsymbol{\beta}^\top \mathbf{v}) = (\boldsymbol{\beta}^\top \mathbf{v} + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1})^{-1}(1 - \boldsymbol{\beta}^\top \mathbf{v}), && \text{(B.39)}\\
\boldsymbol{\Gamma}_{P} &\equiv \Gamma_{C} \cdot (\boldsymbol{\mathcal{M}}_p + \boldsymbol{\mathcal{M}}_\mu) + \Gamma_{S,C} \cdot (\boldsymbol{\beta} \odot \mathbf{v})(1 + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1}),\\
\boldsymbol{\Gamma}_{EX} &\equiv -\Gamma_{C} \cdot (\boldsymbol{\mathcal{M}}_{EX} \oslash \boldsymbol{\theta}_F),\\
\boldsymbol{\Gamma}_{IM} &\equiv -\Gamma_{C} \cdot \boldsymbol{\mathcal{M}}_{IM} + \Gamma_{S,C} \cdot [\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})](1 + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1}).
\end{aligned}$$

In particular, we have $\Gamma_{S,C} = [(\boldsymbol{\mathcal{M}}_{EX} + \boldsymbol{\mathcal{M}}_{IM})^\top \mathbf{1}]^{-1}(\Gamma_C + 1)$.

Step 2: derive $\widehat{W}$ as a function of $\{\widehat{\mathbf{P}}, \widehat{C}, \widehat{\boldsymbol{\xi}}\}$. Substituting $\widehat{P}_C$ in equation (B.38) and $\widehat{L}$ in equation (E.100) into the labor supply equation (E.112), yields

$$\widehat{W}(\boldsymbol{\xi}) = \Gamma_{W,C}\widehat{C}(\boldsymbol{\xi}) + \boldsymbol{\Gamma}_{W,P}^\top \widehat{\mathbf{P}}(\boldsymbol{\xi}) + \boldsymbol{\Gamma}_{W,A}^\top \widehat{\mathbf{A}} + \boldsymbol{\Gamma}_{W,EX}^\top \widehat{\mathbf{D}}_{EX,F}^* + \boldsymbol{\Gamma}_{W,IM}^\top \widehat{\mathbf{P}}_{IM,F}^* + o(\|\widehat{\boldsymbol{\xi}}\|), \quad \text{(B.40)}$$

where the shorthand notations are

$$\begin{aligned}
\Gamma_{W,C} &\equiv \sigma + \frac{\varphi}{\Lambda_L} + \Gamma_C, \qquad \boldsymbol{\Gamma}_{W,P} \equiv \boldsymbol{\Gamma}_P, \qquad \boldsymbol{\Gamma}_{W,A} \equiv -\frac{\varphi}{\Lambda_L}\boldsymbol{\lambda},\\
\boldsymbol{\Gamma}_{W,EX} &\equiv \boldsymbol{\Gamma}_{EX} - \frac{\varphi}{\Lambda_L}[\boldsymbol{\lambda}_{EX} \oslash (\boldsymbol{\theta}_F - \mathbf{1})],\\
\boldsymbol{\Gamma}_{W,IM} &\equiv \boldsymbol{\Gamma}_{IM} + \frac{\varphi}{\Lambda_L}\left[\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v}) + (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})^\top \boldsymbol{\lambda}\right].
\end{aligned}$$

Step 3: substitute $\widehat{W}$ and $\widehat{S}$ into sectoral pricing equation. Substituting the sectoral marginal costs in equation (E.105) into the sectoral inflation in equation (D.89) yields the following pricing equation:

$$\widehat{\mathbf{P}}(\boldsymbol{\xi}) = \boldsymbol{\Delta}\big[\boldsymbol{\alpha}\widehat{W}(\boldsymbol{\xi}) + (\boldsymbol{\Omega} \odot \mathbf{V}_x)\widehat{\mathbf{P}}(\boldsymbol{\xi}) + (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\big(\mathbf{1}\widehat{S}(\boldsymbol{\xi}) + \widehat{\mathbf{P}}^*_{IM,F}\big) - \widehat{\mathbf{A}}\big] + o(\|\widehat{\boldsymbol{\xi}}\|). \tag{B.41}$$

Substituting $\widehat{W}$ and $\widehat{S}$ in equations (B.40) and (B.37) into the pricing equation (B.41) yields the following sectoral Phillips curves in terms of $\widehat{C}$:

$$\widehat{\mathbf{P}}(\boldsymbol{\xi}) = \boldsymbol{\mathcal{B}}\widehat{C}(\boldsymbol{\xi}) + \boldsymbol{\mathcal{V}}_{C,A}\widehat{\mathbf{A}} + \boldsymbol{\mathcal{V}}_{C,EX}\widehat{\mathbf{D}}^*_{EX,F} + \boldsymbol{\mathcal{V}}_{C,IM}\widehat{\mathbf{P}}^*_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{B.42}$$

where the shorthand notations are

$$\begin{aligned}
\boldsymbol{\mathcal{B}} &\equiv \boldsymbol{\Delta}_\Phi\big[\boldsymbol{\alpha}\Gamma_{W,C} + (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\mathbf{1}\Gamma_{S,C}\big],\\
\boldsymbol{\mathcal{V}}_{C,A} &\equiv \boldsymbol{\Delta}_\Phi\big(\boldsymbol{\alpha}\boldsymbol{\Gamma}^\top_{W,A} - \mathbf{1}\big),\\
\boldsymbol{\mathcal{V}}_{C,EX} &\equiv \boldsymbol{\Delta}_\Phi\big[\boldsymbol{\alpha}\boldsymbol{\Gamma}^\top_{W,EX} + (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\mathbf{1}\boldsymbol{\Gamma}^\top_{S,EX}\big],\\
\boldsymbol{\mathcal{V}}_{C,IM} &\equiv \boldsymbol{\Delta}_\Phi\big[\boldsymbol{\alpha}\boldsymbol{\Gamma}^\top_{W,IM} + (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\mathbf{1}\boldsymbol{\Gamma}^\top_{S,IM}\big],\\
\boldsymbol{\Delta}_\Phi &\equiv \big[\boldsymbol{\Delta}^{-1} - \boldsymbol{\Omega} \odot \mathbf{V}_x - \boldsymbol{\alpha}\boldsymbol{\Gamma}^\top_{W,P} - (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\mathbf{1}\boldsymbol{\Gamma}^\top_{S,P}\big]^{-1}.
\end{aligned}$$

To further derive the sectoral Phillips curves in terms of the aggregate output gap $\widehat{C}^{gap}$, we need to solve for the log deviation of the aggregate output in the *flexible-price equilibrium* from the steady state, denoted by $\widehat{C}^{flex}(\boldsymbol{\xi})$. To do so, we derive the flexible-price version of equations (B.41), (E.112), (B.34), (E.100), and (E.108) by setting $\boldsymbol{\Delta} = \mathbf{I}$, which yields the following equations, respectively:

$$\begin{aligned}
\widehat{\mathbf{P}}^{flex}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{flex}(\boldsymbol{\xi}) &= \widetilde{\boldsymbol{\alpha}}(\widehat{W}^{flex}(\boldsymbol{\xi}) - \widehat{S}^{flex}(\boldsymbol{\xi})) - \mathbf{L}_{vx}\widehat{\mathbf{A}} + \mathbf{L}_{vx}(\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\widehat{\mathbf{P}}^*_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|),\\
\widehat{W}^{flex}(\boldsymbol{\xi}) - \widehat{S}^{flex}(\boldsymbol{\xi}) &= \widehat{P}^{flex}_C(\boldsymbol{\xi}) - \widehat{S}^{flex}(\boldsymbol{\xi}) + \sigma\widehat{C}^{flex}(\boldsymbol{\xi}) + \varphi\widehat{L}^{flex}(\boldsymbol{\xi}),\\
\widehat{P}^{flex}_C(\boldsymbol{\xi}) - \widehat{S}^{flex}(\boldsymbol{\xi}) + \widehat{C}^{flex}(\boldsymbol{\xi}) &= -\boldsymbol{\mathcal{M}}^\top_P(\widehat{\mathbf{P}}^{flex}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{flex}(\boldsymbol{\xi})) + (\boldsymbol{\mathcal{M}}_{EX} \oslash \boldsymbol{\theta}_F)^\top\widehat{\mathbf{D}}^*_{EX,F} + \boldsymbol{\mathcal{M}}^\top_{IM}\widehat{\mathbf{P}}^*_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|),\\
\widehat{C}^{flex}(\boldsymbol{\xi}) - \Lambda_L\widehat{L}^{flex}(\boldsymbol{\xi}) &= \boldsymbol{\lambda}^\top\widehat{A} + [\boldsymbol{\lambda}_{EX} \oslash (\boldsymbol{\theta}_F - \mathbf{1})]^\top\widehat{\mathbf{D}}^*_{EX,F} - \big[\boldsymbol{\beta}^\top \odot (\mathbf{1} - \mathbf{v})^\top + \boldsymbol{\lambda}^\top(\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\big]\widehat{\mathbf{P}}^*_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|),\\
\widehat{P}^{flex}_C(\boldsymbol{\xi}) - \widehat{S}^{flex}(\boldsymbol{\xi}) &= (\boldsymbol{\beta} \odot \mathbf{v})^\top(\widehat{\mathbf{P}}^{flex}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{flex}(\boldsymbol{\xi})) + [\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})]^\top\widehat{\mathbf{P}}^*_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|).
\end{aligned}$$

Combining the above five equations yields

$$\widehat{C}^{flex}(\boldsymbol{\xi}) = \boldsymbol{\Gamma}^{flex}_{C,A}\widehat{\mathbf{A}} + \boldsymbol{\Gamma}^{flex}_{C,EX}\widehat{\mathbf{D}}^*_{EX,F} + \boldsymbol{\Gamma}^{flex}_{C,IM}\widehat{\mathbf{P}}^*_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{B.43}$$

where the shorthand notations are

$$\begin{aligned}
\boldsymbol{\Gamma}^{flex}_{C,A} &\equiv (\Delta^{flex}_C)^{-1}\big(\boldsymbol{\mathcal{M}}^\top_L\widetilde{\boldsymbol{\alpha}}\boldsymbol{\lambda}^\top\varphi/\Lambda_L + \boldsymbol{\mathcal{M}}^\top_L\mathbf{L}_{vx}\big),\\
\boldsymbol{\Gamma}^{flex}_{C,IM} &\equiv -(\Delta^{flex}_C)^{-1}\Big\{\boldsymbol{\mathcal{M}}^\top_L\widetilde{\boldsymbol{\alpha}}\big[(\Lambda_L + \varphi)\boldsymbol{\beta}^\top \odot (\mathbf{1} - \mathbf{v})^\top + \boldsymbol{\lambda}^\top(\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\big]/\Lambda_L\\
&\qquad + \boldsymbol{\mathcal{M}}^\top_L\mathbf{L}_{vx}(\boldsymbol{\Omega} \odot \mathbf{V}_{1-x}) - \boldsymbol{\mathcal{M}}^\top_{IM} + \boldsymbol{\beta}^\top \odot (\mathbf{1} - \mathbf{v})^\top\Big\},
\end{aligned}$$

$$\begin{aligned}
\mathbf{\Gamma}_{C,EX}^{flex} &\equiv (\Delta_C^{flex})^{-1}\{\boldsymbol{\mathcal{M}}_L^\top \widetilde{\boldsymbol{\alpha}}[\boldsymbol{\lambda}_{EX} \oslash (\boldsymbol{\theta}_F - \mathbf{1})]^\top \varphi/\Lambda_L + (\boldsymbol{\mathcal{M}}_{EX} \oslash \boldsymbol{\theta}_F)^\top\}, \\
\boldsymbol{\mathcal{M}}_L^\top &\equiv (\boldsymbol{\mathcal{M}}_P + \boldsymbol{\beta} \odot \mathbf{v})^\top \left[\mathbf{I} - \widetilde{\boldsymbol{\alpha}}(\boldsymbol{\beta} \odot \mathbf{v})^\top\right]^{-1}, \\
\Delta_C^{flex} &\equiv 1 + \boldsymbol{\mathcal{M}}_L^\top \widetilde{\boldsymbol{\alpha}}(\sigma + \varphi/\Lambda_L).
\end{aligned}$$

Combining equations (B.42) and (B.43), yields the following sectoral Phillips curves in terms of the aggregate output gap $\widehat{C}^{gap}$:

$$\widehat{\mathbf{P}}(\boldsymbol{\xi}) = \boldsymbol{\mathcal{B}}\widehat{C}^{gap}(\boldsymbol{\xi}) + \boldsymbol{\mathcal{V}}_A\widehat{\mathbf{A}} + \boldsymbol{\mathcal{V}}_{EX}\widehat{\mathbf{D}}_{EX,F}^* + \boldsymbol{\mathcal{V}}_{IM}\widehat{\mathbf{P}}_{IM,F}^* + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{B.44}$$

where the matrices of coefficients of exogenous shocks are as follows:

$$\begin{aligned}
\boldsymbol{\mathcal{V}}_A &\equiv \boldsymbol{\mathcal{V}}_{C,A} + \boldsymbol{\mathcal{B}} \cdot \mathbf{\Gamma}_{C,EX}^{flex}, \\
\boldsymbol{\mathcal{V}}_{EX} &\equiv \boldsymbol{\mathcal{V}}_{C,EX} + \boldsymbol{\mathcal{B}} \cdot \mathbf{\Gamma}_{C,EX}^{flex}, \\
\boldsymbol{\mathcal{V}}_{IM} &\equiv \boldsymbol{\mathcal{V}}_{C,IM} + \boldsymbol{\mathcal{B}} \cdot \mathbf{\Gamma}_{C,IM}^{flex}.
\end{aligned}$$

