How do central banks control inflation? A guide for the perplexed*

Laura Castillo-Martinez — Duke University

Ricardo Reis — LSE

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Abstract

Central banks have a primary goal of price stability. They pursue it using tools that include the interest they pay on reserves, the size and the composition of their balance sheet, and the dividends they distribute. We describe the economic theories that justify the central bank's ability to control inflation and discuss their relative effectiveness, in light of both theory and the historical record. We present alternative approaches as consistent with each other, as opposed to conflicting ideological camps. While interest-rate setting is often superior, having both a monetarist pillar and fiscal support is essential, and at times pegging the exchange rate or monetizing the debt is inevitable.

E31, E52, E61.

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*Contacts: l.castillo-martinez@duke.edu and r.a.reis@lse.ac.uk.

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How do central banks keep inflation on target? How do they prevent episodes of hyperinflation and their tragic consequences for welfare? Can a central bank control inflation if the economy goes through secular stagnation, a liquidity trap, or a fiscal crisis? Why was inflation so high in 2021-24 and will it persist? These are crucial questions that have answers in current economic theory.

Yet, students coming out of a macroeconomics class are often flummoxed by this topic. Undergraduates mostly retain that central banks print money and more money means higher inflation. They are then thoroughly confused when they learn that most central banks barely mention money in their speeches, that they do not actually choose how much currency to print, and that the US monetary base increased five-fold between 2008 and 2014 with no visible dent on inflation. Graduate students learn about the setting of interest rates and the Phillips curve, and perhaps even about the welfare costs of inflation and the links between monetary and fiscal policy. However, when asked to reconcile the Fisher relation (higher interest rates are associated with higher expected inflation one-to-one), the Taylor principle (increasing interest rates more than one-to-one in response to inflation keeps inflation constant), and the empirical evidence (exogenous positive shocks to interest rates lower inflation), they struggle with the apparent paradoxes. Discussions of equilibrium determinacy or active-passive regimes attract theoretically-minded researchers as much as they put off those focused on empirical applications.

The goal of this article is to provide a unified treatment of the theory of how central banks control inflation. The hope is that researchers will have an accessible entry point to this literature, so they can make sense of monetary policies and inflation outcomes. Our approach has three distinctive features.[1] The first is that we highlight the common features of different viewpoints by using a single model of prices. We present alternative theories not as opposing views, but rather as actively relying on some economic mechanisms while passively allowing other economic mechanisms to accommodate.

The second feature is that we put the central bank at the center of all of them. It is the central bank whose liabilities define the price level and that has a mandate to target a value for inflation. The central bank's policies are always the key determinant of inflation, whether they can be described as monetary, fiscal or, more accurately, as a mix of both.

The third feature is that we provide one interpretation of the history of inflation as a result of different strategies followed at different times. Our goal is not to defend it as either the only or the right explanation for inflation's movements, but rather to let the

[1] Previous surveys, taking a different approach, are McCallum (1999) and Woodford (2003).

reader see the different theoretical concepts at work and their relative strengths. We conclude that by setting the interest rate on banks' deposits at the central bank (reserves) in an aggressive and transparent way, while having a monetary pillar to anchor expectations, and fiscal support to prevent runs on its liabilities, the central bank can effectively control inflation. We discuss the circumstances when this is not possible, and how central banks can regain control of inflation by picking from a menu of unconventional policies.

This is not an article about the optimal way to conduct monetary policy or about how to trade off variability in inflation versus real activity. We take as given a target for inflation, and study the central bank's ability to achieve it along three dimensions. The first is *determinacy*, on whether policy can deliver a unique price level. We take multiple or indeterminate equilibria in a model as a sign of an incomplete policy framework. The second is *effectiveness*, on whether one policy leads to a smaller expected variation in the deviations of actual inflation from target. By characterizing the components of this variation, these can be quantified to then choose between policies based on predicted outcomes. The third is *experience*, by comparing different inflation episodes of both advanced economies and emerging markets and linking them to the monetary policy regimes of the time.

Section 1 sets up the general model of prices we will use, as well as a neoclassical and a New Keynesian general equilibrium special cases. The key result is that the classical analysis of supply and demand does not pin down inflation. This is what makes this topic special.

To discuss inflation, one must introduce a central bank, whose liabilities define what prices are in the first place. Section 2 lays out the tools at the central bank's disposal, and sets up a passive strategy where they are not used, and the government only receives dividends in the background. Inflation remains indeterminate. The sections that follow activate one tool at a time, together with different interactions with the fiscal authority. Each tool is associated with one economic force to pin down inflation.

In section 3, we consider the economic force of arbitrage between savings in nominal or real investments. The central bank steers inflation by choosing the interest rate that remunerates reserves, but it must do so with a feedback from inflation to the interest rate. If that feedback is aggressive, if the central bank forms accurate estimates of the state of the economy, and if it communicates them transparently, then it can keep inflation close to its target. The inflation targeting regimes that dominated monetary policy in the 1990s and 2000s testify to the success of this approach, even though testing it econometrically is challenging. This section also shows the importance of having a pillar, in the sense of an escape clause, if self-fulfilling expectations make inflation explode.

Section 4 continues with setting interest rates, but in the version that dominated the actions of the major central banks in the 2010s (and earlier in the case of Japan). We discuss why there may be bounds on interest rates that lead to unconventional times where the approach of the previous section cannot be used. Forward guidance, quantitative easing, or going long are strategies that focus on long-term interest rates, while subsidizing bank credit focuses on private bank rates. All of these put a great burden on the rationality of the economic agents interacting with the central bank, as their effectiveness rely on how far-sighted they are in setting their expectations. The experience with these strategies has been mixed.

Section 5 considers the economic force that brings the market for banknotes into equilibrium. The key equation is the demand for currency, while the policy tool is the central bank's exclusive right to supply banknotes. We consider money growth rules, fiscal rules on the seignorage revenues from this activity, and pegs to either commodities like gold, or to foreign currencies. Determinacy is easy to ensure, but the theory suggests that this policy strategy is usually not effective. The experience using money growth rules in the early 1980s in the UK and US and with the pegs of the Bretton Woods system in the post-war confirms that the monetary approach often leads to volatile inflation. The experience in Latin America in the 1980s and 1990s shows that it can even come hand in hand with hyperinflation.