B.3 Proof of Propositions 7 and 8: the optimal monetary policy

The optimal monetary policy maximizes the welfare loss (up to the second-order approximation) in equation (A.1) subject to the sectoral Phillips curves (up to the first-order approximation) in equation (A.3):

$$\begin{aligned}
&\max_{\widehat{C}^{gap},\widehat{\mathbf{P}}} \left\{ -\frac{1}{2}\Big(\sigma - 1 + \frac{\varphi+1}{\Lambda_L}\Big)\widehat{C}^{gap}(\boldsymbol{\xi})^2 - \frac{1}{2}\widehat{\mathbf{P}}^\top \boldsymbol{\mathcal{L}}\widehat{\mathbf{P}} \right\} \\
&\text{s.t.} \qquad \widehat{\mathbf{P}}(\boldsymbol{\xi}) = \boldsymbol{\mathcal{B}}\widehat{C}^{gap}(\boldsymbol{\xi}) + \boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}}.
\end{aligned}$$

Denote $\boldsymbol{\eta}$ the vector of multipliers for the constraint of sectoral Phillips curves. The first-order conditions with respect to $\widehat{C}^{gap}$ and $\widehat{\mathbf{P}}$, respectively, are

$$-\left[\sigma - 1 + (\varphi+1)/\Lambda_L\right]\widehat{C}^{gap}(\boldsymbol{\xi}) + \boldsymbol{\eta}^\top\boldsymbol{\mathcal{B}} = 0, \tag{B.45}$$

$$-\boldsymbol{\mathcal{L}}\widehat{\mathbf{P}}(\boldsymbol{\xi}) - \boldsymbol{\eta} = 0. \tag{B.46}$$

Substituting equation (B.46) into equation (B.45) to eliminate $\boldsymbol{\eta}$ yields

$$\left[\sigma - 1 + (\varphi+1)/\Lambda_L\right]\widehat{C}^{gap}(\boldsymbol{\xi}) + \boldsymbol{\mathcal{B}}^\top\boldsymbol{\mathcal{L}}\widehat{\mathbf{P}}(\boldsymbol{\xi}) = 0. \tag{B.47}$$

Substituting equations (38) and (36) from Section 3.3 into equation (B.47) yields

$$\left\{\left[\sigma - 1 + (\varphi+1)/\Lambda_L\right]\kappa_C^{-1}\boldsymbol{\mathcal{M}}_{OG}^\top(\boldsymbol{\Delta}^{-1} - \mathbf{I}) + \boldsymbol{\mathcal{B}}^\top\boldsymbol{\mathcal{L}}\right\}\widehat{\mathbf{P}}(\boldsymbol{\xi}) = 0.$$

Substituting the sectoral Phillips curves in equation (A.3) in equation (B.47) yields

$$\left[\sigma - 1 + (\varphi + 1)/\Lambda_L + \boldsymbol{\mathcal{B}}^\top \boldsymbol{\mathcal{L}} \boldsymbol{\mathcal{B}}\right] \widehat{C}^{gap}(\boldsymbol{\xi}) + \boldsymbol{\mathcal{B}}^\top \boldsymbol{\mathcal{L}} \boldsymbol{\mathcal{V}} \widehat{\boldsymbol{\xi}} = 0.$$

C Quantitative analysis

C.1 Data and calibration

We calibrate our model of a small open economy with production networks using the World Input-Output Database (WIOD). The WIOD covers 28 EU countries and 15 other major countries in the world from 2000 to 2014 and provides information for 56 major sectors.[37] We calibrate our model using the Input-Output Tables for the year 2014 for each country. For each sector in each country, the WIOD reports the following sectoral values: (i) intermediate goods expenditures on goods from different domestic and foreign sectors, (ii) labor compensation, (iii) gross output, (iv) value-added, and (v) exports to foreign countries. For each country, we calibrate the parameters as follows: (i) the (i, j) element of the input-output matrix $\mathbf{\Omega}$ is calibrated using the share of customer sector i's intermediate goods expenditure on the supplier sector j (the sum of expenditures on the domestic and foreign sector j) in the customer i's gross output, (ii) the (i, j) element of the home bias in intermediate inputs $\mathbf{V}_x$ is calibrated using the ratio of customer sector i's intermediate goods expenditure on the domestic supplier sector j to the sum of expenditures on the domestic and foreign sector j's goods; (iii) the sectoral labor share of $\boldsymbol{\alpha}$ is calibrated using the share of sectoral labor compensation in sectoral gross output for each sector; (iv) the steady-state values of sectoral demand from foreign countries $\mathbf{D}_H^{*,ss}$ are calibrated such that the sectoral export-to-GDP ratios in the model matches the sector's export-to-GDP ratios in the data; (v) the i-th element of the consumption shares $\boldsymbol{\beta}$ is calibrated using the ratio of the sum of domestic households' and government's consumption expenditures on sector i goods to the value added of sector i; (vi) the i-th element of the home bias in consumption $\mathbf{v}$ is calibrated using the ratio of the sum of domestic household's and government's consumption expenditures on the domestic sector i's goods to the sum of expenditures on the domestic and foreign sector i's goods.

We calibrate the values of other parameters to their standard levels in the literature. The risk aversion parameter and the inverse of the labor supply elasticity of the households are calibrated to $\sigma = 2$ and $\varphi = 1$, respectively, following the business cycle literature (e.g., Corsetti et al., 2010; Arellano et al., 2019). We follow Atkeson and Burstein (2008) and calibrate the within-sector elasticity of substitution to $\varepsilon_i = 8$ for all sectors i. We calibrate the elasticity of substitution between domestic and foreign goods to 5 for both domestic and foreign households and firms, *viz*, $\theta_i = \theta_{Fi} = 5$ for all sectors i, following Head and Mayer (2014). We calibrate the sector-level parameters of price rigidity δ_i using the sector-level price rigidities from Pasten et al.

[37] We use the version of Release 2016 of the World Input-Output Database. Shown in Table C.1 is the list of sectors.

(2024).[38] With the calibrated sector-level price rigidities, the average quarterly frequency of price adjustment across all sectors equals 0.49. We follow Rubbo (2023) and La'O and Tahbaz-Salehi (2022) to introduce wage stickiness by adding a labor sector 0, which uses domestic labor to produce the product of "labor" that is supplied to all other sectors as inputs. We follow Beraja et al. (2019) and Barattieri et al. (2014) to calibrate the parameter of wage rigidity δ_0 such that the quarterly frequency of wage adjustment equals 0.25. Summarized in Table 3 in Section 4.1 is the calibration of different parameters.

Last, we calibrate the exogenous shocks as now described. We calculate the growth rates of sectoral import prices and productivity using the social economic accounts in the WIOD. We compute the covariance matrix between different sectors' import price series and use it to calibrate the covariance matrix of import prices used in the simulation of the model. We use the same method to calibrate the covariance matrix for the sectoral productivity.

Table C.1: Industry classifications in World Input-Output Database

ID	Industry code	Description	ID	Industry code	Description
1	A01	Crop and animal production, hunting and related service	29	G46	Wholesale trade, except of motor vehicles and motorcycles
2	A02	Forestry and logging	30	G47	Retail trade, except of motor vehicles and motorcycles
3	A03	Fishing and aquaculture	31	H49	Land transport and transport via pipelines
4	B	Mining and quarrying	32	H50	Water transport
5	C10-C12	Manufacture of food products, beverages and tobacco products	33	H51	Air transport
6	C13-C15	Manufacture of textiles, wearing apparel and leather products	34	H52	Warehousing and support activities for transportation
7	C16	Manufacture of wood products, plaiting materials	35	H53	Postal and courier activities
8	C17	Manufacture of paper and paper products	36	I	Accommodation and food service activities
9	C18	Printing and reproduction of recorded media	37	J58	Publishing activities
10	C19	Manufacture of coke and refined petroleum products	38	J59_J60	Motion picture, video, and television
11	C20	Manufacture of chemicals and chemical products	39	J61	Telecommunications
12	C21	Manufacture of basic pharmaceutical products	40	J62_J63	Computer programming, consultancy and related activities; information service activities
13	C22	Manufacture of rubber and plastic products	41	K64	Financial service activities, except insurance and pension funding
14	C23	Manufacture of other non-metallic mineral products	42	K65	Insurance, reinsurance and pension funding, except compulsory social security
15	C24	Manufacture of basic metals	43	K66	Activities auxiliary to financial services and insurance activities
16	C25	Manufacture of fabricated metal products	44	L68	Real estate activities
17	C26	Manufacture of computer, electronic and optical products	45	M69_M70	Legal and accounting activities; activities of head offices; management consultancy activities
18	C27	Manufacture of electrical equipment	46	M71	Architectural and engineering activities; technical testing and analysis
19	C28	Manufacture of machinery and equipment n.e.c.	47	M72	Scientific research and development
20	C29	Manufacture of motor vehicles, trailers and semi-trailers	48	M73	Advertising and market research
21	C30	Manufacture of other transport equipment	49	M74_M75	Other professional, scientific and technical activities; veterinary activities
22	C31_C32	Manufacture of furniture; other manufacturing	50	N	Administrative and support service activities
23	C33	Repair and installation of machinery and equipment	51	O84	Public administration and defence; compulsory social security
24	D35	Electricity, gas, steam and air conditioning supply	52	P85	Education
25	E36	Water collection, treatment and supply	53	Q	Human health and social work activities
26	E37-E39	Sewerage; waste management services	54	R_S	Other service activities
27	F	Construction	55	T	Activities of households as employers
28	G45	Wholesale and retail trade,repair motor vehicles	56	U	Activities of extraterritorial organizations and bodies

C.2 Sectoral weights under alternative monetary policies

All of the alternative monetary policies we study in Section 4.3 are implemented by setting the following aggregate inflation index to zero:

$$\chi^\top \cdot (\mathbf{\Delta}^{-1} - \mathbf{I})\widehat{\mathbf{P}}(\widehat{\xi}) = 0, \tag{C.48}$$

where the sectoral weights χ are equal to the following:

$$\text{optimal monetary policy:} \quad \chi^\top = \left\{ [\sigma - 1 + \varphi + 1)/\Lambda_L] \, \kappa_C^{-1} \boldsymbol{\mathcal{M}}_{OG}^\top + \boldsymbol{\mathcal{B}}^\top \boldsymbol{\mathcal{L}} \mathbf{\Delta} (\mathbf{I} - \mathbf{\Delta})^{-1} \right\};$$

$$\text{OG monetary policy:} \quad \chi^\top = \mathcal{M}_{OG}^\top;$$

[38] We thank Michael Weber for kindly providing the sector-level price rigidities.

$$\begin{aligned} \text{Domar-weight policy:} \quad & \boldsymbol{\chi}^\top = \boldsymbol{\lambda}^\top; \\ \text{CPI-weight policy:} \quad & \boldsymbol{\chi}^\top = \boldsymbol{\beta}^\top; \\ \text{OG policy w/o networks:} \quad & \boldsymbol{\chi}^\top = (\mathcal{M}_{OG}^{no\_net})^\top. \end{aligned}$$

Policy weights in the OG monetary policy ignoring the production network are

$$\mathcal{M}_{OG}^{no\_net} = (\boldsymbol{\beta} \odot \mathbf{v}) + \kappa_S \boldsymbol{\rho}_{NX} + \kappa_S \boldsymbol{\lambda}_{\boldsymbol{EX}},$$

where the parameters are equal to the following (using the results in Lemma 8 for $\boldsymbol{\lambda}_{EX}$ and Theorem 1 for $\boldsymbol{\rho}_{NX}$ and κ_S):

$$\begin{aligned} \boldsymbol{\lambda}_{EX} &= (1 - \boldsymbol{\beta}^\top \mathbf{v}) \left[(\boldsymbol{\theta}_F - \mathbf{1}) \oslash \boldsymbol{\theta}_F \odot \mathbf{v}_H^* \right], \\ \boldsymbol{\rho}_{NX} &= (\boldsymbol{\theta}_F - \mathbf{1}) \odot \boldsymbol{\lambda}_{EX} + (\boldsymbol{\theta} - \mathbf{1}) \odot \left[\boldsymbol{\beta} \odot \mathbf{v} \odot (\mathbf{1} - \mathbf{v}) \right], \\ \kappa_S &= \left[(1 - \boldsymbol{\beta}^\top \mathbf{v}) + (\boldsymbol{\rho}_{NX} + \boldsymbol{\lambda}_{EX})^\top \mathbf{1} \right]^{-1} (1 - \boldsymbol{\beta}^\top \mathbf{v}). \end{aligned}$$

Combining the monetary policy rule in equation (C.48) with the sectoral Phillips curves in equation (A.3), yields the aggregate output gap as a function of the specific policy weights $\boldsymbol{\chi}$ and the parameters of the sectoral Phillips curves, *viz*:

$$\widehat{C}^{gap}(\widehat{\boldsymbol{\xi}}) = -\frac{\boldsymbol{\chi}^\top (\boldsymbol{\Delta}^{-1} - \mathbf{I}) \boldsymbol{\mathcal{V}} \widehat{\boldsymbol{\xi}}}{\boldsymbol{\chi}^\top (\boldsymbol{\Delta}^{-1} - \mathbf{I}) \boldsymbol{\mathcal{B}}}. \tag{C.49}$$

Substituting equation (C.49) into the welfare loss function in equation (A.5) of Section A, we obtain the welfare loss under the alternative monetary policy with policy weights $\boldsymbol{\chi}$ and any realized state $\widehat{\boldsymbol{\xi}}$.