In section 6, the key equation is the intertemporal budget constraint of the central bank, and the economic force is the unwillingness of private agents to hold the liabilities of an insolvent institution. The policy tools are the central bank's expenses, the composition of its assets, and especially the dividends it pays to the fiscal authority, all of them affecting the net shortfall of the central bank. In practice, this strategy is more often imposed on, rather than adopted by, central banks when they lose their financial independence. This section makes clear that ensuring fiscal support for the central bank is the way to avoid it, and discusses how this can be done.

Section 7 concludes with a brief discussion of the inflation disaster of 2021-24 and how central banks might have let it happen.

1 The challenge of pinning down inflation

Our starting point is a general model of prices that nests most frameworks used to study inflation. This generality comes from a strict focus on nominal *prices*, while taking *real* outcomes as given. We then incorporate two alternative mappings from nominal to real variables, which require many assumptions but are nested within the general framework: a neoclassical model first, and a New Keynesian model next. The take-away is that, in the absence of an institution like the central bank, inflation is indeterminate.

1.1 A general model of prices

Time is discrete, indexed by t from 0 to infinity. There is a large number of differentiated goods, indexed by i from 0 to I, as well as assets, indexed by j. Agents behave optimally and interact in markets. First, we describe each of three types of agents, together with the measure of the price level, and define an equilibrium. Then, after making some assumptions to simplify the exposition, we try to solve for a unique level of prices.

1.1.1 Building blocks

The first building block is a *definition of the price level*: an index function over the prices of many goods in a basket that captures how much is a dollar's worth in terms of a basket of goods over time. Given individual prices $P_t(i) \geq 0$, the price level is:

$$P_t = \mathcal{P}\left(\{P_t(i)\}_{i=0,...I}\right) \tag{1}$$

The function $\mathcal{P}(.)$ is non-negative and linearly homogeneous, so that the price level doubles when all individual prices double. The index may be the ideal measure of the cost of living in the economy. But, it may as well be any other arbitrary index that we choose to measure inflation over. This is closer to reality, when we use the consumer price index, the consumption deflator, or core versions of either of them.

The second building block is the optimal behavior by households in allocating spending across goods. Namely, a representative *consumer* equates its marginal rate of substitution between good i and good 0 to the relative price of these goods:

$$\frac{P_t(i)}{P_t(0)} = \mathbb{R}_t(i) \quad \text{for } i = 1, ..., I, \tag{2}$$

where $\mathbb{R}_t(i)$ captures how many units of good i the consumer would trade for one unit of good 0. The $\mathbb{R}_t(i)$ may change over time, reflecting shocks to consumer tastes.

Households live for multiple periods and save across time using assets. *The* $\mathbb{E}_t(.)$ operator captures the expectations of the household as of period t, which may not have full information, but we will assume is rational in the sense of being consistent with the other equilibrium equations of the model.[2] An asset j, held between periods t and $t+1$, pays a nominal return of $I_{t+1}(j)$. The asset may also provide a non-pecuniary utility to its holder, which we capture as a convenience yield defined in utility units: $\mathbb{Q}_t(j)$.

In making optimal saving decisions, the representative *saver* equates the marginal rate of substitution between consumption at time t versus $t+1$ to the relative price. This intertemporal marginal rate of substitution is $\mathbb{M}_{t+1} \geq 0$, sometimes also called the stochastic discount factor. If an asset is purely financial, i.e., valued only for the income it provides, then $\mathbb{Q}_t(j) = 0$ and the relative price is $P_t(1 + I_{t+1}(j))/P_{t+1}$, since this is the opportunity cost of consuming an additional unit today in terms of foregone consumption tomorrow. More generally, the corresponding relative price is the (inverse of the) nominal asset return plus its convenience yield:

$$\mathbb{E}_t \left[\mathbb{M}_{t+1} \left(\frac{P_t(1 + I_{t+1}(j))}{P_{t+1}} \right) + \mathbb{Q}_t(j) \right] = 1 \quad \text{for all } j\,. \tag{3}$$

There are two specific (pure) financial assets whose returns provide useful benchmarks for our analysis. The first is a safe bond that pays the same nominal return at $t+1$, no matter the state of the world. We denote its nominal return by $I_{t+1}(nom) = I_t$, to emphasize that it is known at time t. The second is a safe bond that pays the same real return at $t+1$, no matter the state of the world. We use $1 + R_t = P_t(1 + I_{t+1}(real)/P_{t+1}$ to denote this return, again to emphasize that is it known at time t. Note that, from the equation above: $1 + R_t = \mathbb{E}_t\left(\mathbb{M}_{t+1}\right)^{-1}$.

The fourth block refers to *firms* setting prices to maximize the surplus of production under imperfect competition. Denote by $\mathbb{C}_t(i)$ the real marginal cost of producing good i, and $\mathbb{Z}_t(i)$ a desired markup, so the desired price charged by the firm, $\tilde{P}_t(i)$, is:

$$\tilde{P}_t(i) = P_t \mathbb{Z}_t(i) \mathbb{C}_t(i)\,. \tag{4}$$

Again, both $\mathbb{Z}_t(i)$ and $\mathbb{C}_t(i)$ are stochastic and time-varying.

[2] Section 4 will relax this assumption where it especially matters, when forming far-away expectations.

These four blocks are common to most models written by macroeconomists. They define a general model of prices.

Definition 1. *Given a path for real outcomes,* $\mathbb{N}_t = \{\mathbb{R}_t(i), \mathbb{Z}_t(i), \mathbb{C}_t(i), \mathbb{M}_{t+1}, \mathbb{Q}_t(j)\}_{t=0}^{\infty}$*, an equilibrium for prices is a path for* $\{P_t, P_t(i), \tilde{P}_t(i), I_{t+1}(j)\}_{t=0}^{\infty}$ *that satisfies equations* (1)-(4).

Our goal in this paper is to study the sequence $\{P_t\}_{t=0}^{\infty}$ and the associated inflation sequence $\{\Pi_t\}_{t=1}^{\infty}$, where $\Pi_t \equiv P_t/P_{t-1}$.[3]

1.1.2 Uncertainty

Before proceeding, we make two simplifications regarding the nature of uncertainty. We invite the interested reader to check an online appendix for derivations without these simplifications.