C.3 Welfare loss under alternative shocks

Table C.2: Welfare loss under different monetary policies: Shocks to import prices of only manufacturing sectors

	(1) Optimal	(2) OG	(3) Domar	(4) CPI	(5) OG w/o network
Mexico Export-to-GDP ratio: 19%					
Total welfare loss	-3.334	-3.357	-3.428	-6.620	-6.669
Improvement by OG policy towards optimal			75.8%	99.3%	99.3%
Output gap misallocation	-0.003	0.000	-0.004	-0.408	-0.415
Within- and across-sector misallocation					
— output-gap-related	0.026	0.000	-0.068	-2.855	-2.898
— policy-irrelevant	-3.357	-3.357	-3.357	-3.357	-3.357
Luxembourg Export-to-GDP ratio: 83%					
Total welfare loss	-1.595	-1.604	-1.678	-4.545	-3.012
Improvement by OG policy towards optimal			89.2%	99.7%	99.4%
Output gap misallocation	-0.002	0.000	-0.007	-0.481	-0.220
Within- and across-sector misallocation					
— output-gap-related	0.011	0.000	-0.067	-2.460	-1.189
— policy-irrelevant	-1.604	-1.604	-1.604	-1.604	-1.604
US Export-to-GDP ratio: 9.2%					
Total welfare loss	-2.634	-2.734	-2.740	-9.816	-9.248
Improvement by OG policy towards optimal			5.5%	98.6%	98.5%
Output gap misallocation	-0.015	0.000	0.000	-0.832	-0.758
Within- and across-sector misallocation					
— output-gap-related	0.115	0.000	-0.006	-6.250	-5.757
— policy-irrelevant	-2.734	-2.734	-2.734	-2.734	-2.734

Notes: Reported in this table is the welfare loss—expressed in units of percent of steady-state consumption—under different monetary policy designs. Columns (1) to (5) show the welfare losses under the optimal policy, the OG policy, the Domar-weight policy, the CPI-weight policy, and the OG policy that ignores the production network, respectively. The sectoral weights in all of the five policies adjust for sectoral price rigidities.

Table C.3: Welfare loss under different monetary policies: Shocks to sectoral productivity

	(1) Optimal	(2) OG	(3) Domar	(4) CPI	(5) OG w/o network
Mexico Export-to-GDP ratio: 19%					
Total welfare loss	-0.744	-0.754	-0.755	-1.529	-1.527
Improvement by OG policy towards optimal			6.6%	98.7%	98.7%
Output gap misallocation	-0.001	0.000	0.000	-0.093	-0.092
Within- and across-sector misallocation					
— output-gap-related	0.011	0.000	-0.001	-0.682	-0.681
— policy-irrelevant	-0.754	-0.754	-0.754	-0.754	-0.754
Luxembourg Export-to-GDP ratio: 83%					
Total welfare loss	-3.057	-3.061	-3.213	-3.459	-3.833
Improvement by OG policy towards optimal			97.4%	99.0%	99.5%
Output gap misallocation	-0.001	0.000	-0.022	-0.061	-0.123
Within- and across-sector misallocation					
— output-gap-related	0.005	0.000	-0.130	-0.337	-0.648
— policy-irrelevant	-3.061	-3.061	-3.061	-3.061	-3.061
US Export-to-GDP ratio: 9.2%					
Total welfare loss	-1.208	-1.216	-1.216	-2.056	-2.047
Improvement by OG policy towards optimal			2.3%	99.1%	99.1%
Output gap misallocation	-0.001	0.000	0.000	-0.104	-0.103
Within- and across-sector misallocation					
— output-gap-related	0.009	0.000	0.000	-0.737	-0.729
— policy-irrelevant	-1.216	-1.216	-1.216	-1.216	-1.216

Notes: Reported in this table is the welfare loss—expressed in units of percent of steady-state consumption—under different monetary policy designs. Columns (1) to (5) show the welfare losses under the optimal policy, the OG policy, the Domar-weight policy, the CPI-weight policy, and the OG policy that ignores the production network, respectively. The sectoral weights in all of the five policies adjust for sectoral price rigidities.

Supplementary Material

D Basic results of the model

This section derives some basic results of the model, thus preparing for the proofs of our main theoretical results in Sections 3 and Online Appendix A.

D.1 Feasible allocation

The feasible allocation of the economy can be defined at the sector level with the help of an additional variable ι_i that captures the within-sector output dispersion in each sector i, as stated in the following definition:

Definition 12 (Feasible allocation)**.** *Denote the use of labor and intermediate inputs of each sector i and j by*

$$\left(L_i, X_{i,j}, X_{Hi,Hj}, X_{Hi,Fj}\right) \equiv \int_0^1 \left(L_{if}, X_{if,j}, X_{Hif,Hj}, X_{Hif,Fj}\right) df.$$

A feasible allocation is a state-contingent allocation of C, $\{C_i\}_i$, $\{Y_i\}_i$, $\{L_i\}_i$, $\{X_{i,j}\}_{i,j}$, $\{C_{Hi}\}_i$, $\{C_{Fi}\}_i$, $\{X_{Hi,Hj}\}_{i,j}$, $\{X_{Hi,Fj}\}_{i,j}$, L, $\{Y_{EX,i}\}_i$, *and* $\{\iota_i\}_i$ *that satisfies the following equations (D.50)-(D.57) for each* $i, j \in \{1, 2, \cdots, N\}$ *and any realized state* $\boldsymbol{\xi} \equiv \left\{A_i, D^*_{EX,Fi}, P^*_{IM,Fi}\right\}_i \in \boldsymbol{\Xi}$ *:*

$$\text{(consumption basket)} \qquad C = \mathcal{C}\left(\{C_i\}_i\right), \tag{D.50}$$

$$\text{(production function)} \qquad Y_i = A_i \cdot \iota_i \cdot F_i\left(L_i, \{X_{i,j}\}_j\right), \tag{D.51}$$

$$\text{(consumption with import)} \qquad C_i = \mathcal{C}_i\left(C_{Hi}, C_{Fi}\right), \tag{D.52}$$

$$\text{(intermediate inputs with import)} \qquad X_{i,j} = \mathcal{X}_{i,j}\left(X_{Hi,Hj}, X_{Hi,Fj}\right), \tag{D.53}$$

$$\text{(labor market clearing)} \qquad L = \sum_i L_i, \tag{D.54}$$

$$\text{(goods market clearing)} \qquad Y_i = C_{Hi} + \sum_j X_{Hj,Hi} + Y_{EX,i}, \tag{D.55}$$

$$\text{(balance of trade)} \qquad EX \equiv \sum_i \left(D^*_{EX,Fi}\right)^{\frac{1}{\theta_{F,i}}} Y_{EX,i}^{\frac{\theta_{F,i}-1}{\theta_{F,i}}} = \sum_i P^*_{IM,Fi}\left(C_{Fi} + \sum_j X_{Hj,Fi}\right), \tag{D.56}$$

$$\text{(within-sector output dispersion)} \qquad \iota_i \equiv Y_i \Big/ \left(\int_0^1 Y_{if} df\right), \tag{D.57}$$

where the aggregators $F_i = \left(L_{if}/\alpha_i\right)^{\alpha_i} \prod_{j=1}^N \left(X_{i,j}/\omega_{i,j}\right)^{\omega_{i,j}}$ *following equation (1),* $\{\mathcal{X}_{i,j}\}_{i,j}$ *is defined in equation (2), and* $\mathcal{C}$ *and* $\{\mathcal{C}_i\}_i$ *are defined in equation (9).*

For sector-level conditions in equations (D.50) to (D.57) to summarize the feasible allocation of the economy at the firm level, all firms within each sector must share the same marginal product of inputs, which happens to hold in the *first-best allocation*, the *sticky-price equilibrium*, and the *flexible-price equilibrium* under our model setup.

D.2 Proof of Lemma 1: efficient flexible-price equilibrium

To prove Lemma 1, we define the *first-best allocation* (Definition 13), present the conditions for it (Lemma 7), and show that these conditions coincide with those for the *flexible-price equilibrium* when Assumption 1 holds.

The *first-best allocation* is the feasible allocation that solves the social planner's problem, as outlined in the following definition.

Definition 13 (First-best allocation)**.** *The first-best allocation is a feasible allocation that maximizes the representative household's utility $u(C, L)$, i.e., it solves the following social planner's problem:*

$$\max_{\{\iota_i, L_i, \{X_{Hi,Hj}, X_{Hi,Fj}\}_j, C_{Hi}, C_{Fi}\}_i} u(C, L)$$
$$\text{s.t.} \quad \text{equations (D.50) to (D.56) and } \iota_i \in [0, 1] \text{ for all } i.$$

Substituting equations (D.50), (D.52), and (D.54) into the utility function $u(C, L)$ yields

$$u(C, L) = u\Big(\mathcal{C}\big(\{\mathcal{C}_i(C_{Hi}, C_{Fi})\}_i\big), \sum_i L_i\Big). \tag{D.58}$$

Substituting equations (D.51), (D.53), and (D.55) into equation (D.56), yields the consolidated constraint of the social planner's problem in the following:

$$\sum_i \left(D^*_{EX,Fi}\right)^{\frac{1}{\theta_{F,i}}} \Big[A_i \iota_i F_i\big(\{L_i, \mathcal{X}_{i,j}(X_{Hi,Hj}, X_{Hi,Fj})\}_j\big) - C_{Hi} - \sum_j X_{Hj,Hi}\Big]^{\frac{\theta_{F,i}-1}{\theta_{F,i}}}$$
$$= \sum_i P^*_{IM,Fi}\Big(C_{Fi} + \sum_j X_{Hj,Fi}\Big). \tag{D.59}$$

As a result, the first-best allocation is the feasible allocation that maximizes the utility function in equation (D.58)—subject to the constraint in equation (D.59)—which, in turn, satisfies the optimality conditions outlined in Lemma 7.

Lemma 7 (First-best allocation)**.** *The first-best allocation satisfies the following optimality conditions:*

$$\iota_i = 1, \tag{D.60}$$
$$-\frac{\partial u/\partial L}{\frac{\partial u}{\partial \mathcal{C}}\frac{\partial \mathcal{C}}{\partial \mathcal{C}_i}\frac{\partial \mathcal{C}_i}{\partial C_{Hi}}} = A_i \frac{\partial F_i}{\partial L_i}, \tag{D.61}$$
$$\frac{\partial \mathcal{C}/\partial \mathcal{C}_j}{\partial \mathcal{C}/\partial \mathcal{C}_i}\frac{\partial \mathcal{C}_j/\partial C_{Hj}}{\partial \mathcal{C}_i/\partial C_{Hi}} = A_i \frac{\partial F_i}{\partial \mathcal{X}_{i,j}}\frac{\partial \mathcal{X}_{i,j}}{\partial X_{Hi,Hj}}, \tag{D.62}$$

$$\frac{\partial \mathcal{C}_i / \partial C_{Fi}}{\partial \mathcal{C}_i / \partial C_{Hi}} = P^*_{IM,Fi} \cdot \frac{\theta_{F,i}}{\theta_{F,i}-1} \Big(\frac{Y_{EX,i}}{D^*_{EX,Fi}} \Big)^{\frac{1}{\theta_{F,i}}}, \tag{D.63}$$

$$\frac{\partial \mathcal{X}_{i,j} / \partial X_{Hi,Fj}}{\partial \mathcal{X}_{i,j} / \partial X_{Hi,Hj}} = P^*_{IM,Fj} \cdot \frac{\theta_{F,j}}{\theta_{F,j}-1} \Big(\frac{Y_{EX,j}}{D^*_{EX,Fj}} \Big)^{\frac{1}{\theta_{F,j}}}. \tag{D.64}$$

Proof of Lemma 7. To eliminate distortions and maximize welfare, the social planner would close the within-sector dispersion in output, i.e., choosing $\iota_i = 1$. Furthermore, denote κ the multiplier for the constraint (D.59) of the social planner's problem, the first-order conditions w.r.t. L_i, $X_{Hi,Hj}$, $X_{Hi,Fj}$, C_{Hi}, and C_{Fi} are

$$\begin{aligned}
0 &= \frac{\partial u}{\partial L} + \kappa \cdot \frac{\theta_{F,i}-1}{\theta_{F,i}} \Big(\frac{Y_{EX,i}}{D^*_{EX,Fi}} \Big)^{-\frac{1}{\theta_{F,i}}} A_i \frac{\partial F_i}{\partial L_i}, \\
0 &= \frac{\theta_{F,i}-1}{\theta_{F,i}} \Big(\frac{Y_{EX,i}}{D^*_{EX,Fi}} \Big)^{-\frac{1}{\theta_{F,i}}} A_i \frac{\partial F_i}{\partial \mathcal{X}_{i,j}} \frac{\partial \mathcal{X}_{i,j}}{\partial X_{Hi,Hj}} - \frac{\theta_{F,j}-1}{\theta_{F,j}} \Big(\frac{Y_{EX,i}}{D^*_{EX,Fj}} \Big)^{-\frac{1}{\theta_{F,j}}}, \\
0 &= \frac{\theta_{F,i}-1}{\theta_{F,i}} \Big(\frac{Y_{EX,i}}{D^*_{EX,Fi}} \Big)^{-\frac{1}{\theta_{F,i}}} A_i \frac{\partial F_i}{\partial \mathcal{X}_{i,j}} \frac{\partial \mathcal{X}_{i,j}}{\partial X_{Hi,Fj}} - P^*_{IM,Fj}, \\
0 &= \frac{\partial u}{\partial \mathcal{C}} \frac{\partial \mathcal{C}}{\partial \mathcal{C}_i} \frac{\partial \mathcal{C}_i}{\partial C_{Hi}} - \kappa \cdot \frac{\theta_{F,i}-1}{\theta_{F,i}} \Big(\frac{Y_{EX,i}}{D^*_{EX,Fi}} \Big)^{-\frac{1}{\theta_{F,i}}}, \\
0 &= \frac{\partial u}{\partial \mathcal{C}} \frac{\partial \mathcal{C}}{\partial \mathcal{C}_i} \frac{\partial \mathcal{C}_i}{\partial C_{Fi}} - \kappa \cdot P^*_{IM,Fi}.
\end{aligned}$$

Rearranging the above first-order conditions and eliminating the multiplier κ yields equations (D.61)-(D.64) of Lemma 7. □

Proof of Lemma 1. Under $\tau_i = -1/(\varepsilon_i - 1)$ of Assumption 1, in the *flexible-price equilibrium*, combining the optimal pricing conditions of the firms that maximize profits in equation (4)—subject to demand function in equation (6)—with the cost minimization conditions that minimize the total costs in equation (3)—subject to the production technology in equations (1) and (2)—yields the following two conditions:

$$A_i \frac{\partial F_i}{\partial L_i}(\boldsymbol{\xi}) = \frac{W^{flex}(\boldsymbol{\xi})}{P_i^{flex}(\boldsymbol{\xi})}, \tag{D.65}$$

$$A_i \frac{\partial F_i}{\partial \mathcal{X}_{i,j}}(\boldsymbol{\xi}) \frac{\partial \mathcal{X}_{i,j}}{\partial X_{Hi,Hj}}(\boldsymbol{\xi}) = \frac{P_j^{flex}(\boldsymbol{\xi})}{P_i^{flex}(\boldsymbol{\xi})}. \tag{D.66}$$

Under $\tau_{EX,i} = 1/\theta_{F,i}$ of Assumption 1, combining the export demand $Y_{EX,i} = (P_{EX,i}/S)^{-\theta_{F,i}} D^*_{EX,Fi}$ with the no-arbitrage condition $(1 - \tau_{EX,i})P_{EX,i} = P_i$, yields the following equation:

$$\frac{\theta_{F,i}}{\theta_{F,i} - 1} \left(\frac{Y^{flex}_{EX,i}(\xi)}{D^*_{EX,Fi}} \right)^{\frac{1}{\theta_{F,i}}} = \frac{S^{flex}(\xi)}{P^{flex}_i(\xi)}. \tag{D.67}$$

Furthermore, for the households' problem that maximizes utility function (8)—subject to the consumption aggregator (9) and budget constraint (10)—combining the first-order conditions with respect to L and C_{Hi} yields the following condition (D.68), combining the first-order conditions with respect to C_{Hj} and C_{Hi} yields the following condition (D.69), and combining the first-order conditions with respect to C_{Fi} and C_{Hi} yields the following condition (D.70). For the firm's cost minimization problem that minimizes the total costs in equation (3) subject to the production technology in equations (1) and (2), combining the first-order conditions with respect to $X_{Hi,Fj}$ and $X_{Hi,Hj}$, yields the following condition (D.71).

$$-\frac{\partial u/\partial L}{\frac{\partial u}{\partial \mathcal{C}} \frac{\partial \mathcal{C}}{\partial \mathcal{C}_i} \frac{\partial \mathcal{C}_i}{\partial C_{Hi}}}(\xi) = \frac{W^{flex}(\xi)}{P^{flex}_i(\xi)}, \tag{D.68}$$

$$\frac{\partial \mathcal{C}/\partial \mathcal{C}_j}{\partial \mathcal{C}/\partial \mathcal{C}_i}(\xi) \frac{\partial \mathcal{C}_j/\partial C_{Hj}}{\partial \mathcal{C}_i/\partial C_{Hi}}(\xi) = \frac{P^{flex}_j(\xi)}{P^{flex}_i(\xi)} \tag{D.69}$$

$$\frac{\partial \mathcal{C}_i/\partial C_{Fi}}{\partial \mathcal{C}_i/\partial C_{Hi}}(\xi) = \frac{P^*_{IM,Fi} S^{flex}(\xi)}{P^{flex}_i(\xi)}. \tag{D.70}$$

$$\frac{\partial \mathcal{X}_{i,j}/\partial X_{Hi,Fj}}{\partial \mathcal{X}_{i,j}/\partial X_{Hi,Hj}}(\xi) = \frac{P^*_{IM,Fj} S^{flex}(\xi)}{P^{flex}_i(\xi)}. \tag{D.71}$$

Substituting equations (D.65)-(D.67) into equations (D.68)-(D.71) to eliminate all of the equilibrium prices $W^{flex}(\xi)$, $S^{flex}(\xi)$, and $\{P^{flex}_i(\xi)\}_i$, yields *exactly the same* conditions for the *flexible-price equilibrium as* the conditions (D.61)-(D.64) for the first-best allocation in Lemma 7, which proves the efficiency of the *flexible-price equilibrium*. □

The role of export taxes $\{\tau_{EX,i}\}_i$. In closed economies á la La'O and Tahbaz-Salehi (2022) and Rubbo (2023), non-contingent sector-specific subsidies $\tau_i = -1/(\varepsilon_i - 1)$ eliminate sectoral distortions due to monopolistic competition and, therefore, are sufficient to make the *flexible-price equilibrium* efficient. In open economies, however, it is welfare-enhancing for the social planner of the small open economy to fully exploit the monopoly powers of the domestic producers on the international market. As a result, the non-contingent sector-specific subsidies that eliminate the sectoral distortions due to monopolistic competition *alone* are no longer optimal in small open economies, and an additional non-contingent export tax $\tau_{EX,i} = 1/\theta_{F,i}$ is required to retain the monopoly powers of the domestic producers on the international market and make the *flexible-price equilibrium* efficient. Under such export taxes, the sectoral export prices become:

$$P_{EX,i} = \frac{1}{1-\tau_{EX,i}} P_i = \frac{\theta_{F,i}}{\theta_{F,i}-1} P_i, \quad \forall i \in \{1, 2, \cdots, N\}.$$

D.3 Lemma 8: steady-state Domar weights and sectoral export-to-GDP ratios

Lemma 8 (Steady-state Domar weights and sectoral export-to-GDP ratios)**.** *The steady-state Domar weights $\boldsymbol{\lambda}$ and sectoral export-to-GDP ratios $\boldsymbol{\lambda}_{EX}$ are functions of parameters as in the following equations:*

$$\boldsymbol{\lambda}^\top = \left\{\boldsymbol{\beta} \odot \mathbf{v} + (1-\boldsymbol{\beta}^\top \mathbf{v})[(\boldsymbol{\theta}_F - 1) \oslash \boldsymbol{\theta}_F \odot \mathbf{v}_H^*]\right\}^\top \cdot \left\{\mathbf{I} - \boldsymbol{\Omega} \odot \mathbf{V}_x - (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\mathbf{1}[(\boldsymbol{\theta}_F - 1) \oslash \boldsymbol{\theta}_F \odot \mathbf{v}_H^*]^\top\right\}^{-1}, \tag{D.72}$$

$$\boldsymbol{\lambda}_{EX}^\top = \boldsymbol{\lambda}^\top (\mathbf{I} - \boldsymbol{\Omega} \odot \mathbf{V}_x) - (\boldsymbol{\beta} \odot \mathbf{v})^\top, \tag{D.73}$$

where $\mathbf{v}_H^*$ is the vector of the steady-state shares of sectoral exports in the value of the aggregate exports, with the i-th element v_{Hi}^* equal to:

$$v_{Hi}^* \equiv \frac{\left(\frac{\theta_{F,i}}{\theta_{F,i}-1}\right)^{1-\theta_{F,i}} D_{EX,Fi}^{*,ss}}{\sum_{i'} \left(\frac{\theta_{F,i'}}{\theta_{F,i'}-1}\right)^{1-\theta_{F,i'}} D_{EX,Fi'}^{*,ss}}.$$

Proof of Lemma 8. In the steady state, the nominal exchange rate S^{ss} and the sectoral prices P_i^{ss} are both normalized to 1. As a result, for each sector i, the export price $P_{EX,i}^{ss}$ is equal to $\theta_{F,i}/(\theta_{F,i}-1)$, and the foreign demand for domestic sector i's product in terms of quantity and value are equal to

$$Y_{EX,i}^{ss} = \left(\frac{P_{EX,i}^{ss}}{S^{ss}}\right)^{-\theta_{F,i}} D_{EX,Fi}^{*,ss} = \left(\frac{\theta_{F,i}}{\theta_{F,i}-1}\right)^{-\theta_{F,i}} D_{EX,Fi}^{*,ss}, \tag{D.74}$$

and

$$\frac{\theta_{F,i}}{\theta_{F,i}-1} Y_{EX,i}^{ss} = v_{Hi}^* \cdot \sum_{i'} \frac{\theta_{F,i'}}{\theta_{F,i'}-1} Y_{EX,i'}^{ss}, \tag{D.75}$$

respectively. In the steady state, the import price $P_{IM,Fi}^{*,ss}$ is also normalized to 1, which yields the steady-state balance of trade condition $\sum_{i'} \frac{\theta_{F,i'}}{\theta_{F,i'}-1} Y_{EX,i'}^{ss} = \sum_{i'}(C_{Fi'}^{ss} + \sum_j X_{Hj,Fi'}^{ss})$. Combining this steady-state balance of trade condition with equation (D.75), yields the following equation of the quantity of foreign demand:

$$Y_{EX,i}^{ss} = \frac{\theta_{F,i}-1}{\theta_{F,i}} v_{Hi}^* \sum_{i'} \left(C_{Fi'}^{ss} + \sum_j X_{Hj,Fi'}^{ss}\right). \tag{D.76}$$

Substituting equation (D.76) into the steady-state goods market clearing condition $Y_i^{ss} = C_{Hi}^{ss} + \sum_j X_{Hj,Hi}^{ss} + Y_{EX,i}^{ss}$ and dividing both sides by the steady-state aggregate output C^{ss} yields

$$\lambda_i = \beta_i v_i + \sum_j \lambda_j \omega_{j,i} v_{x,j,i} + \frac{\theta_{F,i} - 1}{\theta_{F,i}} v_{Hi}^* \sum_{i'} \Big[\beta_{i'} (1 - v_{i'}) + \sum_j \lambda_j \omega_{j,i'} (1 - v_{x,j,i'}) \Big],$$

which has equation (D.72) as its matrix form.

Dividing both sides of the steady-state goods market clearing condition $Y_i^{ss} = C_{Hi}^{ss} + \sum_j X_{Hj,Hi}^{ss} + Y_{EX,i}^{ss}$ by the steady-state aggregate output C^{ss} and substituting in the definition of the sectoral export-to-GDP ratio $\lambda_{EX,i} \equiv (P_i^{ss} Y_{EX,i}^{ss})/(P_C^{ss} C^{ss})$ with normalized $P_i^{ss} = P_C^{ss} = 1$ yields the following equation:

$$\lambda_{EX,i} = \lambda_i - \Big(\beta_i v_i + \sum_j \lambda_j \omega_{j,i} v_{x,j,i} \Big), \tag{D.77}$$

which has equation (D.73) as its matrix form. □

D.4 Lemma 9: Goods market clearing condition up to the first-order approximation

Lemma 9 (Goods market clearing condition). *Up to the first-order approximation, the following condition holds in the* sticky-price equilibrium.

$$\begin{aligned} \big[\boldsymbol{\lambda} \odot \big(\widehat{\boldsymbol{P}}(\boldsymbol{\xi}) + \widehat{\boldsymbol{Y}}(\boldsymbol{\xi})\big)\big]^\top &= \widetilde{\boldsymbol{\lambda}}_D^\top \big(\widehat{P}_C(\boldsymbol{\xi}) + \widehat{C}(\boldsymbol{\xi})\big) \\ &\quad - \big(\boldsymbol{\lambda} \odot \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})\big)^\top (\mathbf{L}_{vx} - \mathbf{I}) + \big[\boldsymbol{\lambda}_{EX} \widehat{S}(\boldsymbol{\xi}) - \boldsymbol{\rho}_{NX} \odot \big(\widehat{\mathbf{P}}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}(\boldsymbol{\xi})\big)\big]^\top \mathbf{L}_{vx} \\ &\quad + \big\{\boldsymbol{\lambda}_{EX} \odot \widehat{\mathbf{D}}_{EX,F}^* + \big[\boldsymbol{\rho}_{NX} - (\boldsymbol{\theta}_F - \mathbf{1}) \odot \boldsymbol{\lambda}_{EX}\big] \odot \widehat{\mathbf{P}}_{IM,F}^*\big\}^\top \mathbf{L}_{vx} + o(\|\widehat{\boldsymbol{\xi}}\|). \end{aligned} \tag{D.78}$$