First, we assume that all uncertainty regarding the path of real outcomes is resolved in period 1 and is captured by the state of the world s. At date 0, agents do not know which s will be realized the following period. But once period 1 arrives, the world is deterministic, and all exogenous variables are constant. The full path of real outcomes for $t \geq 1$ is then a function of this state: $\mathbb{N}_t = \mathbb{N}(s)$. In turn, the endogenous future prices that we want to solve for are then also a function of the state: $(P_0, \Pi_t(s))$.

Second, we work with a log-linearized version of this economy around a steady state where all the real outcomes are constant (for instance $\mathbb{M}_{t+1} = \beta$), and all prices grow over time at the constant rate $\bar{\Pi}$. The four log-linearized equations are:

$$p_t = \sum_{i=0}^{I} \omega_i p_t(i) \tag{5}$$

$$p_t(i) = p_t(0) + \rho_t(i) \tag{6}$$

$$\tilde{p}_t(i) = p_t + z_t(i) + c_t(i) \tag{7}$$

$$r_t = \mathbb{E}_t(i_{t+1}(j) - \pi_{t+1}) + q_t(j)\,. \tag{8}$$

Lower-case prices denote the log-linearized counterpart of prices in capital letters, and the same applies to the real outcomes: for instance $\rho_t(i)$ is the log-linear counterpart of $\mathbb{R}_t(i)$. For returns, r_t is the log of the gross yield on the real bond and similarly for the nominal rates, while $q_t(j) = \frac{\bar{Q}(j)}{1-\bar{Q}(j)} \log(Q_t(j)/\bar{Q}(j))$. Finally, ω_i represents the elasticity of function $\mathcal{P}(.)$ with respect to its i-th argument evaluated at the steady state. At the start

[3] For convenience, we sometimes write $\Pi_0 = P_0$.

of each section, we will refer to the general non-linear model to highlight the economic forces, but from then on proceed to work with this special log-linear version.

1.1.3 Price level (in)determinacy

Does this model of the economy deliver a unique prediction for inflation? It will if it obeys the following conditions:

Definition 2. *The level of inflation is unique or determinate in equilibrium if:*

1. *There is a unique scalar P_0 in equilibrium.*
2. *If $\Pi'_t(s)$ and $\Pi''_t(s)$ both satisfy equilibrium conditions, then $\Pi'_t(s) = \Pi''_t(s)$.*

In our general model of prices, for a given path of real outcomes, $\mathbb{N}(s)$, and a path for the price level, $(P_0, \Pi_t(s))$, the $I+1$ equations in (1)-(2) pin down the prices $P_t(i)$ for each of the $I+1$ goods. In turn, each indexed version of equations (4) and (3) solves for desired goods' price $\tilde{P}_t(i)$ and the prices of financial assets $I_{t+1}(j)$, respectively.

But what pins down the price level in the first place? Nothing, there are no equations left.

Under the current framework, any level of inflation is consistent with the equilibrium conditions above. If the price level doubles at all dates, then actual and desired prices will double as well and nominal returns will be the same. If the price level grows twice as fast, returns will be twice higher, and actual and desired good prices will grow twice as fast.

This result is what makes the study of inflation so special, and different from other variables in economics. Its importance and generality dates back to Hume (1752): dollars are just a unit of account with which the prices of goods are determined. If people started denominating prices in cents instead of dollars nothing would change. There is no demand or supply that ensures that 100 cents equals one dollar. Nothing in classical economics pins down the price level or inflation, in the same way that nothing determines whether measurements should be in inches or centimeters.

1.2 From prices to quantities

Some readers may only care about inflation insofar as it is associated with changes in real outcomes that affect well being. To make explicit the link between prices and quantities, we now introduce two, more stylized but general equilibrium, models. While they require more structure, they endogenize both nominal and real variables as a function of shocks.

The list of new assumptions, which some readers may choose to skip and jump to the next sub-section, are:

1. No investment: consumption of each good equals output, denoted by $Y_t(i)$.

2. No good, household, or asset-specific shocks: only aggregate shocks.

3. A CES aggregator of individual goods: $\mathbb{R}(i) = (Y_t(i)/Y_t(0))^{-\sigma}$, where $\sigma > 1$ is the elasticity of substitution across goods;

4. An ideal cost-of-living price index: $P_t^{1-\sigma} = \sum_{i=0}^{\infty} P_t(i)^{1-\sigma}$ since with CES preferences the utility-based measure of aggregate output is $Y_t^{\frac{\sigma-1}{\sigma}} = \sum_{i=0}^{\infty} Y_t(i)^{\frac{\sigma-1}{\sigma}}$.

5. A CRRA utility function in consumption: $\mathbb{M}_{t+1} = \beta(Y_t/Y_{t+1})^{1/\theta}$, where $\theta > 0$ is the intertemporal elasticity of substitution and $\beta < 1$ is a subjective impatience factor.

6. Constant elasticity disutility of working separable from consumption, and linear production function in work: marginal cost is $\mathbb{C}_t(i) = Y_t^{\frac{1}{\theta}+\frac{1}{\varphi}}/A_t^{1+\frac{1}{\varphi}}$ where φ is the Frisch elasticity of labor supply.

7. Markups are exogenous: $\mathbb{Z}_t(i)$ is driven by a single aggregate shock.

8. The non-pecuniary benefits from holding an asset depends solely on the asset's supply relative to output: $\mathbb{Q}_t(j) = (B_t(j)/P_t Y_t)^{-1/\eta^j} U_t(j)$, where $B_t(j)$ is the nominal supply of the asset, $U_t(j)$ is an exogenous shock, and $\eta^j > 0$ is the elasticity.

The missing ingredient is the link between desired $\tilde{P}_t(i)$ and actual prices $P_t(i)$. This is where our two stylized models differ. If desired and actual prices are the same, prices are flexible, and we have a neoclassical economy. If, instead, there is a wedge between $\tilde{P}_t(i)$ and $P_t(i)$, the economy features frictions, which can arise in different markets. Among these, we study the most popular type, nominal rigidities, in the context of the New Keynesian model.