Proof of Lemma 9. The goods market clearing condition (D.55) multiplied by the sectoral price P_i is

$$P_i Y_i = P_i C_{Hi} + P_i \sum_j X_{Hj,Hi} + P_i Y_{EX,i}. \tag{D.79}$$

Denote $P_{c,i}$ as the price index of the sectoral consumption goods from sector i and $P_{x,j,i}$ as the price index of the intermediate inputs purchased by sector j from sector i—both of which are weighted averages of domestic price P_i and import price $S \cdot P_{IM,Fi}^*$. Minimizing the costs of purchasing $\mathcal{C}, \{F_i\}_i, \{\mathcal{C}_i\}_i, \{\mathcal{X}_{i,j}\}_{i,j}$ yields the following quantity of the demand for consumption and intermediate inputs as functions of prices:

$$C_{Hi} = \left(\frac{P_i}{P_{c,i}}\right)^{-\theta_i} v_i C_i = \left(\frac{P_i}{P_{c,i}}\right)^{-\theta_i} \frac{v_i \beta_i P_C C}{P_{c,i}},$$

$$X_{Hj,Hi} = \left(\frac{P_i}{P_{x,j,i}}\right)^{-\theta_i} v_{x,j,i} X_{j,i} = \left(\frac{P_i}{P_{x,j,i}}\right)^{-\theta_i} \frac{v_{x,j,i}\omega_{j,i}P_jY_j}{P_{x,j,i}\mu_j}.$$

Substituting equation (13) into equation (14) yields the export demand as follows:

$$Y_{EX,i} = \left(\frac{P_{EX,i}}{S}\right)^{-\theta_{F,i}} D^*_{EX,Fi} = \left(\frac{\theta_{F,i}}{\theta_{F,i}-1}\right)^{-\theta_{F,i}} \left(\frac{P_i}{S}\right)^{-\theta_{F,i}} D^*_{EX,Fi}.$$

Substituting the quantity of consumption, intermediate inputs, and export demand above back to the goods market-clearing condition in equation (D.79) yields

$$P_iY_i = \left(\frac{P_i}{P_{c,i}}\right)^{1-\theta_i} v_i\beta_iP_CC + \sum_j \left(\frac{P_i}{P_{x,j,i}}\right)^{1-\theta_i} \frac{v_{x,j,i}\omega_{j,i}P_jY_j}{\mu_j} + \left(\frac{\theta_{F,i}}{\theta_{F,i}-1}\right)^{-\theta_{F,i}} \left(\frac{P_i}{S}\right)^{1-\theta_{F,i}} S\cdot D^*_{EX,Fi}. \quad \text{(D.80)}$$

Log-linearizing equation (D.80) yields

$$\lambda_i(\widehat{P}_i+\widehat{Y}_i) = \beta_iv_i\Big[(\theta_i-1)(\widehat{P}_{c,i}-\widehat{P}_i)+\widehat{P}_C+\widehat{C}\Big] + \sum_j \lambda_j\omega_{j,i}v_{x,j,i}\Big[(\theta_i-1)(\widehat{P}_{x,j,i}-\widehat{P}_i)+\widehat{P}_j+\widehat{Y}_j-\widehat{\mu}_j\Big] + \lambda_{EX,i}\Big[(\theta_{F,i}-1)(\widehat{S}-\widehat{P}_i)+\widehat{S}+\widehat{D}^*_{EX,Fi}\Big] + o(\|\widehat{\xi}\|). \quad \text{(D.81)}$$

Log-linearizing the price indices $P_{c,i}$ and $P_{x,j,i}$ yields the following:

$$\widehat{P}_{c,i} = v_i\widehat{P}_i + (1-v_i)\big(\widehat{S}+\widehat{P}^*_{IM,Fi}\big) + o(\|\widehat{\xi}\|), \quad \text{(D.82)}$$

$$\widehat{P}_{x,j,i} = v_{x,j,i}\widehat{P}_i + (1-v_{x,j,i})\big(\widehat{S}+\widehat{P}^*_{IM,Fi}\big) + o(\|\widehat{\xi}\|), \quad \text{(D.83)}$$

which implies the following relative prices:

$$\widehat{P}_{c,i}-\widehat{P}_i = (1-v_i)(\widehat{S}+\widehat{P}^*_{IM,Fi}-\widehat{P}_i) + o(\|\widehat{\xi}\|),$$

$$\widehat{P}_{x,j,i}-\widehat{P}_i = (1-v_{x,j,i})(\widehat{S}+\widehat{P}^*_{IM,Fi}-\widehat{P}_i) + o(\|\widehat{\xi}\|).$$

Substituting these relative prices into equation (D.81) yields

$$\lambda_i(\widehat{P}_i+\widehat{Y}_i) = \beta_iv_i\Big[(\theta_i-1)(1-v_i)(\widehat{S}+\widehat{P}^*_{IM,Fi}-\widehat{P}_i)+\widehat{P}_C+\widehat{C}\Big] + \sum_j \lambda_j\omega_{j,i}v_{x,j,i}\Big[(\theta_i-1)(1-v_{x,j,i})(\widehat{S}+\widehat{P}^*_{IM,Fi}-\widehat{P}_i)+\widehat{P}_j+\widehat{Y}_j-\widehat{\mu}_j\Big] + \lambda_{EX,i}\Big[(\theta_{F,i}-1)(\widehat{S}-\widehat{P}_i)+\widehat{S}+\widehat{D}^*_{EX,Fi}\Big] + o(\|\widehat{\xi}\|).$$

Rearranging the above equation and substituting in the definition of the net export elasticity $\rho_{NX,i}$ in equation (35) yield the following:

$$\lambda_i(\widehat{P}_i + \widehat{Y}_i) - \sum_j \lambda_j \omega_{j,i} v_{x,j,i}(\widehat{P}_j + \widehat{Y}_j) = \beta_i v_i(\widehat{P}_C + \widehat{C}) - \sum_j \lambda_j \omega_{j,i} v_{x,j,i} \widehat{\mu}_j + \lambda_{EX,i}\widehat{S} - \rho_{NX,i}(\widehat{P}_i - \widehat{S}) \\ + \lambda_{EX,i}\widehat{D}^*_{EX,Fi} + [\rho_{NX,i} - (\theta_{F,i} - 1)\lambda_{EX,i}]\widehat{P}^*_{IM,Fi} + o(\|\widehat{\xi}\|),$$

which has the following matrix form as in equation (D.78) in Lemma 9:

$$\left[\boldsymbol{\lambda} \odot (\widehat{\boldsymbol{P}} + \widehat{\boldsymbol{Y}})\right]^\top = \widetilde{\boldsymbol{\lambda}}_D^\top (\widehat{P}_C + \widehat{C}) - (\boldsymbol{\lambda} \odot \widehat{\boldsymbol{\mu}})^\top (\mathbf{L}_{vx} - \mathbf{I}) + \left[\boldsymbol{\lambda}_{EX}\widehat{S} - \boldsymbol{\rho}_{NX} \odot (\widehat{\mathbf{P}} - \mathbf{1}\widehat{S})\right]^\top \mathbf{L}_{vx} \\ + \left\{\boldsymbol{\lambda}_{EX} \odot \widehat{\mathbf{D}}^*_{EX,F} + \left[\boldsymbol{\rho}_{NX} - (\boldsymbol{\theta_F} - \mathbf{1}) \odot \boldsymbol{\lambda}_{EX}\right] \odot \widehat{\mathbf{P}}^*_{IM,F}\right\}^\top \mathbf{L}_{vx} + o(\|\widehat{\xi}\|).$$

□

D.5 Lemma 10: household's budget constraint up to first-order approximation

Lemma 10 (Household's budget constraint)**.** *Up to the first-order approximation, the following condition holds in the* sticky-price equilibrium*:*

$$\widehat{P}_C(\boldsymbol{\xi}) + \widehat{C}(\boldsymbol{\xi}) = \left[\boldsymbol{\lambda} \odot \left(\widehat{\boldsymbol{P}}(\boldsymbol{\xi}) + \widehat{\boldsymbol{Y}}(\boldsymbol{\xi})\right)\right]^\top \boldsymbol{\alpha} + \left(\boldsymbol{\lambda} \odot \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})\right)^\top (\mathbf{1} - \boldsymbol{\alpha}) + (1 - \boldsymbol{\lambda}^\top \boldsymbol{\alpha})\widehat{S}(\boldsymbol{\xi}) \\ - \boldsymbol{\lambda}_{EX}^\top \left(\widehat{\mathbf{P}}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}(\boldsymbol{\xi})\right) + \left[\boldsymbol{\lambda}_{EX} \oslash (\boldsymbol{\theta}_F - \mathbf{1})\right]^\top \widehat{\mathbf{D}}^*_{EX,F} + o(\|\widehat{\boldsymbol{\xi}}\|). \quad \text{(D.84)}$$

Proof of Lemma 10. Substituting the profit, total cost of inputs, and lump-sum transfer in equations (4), (3), and (17) into the household budget constraint in equation (10) yields

$$\begin{aligned} P_C C &= WL + \sum_i \int_0^1 \Pi_{if} df + T \\ &= WL + \sum_i \left[(1-\tau_i)P_i Y_i - W L_i - \sum_j \left(P_j X_{Hi,Hj} + S \cdot P^*_{IM,Fj} X_{Xi,Fj}\right)\right] \\ &\qquad + \sum_i \left(\tau_i P_i Y_i + \tau_{EX,i} P_{EX,i} Y_{EX,i}\right) \\ &= \sum_i \left[P_i Y_i - \sum_j \left(P_j X_{Hi,Hj} + S \cdot P^*_{IM,Fj} X_{Xi,Fj}\right)\right] + \sum_i \tau_{EX,i} P_{EX,i} Y_{EX,i}. \end{aligned} \quad \text{(D.85)}$$

Under the Cobb-Douglas production functions, $\sum_j \left(P_j X_{Hi,Hj} + S \cdot P^*_{IM,Fj} X_{Xi,Fj}\right) = P_i Y_i (1 - \alpha_i)/\mu_i$. Therefore, substituting the export tax rate $\tau_{EX,i} = 1/\theta_{F,i}$, the export price $P_{EX,i} = P_i/(1 - \tau_{EX,i})$, and the export demand $Y_{EX,i} = (P_{EX,i}/S)^{-\theta_{F,i}} D^*_{EX,Fi}$ into equation (D.85) yields

$$P_C C = \sum_i P_i Y_i \Big(1 - \frac{1-\alpha_i}{\mu_i}\Big) + \sum_i \Big(\frac{S}{\theta_{F,i}}\Big)^{\theta_{F,i}} \Big(\frac{P_i}{\theta_{F,i}-1}\Big)^{1-\theta_{F,i}} D^*_{EX,Fi}. \tag{D.86}$$

In the steady state, the sectoral markups, prices, and nominal exchange rate are normalized to $\mu_i^{ss} = P_i^{ss} = S^{ss} = 1$. As a result, equation (D.86) becomes

$$1 = \sum_i \lambda_i \alpha_i + \sum_i \frac{\lambda_{EX,i}}{\theta_{F,i}-1}. \tag{D.87}$$

Log-linearizing equation (D.86) around the steady state yields

$$\widehat{P}_C + \widehat{C} = \sum_i \lambda_i \alpha_i \Big(\frac{1-\alpha_i}{\alpha_i}\widehat{\mu}_i + \widehat{P}_i + \widehat{Y}_i\Big) + \sum_i \frac{\lambda_{EX,i}}{\theta_{F,i}-1}[\widehat{S} - (\theta_{F,i}-1)(\widehat{P}_i - \widehat{S}) + \widehat{D}^*_{EX,Fi}] + o(\|\widehat{\xi}\|),$$

which has the following matrix form as in equation (D.84) in Lemma 10:

$$\begin{aligned}\widehat{P}_C + \widehat{C} = {}& \big[\boldsymbol{\lambda} \odot (\widehat{\boldsymbol{P}} + \widehat{\boldsymbol{Y}})\big]^\top \boldsymbol{\alpha} + (\boldsymbol{\lambda} \odot \widehat{\boldsymbol{\mu}})^\top (\mathbf{1} - \boldsymbol{\alpha}) \\ & + (1 - \boldsymbol{\lambda}^\top \boldsymbol{\alpha})\widehat{S} - \boldsymbol{\lambda}_{EX}^\top (\widehat{\mathbf{P}} - \mathbf{1}\widehat{S}) + [\boldsymbol{\lambda}_{EX} \oslash (\boldsymbol{\theta}_F - \mathbf{1})]^\top \widehat{\mathbf{D}}^*_{EX,F} + o(\|\widehat{\boldsymbol{\xi}}\|).\end{aligned}$$

□

D.6 Lemma 11: sectoral markup wedges and sectoral inflation

Lemma 11 (Sectoral markup wedges and sectoral inflation)**.** *Up to the first-order approximation, the following condition holds in the* sticky-price equilibrium*:*

$$\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) = -(\boldsymbol{\Delta}^{-1} - \mathbf{I})\widehat{\mathbf{P}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|). \tag{D.88}$$

Proof of Lemma 11. Under static Calvo-pricing, the vector of sectoral inflation is a function of the sectoral frequency of price adjustment $\boldsymbol{\Delta}$ and the vector of sectoral nominal marginal costs $\boldsymbol{\Phi}$:

$$\widehat{\mathbf{P}}(\boldsymbol{\xi}) = \boldsymbol{\Delta}\widehat{\boldsymbol{\Phi}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|). \tag{D.89}$$

On the other hand, the definition of the sectoral markup wedges $\widehat{\boldsymbol{\mu}}$ yields

$$\widehat{\mathbf{P}}(\boldsymbol{\xi}) = \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + \widehat{\boldsymbol{\Phi}}(\boldsymbol{\xi}). \tag{D.90}$$

Combining the above two conditions to eliminate $\widehat{\boldsymbol{\Phi}}(\boldsymbol{\xi})$ yields equation (D.88). □

E Proofs of the theoretical results in Section 3

This appendix derives the theoretical results associated with the aggregate output gap and the OG policy in Section 3. These theoretical results are all up to the first-order approximation around the efficient steady state under Assumption 1.

E.1 Proof of Lemma 2: the open economy Hulten's theorem

Hulten's theorem in Hulten (1978) characterizes the first-order impact of disaggregated productivity shocks on the aggregate TFP in an efficient closed economy (e.g., Baqaee and Farhi, 2019). Our paper extends the closed-economy version of Hulten's theorem into a small open economy with international trade, exchange rate adjustments, and sector-specific shocks to import prices and export demand besides sectoral productivity.

Under $\tau_i = -1/(\varepsilon_i - 1)$ and $\tau_{EX,i} = 1/\theta_{F,i}$ of Assumption 1 and with all of the prices but $P^{ss}_{EX,i}$ and W^{ss} normalized to 1, the first-order approximation of the conditions in Lemma 12 around the efficient steady state yields the following:

$$C^{ss}\widehat{C}(\boldsymbol{\xi}) = \sum_i C_i^{ss}\widehat{C}_i(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{E.91}$$

$$Y_i^{ss}\widehat{Y}_i(\boldsymbol{\xi}) = Y_i^{ss}\widehat{A}_i + W^{ss}L_i^{ss}\widehat{L}_i(\boldsymbol{\xi}) + \sum_j X_{i,j}^{ss}\widehat{X}_{i,j}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{E.92}$$

$$C_i^{ss}\widehat{C}_i(\boldsymbol{\xi}) = C_{Hi}^{ss}\widehat{C}_{Hi}(\boldsymbol{\xi}) + C_{Fi}^{ss}\widehat{C}_{Fi}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{E.93}$$

$$X_{i,j}^{ss}\widehat{X}_{i,j}(\boldsymbol{\xi}) = X_{Hi,Hj}^{ss}\widehat{X}_{Hi,Hj}(\boldsymbol{\xi}) + X_{Hi,Fj}^{ss}\widehat{X}_{Hi,Fj}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{E.94}$$

$$L^{ss}\widehat{L}(\boldsymbol{\xi}) = \sum_i L_i^{ss}\widehat{L}_i(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{E.95}$$

$$Y_i^{ss}\widehat{Y}_i(\boldsymbol{\xi}) = C_{Hi}^{ss}\widehat{C}_{Hi}(\boldsymbol{\xi}) + \sum_j X_{Hj,Hi}^{ss}\widehat{X}_{Hj,Hi}(\boldsymbol{\xi}) + Y_{EX,i}^{ss}\widehat{Y}_{EX,i}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{E.96}$$

$$\begin{aligned} EX_i^{ss}\widehat{EX}_i(\boldsymbol{\xi}) &= \sum_i Y_{EX,i}^{ss}\Big[(\theta_{F,i}-1)^{-1}\widehat{D}^*_{EX,Fi} + \widehat{Y}_{EX,i}(\boldsymbol{\xi})\Big] \\ &= \sum_i \Big[C_{Fi}^{ss}\big(\widehat{P}^*_{IM,Fi} + \widehat{C}_{Fi}(\boldsymbol{\xi})\big) + \sum_j X_{Hj,Fi}^{ss}\big(\widehat{P}^*_{IM,Fi} + \widehat{X}_{Hj,Fi}(\boldsymbol{\xi})\big)\Big] + o(\|\widehat{\boldsymbol{\xi}}\|). \end{aligned} \tag{E.97}$$

Then, we combine equations (E.91)-(E.97) to prove Lemma 2. Rearranging the balance of trade condition (E.97) to move all endogenous terms to the *LHS* and all exogenous ones to the *RHS* yields the following:

$$\begin{aligned} LHS &\equiv \sum_i \Big(Y_{EX,i}^{ss}\widehat{Y}_{EX,i}(\boldsymbol{\xi}) - C_{Fi}^{ss}\widehat{C}_{Fi}(\boldsymbol{\xi}) - \sum_j X_{Hi,Fj}^{ss}\widehat{X}_{Hi,Fj}(\boldsymbol{\xi})\Big) \\ &= \sum_i \Big(C_{Fi}^{ss}\widehat{P}^*_{IM,Fi} + \sum_j X_{Hj,Fi}^{ss}\widehat{P}^*_{IM,Fi} - \frac{Y_{EX,i}^{ss}}{\theta_{F,i}-1}\widehat{D}^*_{EX,Fi}\Big) + o(\|\widehat{\boldsymbol{\xi}}\|) \equiv RHS. \end{aligned} \tag{E.98}$$

Combined with the goods market clearing condition in equation (E.96), the *LHS* of equation

(E.98) becomes:

$$LHS = \sum_i \left(Y_i^{ss}\widehat{Y}_i(\boldsymbol{\xi}) - C_{Hi}^{ss}\widehat{C}_{Hi}(\boldsymbol{\xi}) - \sum_j X_{Hj,Hi}^{ss}\widehat{X}_{Hj,Hi}(\boldsymbol{\xi}) - C_{Fi}^{ss}\widehat{C}_{Fi}(\boldsymbol{\xi}) - \sum_j X_{Hj,Fi}^{ss}\widehat{X}_{Hj,Fi}(\boldsymbol{\xi}) \right).$$

Further combined with the aggregators in equations (E.91), (E.93), and (E.94), the LHS becomes:

$$LHS = \sum_i \left(Y_i^{ss}\widehat{Y}_i(\boldsymbol{\xi}) - \sum_j X_{i,j}^{ss}\widehat{X}_{i,j}(\boldsymbol{\xi}) \right) - C^{ss}\widehat{C}(\boldsymbol{\xi}).$$

Combined with the production function in equation (E.92),

$$LHS = \sum_i \left(Y_i^{ss}\widehat{A}_i + W^{ss}L_i^{ss}\widehat{L}_i(\boldsymbol{\xi}) \right) - C^{ss}\widehat{C}(\boldsymbol{\xi}).$$

Combined with the labor market clearing condition in equation (E.95),

$$LHS = \sum_i Y_i^{ss}\widehat{A}_i + W^{ss}L^{ss}\widehat{L}(\boldsymbol{\xi}) - C^{ss}\widehat{C}(\boldsymbol{\xi}).$$

Substituting LHS back into equation (E.98) yields

$$C^{ss}\widehat{C}(\boldsymbol{\xi}) - W^{ss}L^{ss}\widehat{L}(\boldsymbol{\xi}) = \sum_i \Bigg(Y_i^{ss}\widehat{A}_i - C_{Fi}^{ss}\widehat{P}_{IM,Fi}^{*} - \sum_j X_{Hj,Fi}^{ss}\widehat{P}_{IM,Fi}^{*} + \frac{Y_{EX,i}^{ss}}{\theta_{F,i}-1}\widehat{D}_{EX,Fi}^{*} \Bigg) + o(\|\widehat{\boldsymbol{\xi}}\|). \quad \text{(E.99)}$$

In the steady state, the sectoral output prices and the CPI are normalized to 1. Therefore, dividing both sides of equation (E.99) by the steady-state aggregate output C^{ss} yields the following:

$$\widehat{C}(\boldsymbol{\xi}) - \Lambda_L\widehat{L}(\boldsymbol{\xi}) = \sum_i \Bigg\{ \lambda_i\widehat{A}_i + \frac{\lambda_{EX,i}}{\theta_{F,i}-1}\widehat{D}_{EX,Fi}^{*} - \Big[\beta_i(1-v_i) + \sum_j \lambda_j\omega_{j,i}(1-v_{x,j,i})\Big]\widehat{P}_{IM,Fi}^{*} \Bigg\} + o(\|\widehat{\boldsymbol{\xi}}\|). \quad \text{(E.100)}$$

E.2 Proof of Proposition 1: efficiency and labor wedges

Efficiency wedge. Log-linearizing the efficiency wedge $A_{agg}(\boldsymbol{\xi})$ in Definition 3 around the steady state yields

$$\widehat{A}_{agg}(\boldsymbol{\xi}) = \widehat{C}(\boldsymbol{\xi}) - \Lambda_L^{flex}(\boldsymbol{\xi})\widehat{L}(\boldsymbol{\xi}).$$

Substituting $\Lambda_L^{flex}(\boldsymbol{\xi}) = \Lambda_L + O(\|\widehat{\boldsymbol{\xi}}\|)$ into the above equation yields

$$\widehat{A}_{agg}(\boldsymbol{\xi}) = \widehat{C}(\boldsymbol{\xi}) - \Lambda_L \widehat{L}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|),$$

where $\widehat{C}(\boldsymbol{\xi}) - \Lambda_L \widehat{L}(\boldsymbol{\xi})$ are functions of only exogenous shocks up to the first-order approximation, as shown in equation (E.100) of Section E.1. Therefore, taking the difference of equation (E.100) in the *sticky-price equilibrium* and in the *flexible-price equilibrium* yields the following:

$$\begin{aligned} \widehat{A}_{agg}(\boldsymbol{\xi}) - \widehat{A}_{agg}^{flex}(\boldsymbol{\xi}) &= \left(\widehat{C}(\boldsymbol{\xi}) - \Lambda_L \widehat{L}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|)\right) - \left(\widehat{C}^{flex}(\boldsymbol{\xi}) - \Lambda_L \widehat{L}^{flex}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|)\right) \\ &= \widehat{C}^{gap}(\boldsymbol{\xi}) - \Lambda_L \widehat{L}^{gap}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|) = o(\|\widehat{\boldsymbol{\xi}}\|). \end{aligned} \tag{E.101}$$

In sum, sectoral markup wedges under price rigidities have no first-order impact on the efficiency wedge.

Labor wedge. Consider a prototype economy similar to the closed economy á la Chari et al. (2007), except that the aggregate production function defined on domestic labor inputs has state-contingent aggregate TFP and returns-to-scale, as in the following equation:

$$F(L, \boldsymbol{\xi}) = A_{agg}(\boldsymbol{\xi}) \cdot L^{\Lambda_L^{flex}(\boldsymbol{\xi})},$$

where $\Lambda_L^{flex}(\boldsymbol{\xi})$ is the economy-wise labor share in the *flexible-price equilibrium* of the small open economy that is contingent on the states of exogenous shocks. According to Definition 3, $C(\boldsymbol{\xi}) = F(L(\boldsymbol{\xi}), \boldsymbol{\xi})$ and, therefore, the labor wedge $\Gamma_L(\boldsymbol{\xi})$ satisfies

$$-\frac{\partial u/\partial L}{\partial u/\partial C}(C(\boldsymbol{\xi}), L(\boldsymbol{\xi})) = \Gamma_L(\boldsymbol{\xi}) \cdot \frac{\partial F}{\partial L}(L(\boldsymbol{\xi}), \boldsymbol{\xi}), \tag{E.102}$$

where the marginal product of labor in the *sticky-price equilibrium* is equal to

$$\frac{\partial F}{\partial L}(L(\boldsymbol{\xi}), \boldsymbol{\xi}) = A_{agg}(\boldsymbol{\xi}) \cdot \Lambda_L^{flex}(\boldsymbol{\xi}) \cdot L^{\Lambda_L^{flex}(\boldsymbol{\xi})-1} \equiv \frac{\partial C}{\partial L}(\boldsymbol{\xi}).$$

Therefore, substituting the utility function in equation (8) into equation (E.102) and log-linearizing it around the steady state yields

$$\widehat{\Gamma}_L(\boldsymbol{\xi}) = \sigma \widehat{C}(\boldsymbol{\xi}) + \varphi \widehat{L}(\boldsymbol{\xi}) - \widehat{A}_{agg}(\boldsymbol{\xi}) - \widehat{\Lambda}_L^{flex}(\boldsymbol{\xi}) - \left(\Lambda_L^{flex}(\boldsymbol{\xi}) - 1\right)\widehat{L}(\boldsymbol{\xi}). \tag{E.103}$$

Taking the difference of equation (E.103) in the *sticky-price equilibrium* and in the *flexible-price equilibrium* yields

$$\begin{aligned} \widehat{\Gamma}_L(\boldsymbol{\xi}) - \widehat{\Gamma}_L^{flex}(\boldsymbol{\xi}) = \sigma \widehat{C}^{gap}(\boldsymbol{\xi}) + \varphi \widehat{L}^{gap}(\boldsymbol{\xi}) \\ - \left(\widehat{A}_{agg}(\boldsymbol{\xi}) - \widehat{A}_{agg}^{flex}(\boldsymbol{\xi})\right) - (\Lambda_L - 1)\widehat{L}^{gap}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|) \end{aligned} \tag{E.104}$$

Combining equation (E.104) with equation (E.101) yields the labor wedge as

$$\widehat{\Gamma}_L(\xi) - \widehat{\Gamma}_L^{flex}(\xi) = \left(\sigma - 1 + \frac{\varphi + 1}{\Lambda_L}\right) \cdot \widehat{C}^{gap}(\xi) + o(\|\widehat{\xi}\|).$$