1.2.1 A neoclassical view

Under flexible prices, the (exact) log-linear solutions for real outcomes is: $y_t^n = (a_t - z_t)/(\varphi^{-1} + \theta^{-1})$ and $r_t^n = -\log(\beta) + (1/\theta)(\mathbb{E}_t\, y_{t+1}^n - y_t^n)$, where we use the superscript n to denote the neoclassical, or natural, unique equilibrium.

Real outcomes in equilibrium are independent of nominal variables. This is commonly referred to as the classical dichotomy: real trade-offs are unchanged regardless of the price level. That is, in a neoclassical world, $\mathbb{N}_t$ is exogenous with respect to inflation. This implies that the indeterminacy of the price level does not affect real variables, which are uniquely determined.

1.2.2 The New Keynesian model

This is no longer the case once we move away from a frictionless world. There are many ways to do so, and we will focus on a single one: nominal rigidities driving a wedge between desired and actual prices. There are also many models of nominal rigidities but will now describe, arguably, the most popular one: the Calvo price-setting model. Note, however, that in each section we will also briefly introduce alternative frictions and describe their impact on inflation.

We refer the reader to any standard textbook for the derivation of the equations (e.g., Gali (2008)). The optimality condition for a safe nominal bond in equation (8) becomes:

$$\tilde{y}_t = \mathbb{E}_t(\tilde{y}_{t+1}) - \theta(i_t - \mathbb{E}_t(\pi_{t+1}) - r_t^n)\,, \tag{9}$$

where $\tilde{y}_t \equiv y_t - y_t^n$ is the output gap. This is sometimes referred to as the IS curve.

The key assumption of the Calvo price-setting model is that, every period, only a share of firms are able to reset prices. Thus, when given the opportunity to do so, firms set their price equal to the discounted sum of expected future desired prices over the period they expect their current price to be binding. The other three equations combined—(1), (2) and (4)—lead to a Phillips curve:

$$\pi_t = \beta\, \mathbb{E}_t(\pi_{t+1}) + \kappa\tilde{y}_t + z_t\,. \tag{10}$$

The parameter κ captures the inverse of the degree of nominal rigidities, so that when $\kappa \to \infty$ we are back at the classical dichotomy. Now, changes in inflation affect real outcomes, which in turn affect inflation as well.

The New Keynesian equilibrium features two equations, (9)-(10) but three unknowns: i_t, y_t and π_t. For a given π_t, the i_t is determined, just as before. But now, for a given π_t, then y_t is also solved for and, from there, all other real variables follow. Nominal outcomes now feed back into real outcomes, without a dichotomy between the two.

There is still nothing in the economy to pin down inflation in the first place. Not only the indeterminacy of inflation remains, it is now worse, as there is indeterminacy of output and all other real variables. Intuitively, for any given path for inflation, firms and workers produce whatever is demanded at these prices. In turn, demand dictates an amount of savings that determines the real interest rate. Other models of nominal rigidities will have different mechanisms, but they share this joint nominal and real indeterminacy.[4]

2 Introducing a central bank

In modern digital economies, people use electronic means of payment, like debit and credit cards, to settle their transactions. For any given transaction, the seller may have an account in bank A while the buyer has an account in bank B, so there must be a settlement whereby bank A collects payments from bank B. The central bank is the clearing house where payments between banks take place. We formally describe the central bank in this section.

2.1 Central bank tools

The central bank's assets and liabilities give it a set of monetary policy tools.

2.1.1 Liabilities: reserves

The payments between banks are made using their deposits at the central bank, a digital means of payment. Often named reserves, these are nothing but liabilities of the central bank vis-à-vis banks. Because reserves are the ultimate form of payment, they are the unit of account of the economy. Since reserves in the United States are denominated in dollars, firms and people choose to denominate their prices in dollars as well. The price of a good is simply how many units of reserves must be exchanged to obtain such good.

[4]Carlstrom and Fuerst (2002) and Nakajima and Polemarchakis (2005) provide further discussion on sticky prices and real indeterminacy.

The current stock of reserves is a list of entries in a spreadsheet at the central bank, one for each bank. Given its control over the spreadsheet, the central bank has two tools: it can choose the amount of reserves, V_t, or the rate at which it remunerates them, $I_{t+1}(V) = I_t^v$, which determines their return at $t + 1$ since they are safe assets. Both are decisions, one on the sum of the entries in the spreadsheet at a date, and the other on the factor by which each entry gets multiplied across dates.

From the perspective of households, reserves are just another asset that can be held (via banks). The savers' optimality condition to hold them is also given by equation (3). Since the great financial crisis, central banks have satiated the demand for reserves driving their convenience yield to zero.[5] We will assume this is the case for our main analysis and then revisit the pre-2008 experience in section 5. Combining equation (3) for both bonds and reserves implies that $I_t = I_t^v$. Intuitively, any discrepancy between these returns would lead savers to want to borrow an infinite amount of the nominal safe bond to invest an infinite amount in reserves, or the other way around, and the markets for neither bonds nor reserves would clear.

Through the power of arbitrage, by choosing the nominal return on reserves, the central bank controls the nominal interest rate on safe bonds.

2.1.2 Liabilities: Banknotes

Central banks also issue physical banknotes in the nominal amount $H_t \geq 0$.

Banknotes are distinct from reserves in a few ways. To start, banknotes pay no interest. Therefore, $I_{t+1}(H) = 0$, and the opportunity cost of using banknotes as opposed to digital means of payment is equal to the interest rate paid on reserves.

Moreover, banknotes can be freely held by anyone in the economy, not just banks, and they are anonymous as people do not have to declare to the government how much currency they have or from whom they got it. This is because banknotes are physical, which in turn may make them easier to use for some payments, as opposed to the electronic means backed by reserves (while for others the opposite is true). These properties create a demand for the services provided by banknotes separate to the demand for reserves.

Economic agents are willing to sacrifice returns when holding banknotes because some prefer not to use the banking system when making payments, some prefer physical to digital payments, some want anonymity in their transactions, and some find cash

[5] Reis (2016) shows evidence for satiation of reserves in the US and argues this is desirable.

easier to use. The convenience yield for banknotes in log-linear terms is:

$$q_t(h) = -\frac{h_t - p_t - y_t}{\eta^h} + u_t(h)\,. \tag{11}$$

As banknotes provide a convenience yield, but no financial return, they are often described as durable goods that the central bank produces and sells for its value $1/P_t$, rather than liabilities. The payoff from these sales of banknotes is called seignorage, and given by $S^H_{t+1} = (H_{t+1} - H_t)/P_{t+1}$.