□

E.3 Proof of Lemma 3: impacts of sectoral markup wedges on CPI

Under the production technology and the total cost of inputs in equations (1), (2), and (3), deriving the sectoral nominal marginal costs $\mathbf{\Phi}(\xi)$ from the producers' cost minimization problem and log-linearizing it around the steady state, yields the following:

$$\widehat{\mathbf{\Phi}}(\xi) = \boldsymbol{\alpha}\widehat{W}(\xi) + (\mathbf{\Omega} \odot \mathbf{V}_x)\widehat{\mathbf{P}}(\xi) + (\mathbf{\Omega} \odot \mathbf{V}_{1-x})\big(\mathbf{1}\widehat{S}(\xi) + \widehat{\mathbf{P}}^*_{IM,F}\big) - \widehat{\mathbf{A}} + o(\|\widehat{\xi}\|), \tag{E.105}$$

which, substituted into equation (D.90) yields

$$\widehat{\mathbf{P}}(\xi) = \boldsymbol{\alpha}\widehat{W}(\xi) + (\mathbf{\Omega} \odot \mathbf{V}_x)\widehat{\mathbf{P}}(\xi) + (\mathbf{\Omega} \odot \mathbf{V}_{1-x})\big(\mathbf{1}\widehat{S}(\xi) + \widehat{\mathbf{P}}^*_{IM,F}\big) - \widehat{\mathbf{A}} + \widehat{\boldsymbol{\mu}}(\xi) + o(\|\widehat{\xi}\|). \tag{E.106}$$

Taking the difference of equation (E.106) in the *sticky-price equilibrium* and in the *flexible-price equilibrium* to eliminate the exogenous shocks, yields

$$\begin{aligned}\widehat{\mathbf{P}}^{gap}(\xi) &= \boldsymbol{\alpha}\widehat{W}^{gap}(\xi) + (\mathbf{\Omega} \odot \mathbf{V}_x)\widehat{\mathbf{P}}^{gap}(\xi) + (\mathbf{\Omega} \odot \mathbf{V}_{1-x})\mathbf{1}\widehat{S}^{gap}(\xi) + \widehat{\boldsymbol{\mu}}(\xi) + o(\|\widehat{\xi}\|). \\ &= \mathbf{L}_{vx}\big(\boldsymbol{\alpha}\widehat{W}^{gap}(\xi) - \boldsymbol{\alpha}\widehat{S}^{gap}(\xi) + \widehat{\boldsymbol{\mu}}(\xi)\big) + \mathbf{1}\widehat{S}^{gap}(\xi) + o(\|\widehat{\xi}\|),\end{aligned} \tag{E.107}$$

where the second equality is derived using the Leontief inverse matrix $\mathbf{L}_{vx} \equiv (\mathbf{I} - \mathbf{\Omega} \odot \mathbf{V}_x)^{-1}$ and the identity $\boldsymbol{\alpha} = \mathbf{1} - \mathbf{\Omega}\mathbf{1}$.

Log-linearizing the CPI in equation (12) around the steady state yields

$$\widehat{P}_C(\xi) = (\boldsymbol{\beta} \odot \mathbf{v})^\top \widehat{\mathbf{P}}(\xi) + [\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})]^\top \big(\mathbf{1}\widehat{S}(\xi) + \widehat{\mathbf{P}}^*_{IM,F}\big) + o(\|\widehat{\xi}\|). \tag{E.108}$$

Taking the difference of equation (E.108) in the *sticky-price equilibrium* and in the *flexible-price equilibrium* to eliminate the exogenous shocks yields

$$\widehat{P}_C^{gap}(\xi) = (\boldsymbol{\beta} \odot \mathbf{v})^\top \widehat{\mathbf{P}}^{gap}(\xi) + [\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})]^\top \mathbf{1}\widehat{S}^{gap}(\xi) + o(\|\widehat{\xi}\|). \tag{E.109}$$

Substituting equation (E.107) into equation (E.109) and using the identity $(\boldsymbol{\beta} \odot \mathbf{v})^\top \mathbf{L}_{vx} = \widetilde{\boldsymbol{\lambda}}_D^\top$ yields

$$\widehat{P}_C^{gap}(\xi) = \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}\widehat{W}^{gap}(\xi) + (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})\widehat{S}^{gap}(\xi) + \widetilde{\boldsymbol{\lambda}}_D^\top \widehat{\boldsymbol{\mu}}(\xi) + o(\|\widehat{\xi}\|),$$

which is equation (37) in Lemma 3.

For interpretation, we re-arrange equation (37) to highlight the real wage and exchange rate:

$$\widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}\left(\widehat{W}^{gap}(\boldsymbol{\xi}) - \widehat{P}_C^{gap}(\boldsymbol{\xi})\right) + (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})\left(\widehat{S}^{gap}(\boldsymbol{\xi}) - \widehat{P}_C^{gap}(\boldsymbol{\xi})\right) = -\widetilde{\boldsymbol{\lambda}}_D^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|). \tag{E.110}$$

In Lemma 12 and Lemma 13 below, we further relate the real wage gap and real exchange rate gap in equation (E.110) to the aggregate output gaps. □

E.4 Lemma 12: real wage gap and aggregate output gap

Lemma 12 (Real wage gap and aggregate output gap). *Up to the first-order approximation, the real wage gap is proportional to the aggregate output gap as in the following equation:*

$$\widehat{W}^{gap}(\boldsymbol{\xi}) - \widehat{P}_C^{gap}(\boldsymbol{\xi}) = \sigma \widehat{C}^{gap}(\boldsymbol{\xi}) + \varphi \widehat{L}^{gap}(\boldsymbol{\xi}) = \left(\sigma + \varphi/\Lambda_L\right)\widehat{C}^{gap}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|). \tag{E.111}$$

Proof of Lemma 12. For the households' problem that maximizes utility function (8) subject to the budget constraint (10), combining the first-order conditions with respect to L and C and log-linearizing it yield

$$\widehat{W}(\boldsymbol{\xi}) - \widehat{P}_C(\boldsymbol{\xi}) = \sigma \widehat{C}(\boldsymbol{\xi}) + \varphi \widehat{L}(\boldsymbol{\xi}). \tag{E.112}$$

Taking the difference of equation (E.112) in the *sticky-price equilibrium* and in the *flexible-price equilibrium* yields the first equality in equation (E.111). Further substituting in equation (E.101) from Section E.2 yields the second equality in equation (E.111). □

Interpreting Lemma 12. Equation (E.111) shows that the lower CPI in the sticky-price equilibrium than in the efficient, flexible-price equilibrium (i.e., $\widehat{P}_C^{gap} < 0$ on the LHS) increases the real wage (i.e., $\widehat{W}^{gap} - \widehat{P}_C^{gap}$) and induces a higher supply of domestic labor (i.e., $\widehat{L}^{gap} > 0$ in the middle), thereby fostering production and generating a positive aggregate output gap (i.e., $\widehat{C}^{gap} > 0$ on the RHS).

E.5 Lemma 13: real exchange rate gap and aggregate output gap

Lemma 13 (Real exchange rate gap and aggregate output gap). *Up to the first-order approximation, the aggregate output gap is a linear function of the real exchange rate gap and sectoral markup wedges, as reflected in the following equation:*

$$\Big[\underbrace{1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}}_{\text{cost of imported factors}} + \underbrace{\left(\boldsymbol{\rho}_{NX} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX}\right)^\top \widetilde{\boldsymbol{\alpha}}}_{\text{income from net exports}}\Big]\left(\widehat{S}^{gap}(\boldsymbol{\xi}) - \widehat{P}_C^{gap}(\boldsymbol{\xi})\right)$$
$$- \Big\{\underbrace{\widetilde{\boldsymbol{\rho}}_{NX}^\top}_{\text{net export income channel}} + \underbrace{\left[\widetilde{\boldsymbol{\lambda}}_F - \boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})\right]^\top}_{\text{net profit income channel}}\Big\}\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$$

$$= \Big[1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha} + \big(\boldsymbol{\rho}_{NX} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX}\big)^\top \widetilde{\boldsymbol{\alpha}}\Big(\sigma + \frac{\varphi}{\Lambda_L}\Big)\Big] \widehat{C}^{gap}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|). \quad \text{(E.113)}$$

Proof of Lemma 13. Taking the difference of equation (D.78) from Lemma 9 in the *sticky-price equilibrium* and in the *flexible-price equilibrium* yields

$$\big[\boldsymbol{\lambda} \odot \big(\widehat{\boldsymbol{P}}^{gap}(\boldsymbol{\xi}) + \widehat{\boldsymbol{Y}}^{gap}(\boldsymbol{\xi})\big)\big]^\top = \widetilde{\boldsymbol{\lambda}}_D^\top \big(\widehat{P}_C^{gap}(\boldsymbol{\xi}) + \widehat{C}^{gap}(\boldsymbol{\xi})\big) - \big(\boldsymbol{\lambda} \odot \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})\big)^\top (\mathbf{L}_{vx} - \mathbf{I}) \\ + \big[\boldsymbol{\lambda}_{EX} \widehat{S}^{gap}(\boldsymbol{\xi}) - \boldsymbol{\rho}_{NX} \odot \big(\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}(\boldsymbol{\xi})^{gap}\big)\big]^\top \mathbf{L}_{vx} + o(\|\widehat{\boldsymbol{\xi}}\|). \quad \text{(E.114)}$$

Taking the difference of equation (D.84) from Lemma 10 in the *sticky-price equilibrium* and in the *flexible-price equilibrium* yields

$$\widehat{P}_C^{gap}(\boldsymbol{\xi}) + \widehat{C}^{gap}(\boldsymbol{\xi}) = \big[\boldsymbol{\lambda} \odot \big(\widehat{\boldsymbol{P}}^{gap}(\boldsymbol{\xi}) + \widehat{\boldsymbol{Y}}^{gap}(\boldsymbol{\xi})\big)\big]^\top \boldsymbol{\alpha} + \big(\boldsymbol{\lambda} \odot \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})\big)^\top (\mathbf{1} - \boldsymbol{\alpha}) \\ + (1 - \boldsymbol{\lambda}^\top \boldsymbol{\alpha}) \widehat{S}^{gap}(\boldsymbol{\xi}) - \boldsymbol{\lambda}_{EX}^\top \big(\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi})\big) + o(\|\widehat{\boldsymbol{\xi}}\|). \quad \text{(E.115)}$$

Substituting equation (E.114) into equation (E.115) and using the identity equations $\widetilde{\boldsymbol{\alpha}} = \mathbf{L}_{vx}\boldsymbol{\alpha}$ and $\widetilde{\boldsymbol{\lambda}}_F = \boldsymbol{\lambda}_{EX}^\top \mathbf{L}_{vx}$ yield

$$\widehat{P}_C^{gap}(\boldsymbol{\xi}) + \widehat{C}^{gap}(\boldsymbol{\xi}) = \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha} \big(\widehat{P}_C^{gap}(\boldsymbol{\xi}) + \widehat{C}^{gap}(\boldsymbol{\xi})\big) + [\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})]^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + (1 - \boldsymbol{\lambda}^\top \boldsymbol{\alpha} + \widetilde{\boldsymbol{\lambda}}_F^\top \boldsymbol{\alpha}) \widehat{S}^{gap}(\boldsymbol{\xi}) \\ - (\boldsymbol{\rho}_{NX} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \big(\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi})\big) + o(\|\widehat{\boldsymbol{\xi}}\|), \quad \text{(E.116)}$$

which implies that

$$(1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}) \big(\widehat{P}_C^{gap}(\boldsymbol{\xi}) - \widehat{S}^{gap}(\boldsymbol{\xi}) + \widehat{C}^{gap}(\boldsymbol{\xi})\big) = [\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})]^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) \\ + (\boldsymbol{\rho}_{NX} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \big(-\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) + \mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi})\big) + o(\|\widehat{\boldsymbol{\xi}}\|), \quad \text{(E.117)}$$

where the last equality uses $\boldsymbol{\lambda} = \widetilde{\boldsymbol{\lambda}}_D + \widetilde{\boldsymbol{\lambda}}_F$ from equation (42) in Lemma 5.

Combining equation (E.107) in Section E.3 and the identity $\widetilde{\boldsymbol{\alpha}} = \mathbf{L}_{vx}\boldsymbol{\alpha}$, yields

$$\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi}) \\ = \widetilde{\boldsymbol{\alpha}} \big(\widehat{W}^{gap}(\boldsymbol{\xi}) - \widehat{P}_C^{gap}(\boldsymbol{\xi}) + \widehat{P}_C^{gap}(\boldsymbol{\xi}) - \widehat{S}^{gap}(\boldsymbol{\xi})\big) + \mathbf{L}_{vx} \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|). \quad \text{(E.118)}$$

Substituting equations (E.118) and (E.111) into equation (E.117) and using the following identity for the coefficients of sectoral markup wedges $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$

$$(\boldsymbol{\rho}_{NX} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \mathbf{L}_{vx} - [\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})]^\top = \widetilde{\boldsymbol{\rho}}_{NX}^\top + [\widetilde{\boldsymbol{\lambda}}_F - \boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})]^\top,$$

we obtain equation (E.113). □

Interpreting Lemma 13. Equation (E.113) shows that a lower CPI in the sticky-price equilibrium than the efficient, flexible-price level (i.e., $\widehat{P}_C < 0$ in the first term on the LHS) increases the real exchange rate (i.e., $\widehat{S}^{gap} - \widehat{P}_C^{gap}$ increases and domestic currency depreciates) and improves the current account. The improvement in the current account induces more use of imported foreign factors as inputs, thereby fostering production and generating a positive aggregate output gap (i.e., $\widehat{C}^{gap} > 0$ on the RHS).

Specifically, the increase in the real exchange rate gap improves the current account in two ways: first, the real expenditures on imported inputs in units of foreign currencies decline. This effect is proportional to the share of imported inputs in the aggregate output (i.e., $1 - \boldsymbol{\lambda}^\top \boldsymbol{\alpha}$) in the brackets of the first term on the LHS); second, the depreciation in domestic currency lowers the export price and increases the (after-tax) income from net exports. This effect is proportional to the elasticity of net export income to exchange rate (i.e., $(\boldsymbol{\rho}_{NX} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \widetilde{\boldsymbol{\alpha}}$) in the brackets of the first term on the LHS).