At the same time, because the central bank commits to exchange them for reserves one for one, at all times, they are a durable with unlimited refunds. So, they can also be described as a liability. Either way, the central bank has a tool, the amount of H_t it prints.

2.1.3 Assets

On its asset side, the central bank's balance sheet holds a portfolio of some of the assets in the economy. Letting $A_t(j)$ be its holdings of asset j, then the real value of these holdings is $A_t = \sum_j A_t(j)/P_t$. Another of its tools is the composition of these assets.

2.1.4 Net shortfall

The central bank earns the return on its assets together with seignorage from printing banknotes. It also has expenses, net of the surcharges it imposes on banks, for instance by not paying interest on required reserves. We denote their real value by E_t. Finally, it pays a real dividend to the government, D_t. Combining all of these elements, the real resource constraint of the central bank in period $t+1$ is:

$$S^H_{t+1} + \sum_j \left[\frac{1 + I_{t+1}(j)}{P_{t+1}}\right] P_t A_t(j) + \frac{V_{t+1}}{P_{t+1}} = E_{t+1} + D_{t+1} + A_{t+1} + \frac{(1 + I^v_t)V_t}{P_{t+1}} \tag{12}$$

The expenses E_t and the dividends D_t are choices of the central bank. Together they determine the income shortfall of the central bank:

$$S_{t+1} = E_{t+1} + D_{t+1} - \sum_j \left[\frac{I_{t+1}(j) - I^v_t}{P_{t+1}}\right] A_t(j) - S^H_{t+1}\,. \tag{13}$$

This shortfall is an extra monetary policy tool, through dividends, expenses, and the choice of assets.

2.1.5 Net worth

Given its unconstrained ability to issue reserves, the central bank could be tempted to increase V_{t+1} without limit and pay an unlimited dividend D_{t+1} to the Treasury. Private agents would not want to participate in this Ponzi scheme. Their refusal to do so is captured by the condition:

$$\lim_{T\to\infty} \mathbb{E}_0 \left[\mathbb{M}(s)^T W_T\right] = 0\,. \tag{14}$$

where $W_t = A_t - V_t/P_t$ is the central bank's real net worth, i.e., the difference between its assets and liabilities.

This is not a policy tool, but a constraint that the central bank faces. Note that, with this definition of net worth, the resource constraint can be written as $W_{t+1} = ((1 + I_t^v)P_t/P_{t+1})W_t - S_{t+1}$ or, in its log-linear version:

$$\beta w_{t+1} = i_t^v - \pi_{t+1} + w_t - (1-\beta)s_{t+1}\,. \tag{15}$$

2.2 The policy target

The central bank uses these tools to achieve an objective: to keep inflation $(P_0, \Pi_t(s))$ close to a target $(P_0^*, \Pi^*(s))$. The target depends on the real state of the economy, and it may be arbitrary or optimal given some objectives of policy.[6] The key assumption is that, again, we solve for prices taking as given the target.

The information of the central bank is limited by imperfect real-time estimates of the state of the economy. While $\mathbb{E}_t(r_{t+j})$ denoted the public's expectation at t of what the real interest rate will be at $t+j$, we use $\hat{r}_{t+j}$ to denote the central bank's expectation at t, and these may not be the same.

If the central bank is able to determine inflation, its effectiveness is assessed by the size of the deviations between the log price level and its target. In order words, $\varepsilon(0) \equiv p_0 - p_0^*$ and $\varepsilon_t(s) \equiv \Pi_t(s) - \Pi^*(s)$.

2.3 A formal definition of a central bank

A central bank is therefore the manager of a spreadsheet of payments on reserves, a seller of an infinitely-lived durable good in currency, and a borrower and lender from households through its balance sheet. We sum up the description of the central bank with a

[6] Readers interested in the choice of P_t^* can see Khan, King and Wolman (2003) or Woodford (2010).

final definition.

Definition 3. *The central bank uses its tools* $(V_t, I_t^v, H_t, A_t(j), S_t)$ *to select an equilibrium price level path* $(P_0, \Pi_t(s))$ *to get as close as possible to the target* $(P_0^*, \Pi^*(s))$*, given its perceptions of real variables* $\hat{\mathbb{N}}$*.*

Take then the following benchmark where the central bank *passively* sets its tools:

(i) the interest rate on reserves to satisfy equation (8);

(ii) banknotes to satisfy whatever demand in equation (11);

(iii) shortfalls to ensure net worth is constant in equation (15).

Under this passive regime, the new equations in this section pin down the central bank tools as endogenous variables. In turn, the choices on the quantity of reserves and the composition of the balance sheet are irrelevant. Inflation continues to be indeterminate.

Yet, in exercising each of these three functions, the central bank has tools to affect the price level through each of the new equations. From this benchmark of indeterminacy, each section will separately relax one of (i) to (iii), while keeping the others unchanged. Each of them reveals an approach the central bank can follow to achieve its inflation target.

3 Steering inflation using the interest rate on reserves

For many decades, most major central banks have conducted monetary policy by actively steering safe interest rates, more recently by setting the interest rate on reserves. We study this policy regime in this section, while keeping all other policies passive.

3.1 The economic force that drives inflation

Combining the saving optimality condition in equation (3) for the nominal and real safe bonds, and recalling that by no-arbitrage nominal rates are equal to the rate on reserves, gives a no-arbitrage relation between real safe bonds and reserves:

$$\mathbb{E}_t \left[\mathbb{M}_{t+1} \left(1 + R_t - \frac{1 + I_t^v}{\Pi_{t+1}} \right) \right] = 0. \tag{16}$$

This states that, once adjusted by the stochastic discount factor, savings in real safe bonds or in reserves at the central bank must yield the same expected return. It is often called the Fisher equation and it is the key equation of the approach in this section. Its log-linearized version is:

$$r_t = i_t^v - \mathbb{E}_t(\pi_{t+1}). \tag{17}$$

The equation captures the central economic force that determines inflation when central banks set interest rates and which relies on the power of arbitrage. It works as follows: the private sector can choose to hold reserves or real safe bonds. Suppose the price level today was too low. All else equal, then expected inflation would be too high. Therefore, the return on reserves would be lower than the return on real safe bonds. In other words, by holding reserves at the central bank, the private sector gets fewer goods in return than if they had invested them privately.

Agents would want to hold zero reserves, and invest all of their resources in real terms. This would not be an equilibrium given a positive supply of reserves.

Rather, as they demand fewer reserves, their value falls. Because reserves are the unit of account, their real value is $1/P_t$, so the price level rises back into equilibrium. A higher price level means that expected inflation is lower and the real return on reserves is higher, rising until the point where agents are, once again, indifferent between real safe bonds and reserves.[7]

This movement from nominal reserves to real investment can come not only through real safe bonds as here discussed, but also through the purchase of durables. In a limit economy without real assets, it would show up as a desire to save less and consume more. A common saying is that inflation results from too much money chasing too few goods. From this perspective, it arises rather from economic agents wanting to substitute nominal assets for real goods or assets.

3.2 Exogenous interest rates

3.2.1 Nominal pegs

By itself, relying on arbitrage and setting interest rates does not determine inflation: it depends on how it is done. Say the central bank chooses to set the interest rate on reserves to follow some exogenous sequence $i_t^v = x_t$. The literature has traditionally referred to

[7] A small literature has studied inflation using the no-arbitrage approach but when incomplete markets lead to variations of equation (16), see Benassy (2000) and Den Haan, Rendahl and Riegler (2017).

this as an interest rate peg. The Fisher equation at all dates then implies that:

$$\mathbb{E}(\pi_1(s)) = x_0 - r_0 \tag{18}$$

$$\pi_t(s) = x(s) - r(s) \quad \text{for } t \geq 2. \tag{19}$$

In the second equation, by choosing the right-hand side, the minimal central bank is able to uniquely pin down inflation at all periods from 2 forwards. Given the target $\pi^*(s)$ it can choose $x(s)$ to make sure actual inflation is always equal to it.

However, in the first equation, by setting the right-hand side, the central bank only pins down expected inflation. Inflation at date 1 itself is not determinate. The second condition for determinacy is not satisfied, as there are an infinite number of inflation rates at different states of the world that satisfy this equation.[8]

Moreover, there is no condition pinning down the initial price level p_0. If people expect higher prices in the future, the price level at date 0 will simply jump up today. The first condition for determinacy is also not satisfied.[9]

3.2.2 Real payment on reserves

Suppose instead that the central bank promises to remunerate reserve holders with a payment in real goods.[10] Governments around the world have issued indexed bonds for a long time, and so could central banks; this is what promising a real payment of goods amounts to. The nominal return on reserves in dollars would then be $i_t^v = x_t + p_{t+1}$ for some exogenous x_t as before.

Plugging the above into equation (17) and rearranging delivers:

$$r_t = \mathbb{E}_t(x_t + p_{t+1} - \pi_{t+1}) \quad \Rightarrow \quad p_t = r_t - x_t\,. \tag{20}$$

At date 0 then $p_0 = r_0 - x_0$, and at all future dates $p(s) = r(s) - x(s)$. Since $(x_0, x(s))$ is uniquely chosen by policy, and $(r_0, r(s))$ is uniquely shaped by real forces, then the above equation delivers a determinate price level.

No central bank does this, but this peg highlights the no-arbitrage forces behind inflation control. The real return on any investment is exogenously fixed by the stochastic

[8]Nakajima and Polemarchakis (2005) provide a thorough discussion across different economic environments.

[9]This classic result is due to Sargent and Wallace (1975).

[10]This was studied by Hall and Reis (2016), building on earlier work by Hall (1997), which in turn formalized a proposal by Irving Fisher.

discount factor. If the central bank promises a real payment on reserves, then arbitrage determines how many goods reserves are worth today. The economic force behind the Fisher equation is that, since real bonds and reserves both deliver the same payment tomorrow, they must be worth the same today. But, since reserves are denominated in dollars, not goods, then this pins down the price level today.

Using its estimates of the future log real rate $\hat{r}(s)$, the central bank will achieve an effectiveness of $\varepsilon(s) = r(s) - \hat{r}(s)$. The better the estimates of the real interest rate, the more effective this policy will be.

3.3 Interest rate feedback rules

While picking interest rates on reserves, the central bank can choose a feedback rule to adjust the interest rate to inflation (or the price level):

$$i_t^v = x_t + \phi \pi_t \,, \tag{21}$$

where $\phi > 1$, so the response to inflation is more than one-to-one.

Combining with the log-linearized Fisher equation (17) gives a difference equation:

$$\phi \pi_t = \mathbb{E}_t(\pi_{t+1}) + r_t - x_t \tag{22}$$

Iterating forwards, even though $r(s), x(s)$ are constant, we cannot just rule out that endogenous inflation rises or falls over time. For now, we impose a terminal condition that inflation will not explode at a rate higher than ϕ: $\lim_{T\to\infty} \phi^{-T}\, \mathbb{E}\, \pi_T = 0$. Then, the difference equation has a unique solution:

$$p_0 = \frac{r_0 - x_0}{\phi} + \left(\frac{1}{\phi(\phi-1)}\right) \mathbb{E}(r(s) - x(s))\,, \tag{23}$$

$$\pi(s) = \frac{r(s) - x(s)}{\phi - 1} \tag{24}$$

The price level is determinate at all dates, including date 0.

By having the interest rate respond to inflation as well as to the central bank's forecast of real interest rates and the inflation target: $x(s) = \hat{r}(s) - (\phi - 1)\pi^*(s)$, the central bank can achieve its target in expectation. This makes transparently simple what are the main tasks of economists at central banks. Some will calculate the optimal target for inflation

$\pi^*(s)$, using models that interpret the mandate of the central bank. Others will evaluate the "state of the economy" in terms of the sufficient statistic $\hat{r}(s)$. Finally, as reasonable people disagree on both the state of the economy and the short-run goal for inflation, committees of policymakers will discuss different views when choosing the right interest rate $x(s)$.