Moreover, equation (E.113) shows that, besides the effects through CPI, negative sectoral markup wedges also directly improve the current account (the second term on the LHS) and induce more use of imported foreign factors as inputs, thereby fostering production and generating a positive aggregate output gap (i.e., $\widehat{C}^{gap} > 0$ on the RHS).

Specifically, the second term on the LHS of equation (E.113) shows that negative sectoral markup wedges (i.e., $\widehat{\boldsymbol{\mu}}$) improve the current account in two different channels: the *net export income channel* and the *net profit income channel*. In the *net export income channel*, negative markup wedges—which lead to lower export prices than in the flexible-price equilibrium—increase the net export income and improve the current account, as summarized by the net export centrality (i.e., $\widetilde{\boldsymbol{\rho}}_{NX}$ in the braces of the second term on the LHS). In the *net profit income channel*, negative markup wedges affect the current account through the net profit income of the home country in two opposite directions: 1) they increase the exports and thus profits of the home country on the international market, summarized by the foreign supplier centrality (i.e., $\widetilde{\boldsymbol{\lambda}}_F$ in the brackets of the second term on the LHS); 2) they decrease the profits by containing the cost of imported intermediate inputs, summarized by the share of imported inputs in the total costs of domestic production (i.e., $-\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})$) in the brackets of the second term on the LHS).[39]

As stated above, both negative CPI gaps and negative sectoral markup wedges improve the current account according to the LHS of equation (E.113) and allow more imports of foreign factors from the international markets—akin to increasing the supply of imported factors for domestic households and firms. The increased "supply" of imported factors leads to a positive aggregate output gap, as captured by the positive coefficient of the aggregate output gap on the RHS of equation (E.113).

[39] The *net profit income channel*, as we show later in Figure 1, is quantitatively negligible, thus making the *net export income channel* dominate and allowing negative sectoral markup wedges to improve the current account in general.

E.6 Proof of Theorem 1: aggregate output gap and sectoral markup wedges

Substituting equation (E.111) from Section E.4 and equation (E.113) from Section E.5 into equation (E.110) from Section E.3 to eliminate the real wage gap $\widehat{W}^{gap}(\boldsymbol{\xi}) - \widehat{P}_C^{gap}(\boldsymbol{\xi})$ and real exchange rate gap $\widehat{S}^{gap}(\boldsymbol{\xi}) - \widehat{P}_C^{gap}(\boldsymbol{\xi})$, yields

$$\widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}(\sigma + \varphi/\Lambda_L)\widehat{C}^{gap}(\boldsymbol{\xi}) + (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})\left[\kappa_S + (1-\kappa_S)(\sigma + \varphi/\Lambda_L)\right]\widehat{C}^{gap}(\boldsymbol{\xi}) \\ = -\widetilde{\boldsymbol{\lambda}}_D^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) - \kappa_S \cdot \widetilde{\boldsymbol{\rho}}_{NX}^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) - \kappa_S \cdot \left[\widetilde{\boldsymbol{\lambda}}_F - \boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})\right]^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|).$$

Defining the grouped parameter

$$\kappa_C \equiv \kappa_S(1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}) + \left[1 - \kappa_S(1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})\right](\sigma + \varphi/\Lambda_L),$$

yields the following matrix form of equation (38) of Theorem 1:

$$\kappa_C \cdot \widehat{C}^{gap}(\boldsymbol{\xi}) = -\left\{\widetilde{\boldsymbol{\lambda}}_D + \kappa_S \cdot \widetilde{\boldsymbol{\rho}}_{NX} + \kappa_S \cdot \left[\widetilde{\boldsymbol{\lambda}}_F - \boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})\right]\right\}^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|) = -\boldsymbol{\mathcal{M}}_{OG}^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|).$$

□

E.7 Proof of Propositions 3: centralities and import shares

As preparation, we derive the partial derivatives of the Leontief inverse matrix $\mathbf{L}_{vx}$ with respect to the home bias in intermediate inputs, as in the following equation:

$$\frac{\partial \mathbf{L}_{vx}}{\partial v_{x,r,s}} = -\mathbf{L}_{vx} \frac{\partial \mathbf{L}_{vx}^{-1}}{\partial v_{x,r,s}} \mathbf{L}_{vx} = -\mathbf{L}_{vx} \frac{\partial(\mathbf{I} - \boldsymbol{\Omega} \odot \mathbf{V}_x)}{\partial v_{x,r,s}} \mathbf{L}_{vx} = \left\{\ell_{vx,j,r}\omega_{r,s}\ell_{vx,s,i}\right\}_{j,i},$$

where $\left\{\ell_{vx,j,r}\omega_{r,s}\ell_{vx,s,i}\right\}_{j,i}$ is the (j,i)-th element of the partial derivative matrix.

Because $\mathbf{L}_{vx} = (\mathbf{I} - \boldsymbol{\Omega} \odot \mathbf{V}_x)^{-1} = \mathbf{I} + \sum_{n=1}^{+\infty} (\boldsymbol{\Omega} \odot \mathbf{V}_x)^n$, $\omega_{j,i} \geq 0$ and $v_{x,j,i} \geq 0$ for all j and i, we have:

$$\ell_{vx,j,i} \begin{cases} > 0 & \forall\, j = i, \\ \geq 0 & \forall\, j \neq i. \end{cases}$$

Proof of Proposition 3. According to $\widetilde{\boldsymbol{\lambda}}_D^\top \equiv (\boldsymbol{\beta} \odot \mathbf{v})^\top \mathbf{L}_{vx}$ in equation (31) of Definition 5, the partial derivatives of the domestic supplier centrality in sector i ($\widetilde{\lambda}_{D,i}$) with respect to the import shares of consumption goods and intermediate inputs are as follows:

$$\frac{\partial \widetilde{\lambda}_{D,i}}{\partial(1 - v_j)} = -\beta_j \ell_{vx,j,i}, \qquad \forall\, j, \tag{E.119}$$

$$\frac{\partial \widetilde{\lambda}_{D,i}}{\partial(1-v_{x,r,s})} = -\Big(\sum_j \beta_j v_j \ell_{vx,j,r}\Big)\omega_{r,s}\ell_{vx,s,i} = -\widetilde{\lambda}_{D,r}\omega_{r,s}\ell_{vx,s,i}, \qquad \forall\, r,s. \tag{E.120}$$

Equation (E.119) implies that, the domestic supplier centrality of sector i strictly decreases in its own import share of consumption—*viz*, $\frac{\partial \widetilde{\lambda}_{D,i}}{\partial(1-v_i)} < 0$—if and only if $\beta_i > 0$, because $\ell_{vx,i,i} > 0$.

Equation (E.120) implies that, the domestic supplier centrality of sector i strictly decreases in its direct downstream sector r's import share of sector i's goods (i.e., $\omega_{r,i} > 0$ and $v_{x,r,i} > 0$), if and only if sector r, directly and indirectly, supplies to domestic aggregate output (i.e., $\sum_j \beta_j v_j \ell_{vx,j,r} > 0$); that is,

$$\frac{\partial \widetilde{\lambda}_{D,i}}{\partial(1-v_{x,r,i})} = -\widetilde{\lambda}_{D,r}\omega_{r,i}\ell_{vx,i,i} < 0.$$

Equation (E.120) also implies that, the domestic supplier centrality of sector i strictly decreases in its indirect downstream sector s's import share of sector r goods if and only if both of the following two conditions hold: (i) sector s, directly and indirectly, supplies to domestic aggregate output (i.e., $\sum_j \beta_j v_j \ell_{vx,j,s} > 0$); and (ii) sector i indirectly supplies inputs to sector s via sector r (i.e., $\omega_{s,r} > 0$ and $\ell_{vx,r,i} > 0$); that is,

$$\frac{\partial \widetilde{\lambda}_{D,i}}{\partial(1-v_{x,s,r})} = -\widetilde{\lambda}_{D,s}\omega_{s,r}\ell_{vx,r,i} < 0.$$

□

E.8 Proof of Lemma 4: OG reduces to Domar weight in closed economies

Recall the expression of OG weights (38) in Theorem 1 in the following:

$$\boldsymbol{\mathcal{M}}_{OG} = \widetilde{\boldsymbol{\lambda}}_D + \kappa_S \cdot \widetilde{\boldsymbol{\rho}}_{NX} + \kappa_S \cdot \big[\widetilde{\boldsymbol{\lambda}}_F - \boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})\big].$$

As shown in Table 2, the centrality measures reduce to the following values in closed economies:

$$\widetilde{\boldsymbol{\lambda}}_D = \boldsymbol{\lambda}, \qquad \widetilde{\boldsymbol{\rho}}_{NX} = \mathbf{0}, \qquad \widetilde{\boldsymbol{\lambda}}_F = \mathbf{0}, \qquad \widetilde{\boldsymbol{\alpha}} = \mathbf{1},$$

which, substituted into the OG weights in equation (38) of Theorem 1, yields $\boldsymbol{\mathcal{M}}_{OG} = \boldsymbol{\lambda}$. □

E.9 Proof of Lemma 5: Domar weight is the sum of domestic and foreign supplier centralities

Multiplying both sides of equation (D.73) in Lemma 8 by the Leontief inverse matrix $\mathbf{L}_{vx} \equiv (\mathbf{I} - \boldsymbol{\Omega} \odot \mathbf{V}_x)^{-1}$, yields the following:

$$\begin{aligned} \boldsymbol{\lambda}_{EX}^\top \mathbf{L}_{vx} &= \boldsymbol{\lambda}^\top - (\boldsymbol{\beta} \odot \mathbf{v})^\top \mathbf{L}_{vx} \\ \Longleftrightarrow \widetilde{\boldsymbol{\lambda}}_F^\top &= \boldsymbol{\lambda}^\top - \widetilde{\boldsymbol{\lambda}}_D^\top, \end{aligned}$$

where the last equality holds due to the definitions of domestic and foreign suppliers centralities in equations (31) and (32), respectively. □

E.10 Lemma 14: output strictly increases in money supply

Lemma 14 (aggregate output increases in money supply). *In the* sticky-price equilibrium *where $\delta_i > 0$ for all $i \in \{1, 2, \cdots, N\}$, for any realized state $\boldsymbol{\xi} \in \boldsymbol{\Xi}$, a rise in $\widehat{M}$ strictly increases $\widehat{C}(\boldsymbol{\xi})$ up to the first-order approximation.*

Proof of Lemma 14. Up to the first-order approximation, given the shock to the money supply $\widehat{M}$, we have the following five conditions: (i) decomposition of CPI in equation (E.108):

$$\widehat{P}_C = (\boldsymbol{\beta} \odot \mathbf{v})^\top \widehat{\mathbf{P}} + [\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})]^\top \mathbf{1}\widehat{S} + \mathbf{Y}_1^\top \widehat{\boldsymbol{\xi}} + o(\|\widehat{M}\|);$$

(ii) the determination of the exchange rate in equation (E.117):

$$(1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})(\widehat{P}_C - \widehat{S} + \widehat{C}) = [\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})]^\top \widehat{\boldsymbol{\mu}} - (\boldsymbol{\rho}_{NX} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top (\widehat{\mathbf{P}} - \mathbf{1}\widehat{S}) + \mathbf{Y}_2^\top \widehat{\boldsymbol{\xi}} + o(\|\widehat{M}\|);$$

(iii) the sectoral Phillips curves in equation (B.42):

$$\widehat{\mathbf{P}} = \boldsymbol{\mathcal{B}} \widehat{C} + \mathbf{Y}_3 \widehat{\boldsymbol{\xi}} + o(\|\widehat{M}\|);$$

(iv) the relationship of sectoral markup wedges and inflation in equation (D.88):

$$\widehat{\boldsymbol{\mu}} = -(\boldsymbol{\Delta}^{-1} - \mathbf{I})\widehat{\mathbf{P}} + o(\|\widehat{M}\|);$$

(v) the money demand equation (11):

$$\widehat{M} = \widehat{P}_C + \widehat{C}.$$

Combining the above five equations yields the following:

$$\widehat{C} = \frac{\boldsymbol{\beta}^\top \mathbf{v} + \boldsymbol{\mathcal{M}}_P^\top \mathbf{1}}{(1+\boldsymbol{\mathcal{M}}_P^\top \mathbf{1})[1+(\boldsymbol{\beta}\odot\mathbf{v})^\top \boldsymbol{\mathcal{B}}] + (1-\boldsymbol{\beta}^\top\mathbf{v})\left[(\boldsymbol{\Delta}^{-1}-\mathbf{I})\frac{\boldsymbol{\lambda}\odot(\mathbf{1}-\widetilde{\boldsymbol{\alpha}})}{1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha}} + \boldsymbol{\mathcal{M}}_P\right]^\top \boldsymbol{\mathcal{B}}}\widehat{M}$$
$$+ \mathbf{Y}^\top \widehat{\boldsymbol{\xi}} + o(\|\widehat{M}\|), \tag{E.121}$$

where vector $\mathbf{Y}$ is a linear combination of $\{\mathbf{Y}_i\}_{i=1,2,3}$ and $\boldsymbol{\mathcal{M}}_p \equiv (1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})^{-1}(\boldsymbol{\rho}_{NX}\odot\widetilde{\boldsymbol{\alpha}}+\boldsymbol{\lambda}_{EX})$. In particular, we need $\delta_i > 0$ for all $i \in \{1, 2, \cdots, N\}$ to ensure that the slopes $\boldsymbol{\mathcal{B}}$ of the sectoral Phillips curves will be finite. □