3.3.1 The virtues of transparency

In period 0, the effectiveness of the interest rate rule is:

$$\varepsilon_0 = \frac{r_0 - x_0}{\phi} + \left(\frac{\phi}{\phi - 1}\right) \mathbb{E}(r(s) - \hat{r}(s)) + \frac{\mathbb{E}\left(\pi^*(s)\right)}{\phi} - p_0^* \tag{25}$$

To choose policy today x_0, the central bank must take into account not just the state of the economy r_0, but also the public's expectations of the estimation mistakes that the central bank will make on the future state of the economy. Even if neither the central bank nor the public know what $r(s)$ is, and even if their estimates are poor, as long as these estimates coincide, policy will be effective.

The central bank's communication is then an important input into its effectiveness. Speeches and statements of what it thinks the future states of the economy will be are crucial to keep inflation on target. They will work as long as the public agrees with these views, a form of transparency. This is sometimes referred to as Delphic forward guidance, as the central bank works like an oracle that convinces believers.

The Federal Reserve started to release lightly edited transcripts of previous FOMC meetings in 1993. In 1999, it began issuing statements at the conclusion of every policy meeting, and including a balance of risks in that statement in 2000. Orphanides (2019) summarizes some of this evolution in the context of anchoring inflation expectations when setting interest rates.

Across the world and since the early 1990s, central banks have adopted inflation targeting frameworks. More than announcements of official inflation targets, these have consisted primarily of transparency and communication efforts with the public about the central banks' objectives, plans, and actions (Bernanke and Mishkin, 1997). Empirical work in this area has shown repeatedly that communication under inflation targeting works by moving financial markets, the economic force behind feedback rules (Blinder et al., 2008). Data for 112 countries from 1998 until 2019 shows an almost uniform increase in transparency in the sense of central banks releasing data, sharing their internal forecasts, explaining their framework and deliberations, and disclosing policy decisions and their rationale (Dincer et al., 2022). Designing a central bank today is as much defining goals and strategies as it is setting a framework for transparency and accountability (Reis, 2013). In this line, the IMF has even published a Central Bank Transparency Code setting international standards (IMF, 2020).

3.4 The Taylor principle

The assumption $\phi > 1$ played a crucial role in delivering the determinacy of inflation. An aggressive enough response of interest rates to inflation ensured that the sum of future real interest rates and policies stayed finite when we iterated the difference equation forward. Also, $\phi > 1$ motivated the terminal condition to rule out fast exploding paths for inflation. This section discusses its role.

3.4.1 Different rules and the aggressiveness of policy

There are broader classes of interest rate feedback rules, partly motivated by the actions and experience of central banks across countries and times.[11]

First, most estimates of policy rules also show that interest rates are inertial. Central banks typically break a desired change in interest rates into 0.25% or 0.5% steps over successive policy meetings. We can represent this by having current interest rates responding to their own past value.

Second, convinced by estimates that monetary policy only affects inflation with a lag, many central banks adjust interest rates in response to public forecasts of future inflation. These are obtained from surveys, financial prices or internal models of the central bank. We can capture this by adding the public's expectation of future inflation to the interest rate rule.

Third, many central banks respond to core measures, in order to smooth out the noisy real-time measures of inflation and capture its permanent trends. Following Muth (1960), we can model core inflation as a weighted average of past inflation, which is the optimal estimate if actual inflation follows a random walk contaminated with white noise measurement error.

Fourth, studies of optimal monetary policy often suggest that the central bank should

[11] McCallum (1981) introduced these rules and first showed that they lead to determinacy. Taylor (1999), Clarida, Gali and Gertler (2000) and Woodford (2003) are classic analyses.

Table 1: Determinacy conditions

Rule	*Condition*
Benchmark: $x_t + \phi\pi_t$	$\phi > 1$
Inertial: $x_t + \phi\pi_t + \chi i^v_{t-1}$	$\phi + \chi > 1$
Forecast targeting: $x_t + \phi\pi_t + \chi\, \mathbb{E}_t(\pi_{t+1})$	$\phi + \chi > 1$
Core inflation: $x_t + \phi(1-\chi)\sum_{j=0}^{\infty} \chi^j \pi_{t-j}$	$\phi > 1$
Wicksellian: $x_t + \phi p_t$	$\phi > 0$

target the price level rather than inflation. These Wicksellian rules replace π_t with p_t in the policy rule.

The mathematics and economic logic of all these cases are similar to the ones in the analysis of the Taylor rule. Table 1 formalizes them and shows the determinacy conditions derived from the same steps as in the previous section. In all of them, the response of interest rates to inflation must be large enough, although the thresholds differ. In fact, for all but the last rule, the condition states that the cumulative response of the interest rate to a persistent unit increase in inflation exceeds one.

3.4.2 Dealing with unobservables

In our general model of prices, the real interest rate $r(s)$ and the optimal short-run target $\pi^*(s)$ are the sufficient statistics to implement effective policy. In practice, neither can be observed in real time.[12] The two stylized models of section 1.2 provide a mapping from observables into the real outcomes that our general model of prices takes as given.

First, recall the neoclassical economy where $r^n_t = -\log(\beta) + (1/\theta)(\mathbb{E}_t\, y^n_{t+1} - y^n_t)$. This has motivated multiple implementations of the Taylor rule to have $x(s)$ respond to estimates of real activity. It also matches well the experience of central banks responding to recessions by cutting interest rates.

Since our analysis already allowed for $r(s)$ to be a general stochastic process—the only restriction was that it was exogenous with respect to inflation—then this does not change

[12] Adão, Correia and Teles (2011) and Holden (2024) suggest using private-sector forecasts of inflation and returns of real indexed bonds, respectively, as measures of the real interest rate.

anything when it comes to the determinacy of inflation. The effectiveness of the policy rule will differ according to different measures of activity, and their relative variance and correlation with the short-term inflation target.

Second, in the New Keynesian model, consider a Taylor rule that makes explicit that the interest rate feedback rule in (21) includes the output gap:

$$i_t^v = x_t + \phi\pi_t + \phi_y\tilde{y}_t. \tag{26}$$

Combining this with equations (9)-(10) to eliminate the interest rate gives a system of two equations in two unknowns:

$$\begin{pmatrix}\tilde{y}_t\\ \pi_t\end{pmatrix} = \Phi\,\mathbb{E}_t\begin{pmatrix}\tilde{y}_{t+1}\\ \pi_{t+1}\end{pmatrix} + \Omega\begin{pmatrix}r_t^n - x_t\\ z_t\end{pmatrix}, \tag{27}$$

$$\text{where } \Phi \equiv \Omega\begin{pmatrix}1 & 1-\beta\phi\\ \kappa & \kappa+\beta(1+\phi_y)\end{pmatrix} \quad \text{and} \quad \Omega \equiv \frac{1}{1+\phi_y+\kappa\phi}.$$

A system of linear difference equations has a unique non-explosive solution if the number of eigenvalues of the matrix outside the unit circle is equal to the number of non-predetermined variables. In this case, both output and inflation can in principle jump, so both eigenvalues have to have modulus larger than 1. Standard linear algebra shows that this is the case if the following condition holds:

$$\phi > 1 - \frac{\phi_y(1-\beta)}{\kappa}. \tag{28}$$

This is a generalized version of the earlier Taylor principle condition. The coefficient on the output gap relaxes the responsiveness with respect to inflation because output covaries with inflation in the long run. By responding to output, the central bank is indirectly further responding to inflation.[13] The same intuition carries through, together with the reliance on a terminal condition. Likewise, the characterization of the effectiveness of the rule, and how it depends on the public's perceived deviations between natural rates of interest, inflation targets, and, now, markups over time, is the same. There is an extensive literature that considers many variants of general equilibrium models with nominal

[13]Meyer-Gohde and Tzaawa-Krenzler (2023) show that in models of the Phillips curve, like under sticky information, where fully anticipated long-run monetary policy has no effect on output, the condition reverts back to $\phi > 1$.

rigidities to derive a variety of lower bounds on ϕ to ensure determinacy.[14]

The prevalence of a real indeterminacy, in addition to the nominal one, brings a further economic force at play. Together with the no-arbitrage channel that is specific to this approach, there is an aggregate demand channel as well. Changes in the returns of financial assets affect households' desire to save, while nominal rigidities make output demand determined. Therefore, changes in the interest rate now also affect inflation through changes in consumption.[15]

3.4.3 Testing the Taylor principle

Since different rules put different lower bounds on ϕ, can we not estimate ϕ and test this mechanism? If one goes by the speeches, reports, and statements of central banks, one would think that they all follow feedback rules and subscribe to the Taylor principle. But central banks say many other things as well, and it turns out that it is hard to empirically verify the condition for determinacy.

Going back to the solution for inflation in equations (23)–(24), imagine that the central bank manages to be fully effective, so $\pi_t = \pi_t^*$ at all dates. In that case, the interest rate on reserves will be $i_t^v = x_t + \phi\pi_t^* = r_t + \pi_{t+1}^*$. Since there is no feedback anymore, this rule is observationally equivalent to a peg. Even if the econometrician had data allowing her to separate the state of the economy r_t from desired inflation π_{t+1}^*, she could not estimate ϕ.

Imagine instead $r_t = \pi_{t+1}^* = 0$, so that there are no shocks to the economy or to the policy goal, but only to monetary policy (mistakes) that follow the stationary process $x_t = \rho x_{t-1} + \epsilon_t$, where ϵ_t is iid mean zero. Then, the solution in equation (23) reduces to $\pi_t = -x_t/(\phi - \rho)$ so inflation is also autoregressive of order 1. Solving for the interest rate on reserves: $i_t^v = x_t + \phi\pi_t = -(\phi - \rho)\pi_t + \phi\pi_t = \rho\pi_t$. Therefore, a regression of the policy rate on inflation would recover the parameter ρ. Since $\rho < 1$ this estimate

[14] A few examples are the inclusion of capital accumulation (Sveen and Weinke, 2005), trend inflation (Ascari and Ropele, 2009, Khan, Phaneuf and Victor, 2020), constraints to firm credit (Lewis and Roth, 2018), multiple policy regimes (Barthélemy and Marx, 2019), and dispersion of information (Lubik, Matthes and Mertens, 2019)

[15] Allowing for incomplete markets in the presence of nominal rigidities, as we eventually do in section 6, does not by itself change the intuition underlying feedback rules, even if it changes the condition (Acharya and Dogra, 2020, Bilbiie, 2024). However, Acharya and Benhabib (2024) show that in heterogeneous agent New Keynesian models, if precautionary savings rise in recessions, this is a source of real indeterminacy in real outcomes that requires interest rates to respond not just to inflation but also to the natural rate of interest.

would mislead the econometrician to think the Taylor principle is violated (Lubik and Schorfheide, 2004, Cochrane, 2011).

The general result is that since shocks to the feedback rule affect inflation, regressions of policy rates on inflation inevitably give biased estimates of the feedback coefficient. To estimate ϕ one needs to measure (or instrument) for changes in inflation that are uncorrelated with the monetary policy shocks. But, since an effective monetary policy will respond to all shocks to inflation, in principle there are no such instruments.[16]

3.4.4 The 1990s and 2000s experience

Monetary policy in the United States during the tenure of Alan Greenspan (1987–2006) closely conformed to what was prescribed by the rule of Taylor (1993). A version of equation (21) that includes the difference between the unemployment rate and a time-varying natural rate plus two lags of the Federal Funds rate has an R^2 of 0.97 on quarterly data during the Greenspan era, but much less during the time of his predecessor, Paul Volcker (Blinder and Reis, 2005). More broadly, feedback interest rate rules that satisfied the Taylor principle became the established way to conduct monetary policy across the world during the 1990s and 2000s (Leeson, Koenig and Kahn, 2013). The actions of the ECB, which started setting monetary policy during this period, can be well described in reference to a feedback rule for interest rates right up until 2013 (Hartmann and Smets, 2018).

During these close to twenty years, inflation was low and stable. Comparing every twenty-year period over eight centuries of UK inflation, Reis (2023) finds that the period 1997-2016 had the best inflation outcomes. It had not been so before in all of the G-7 countries, and the most likely explanation was the new monetary regime (Cecchetti et al., 2007).

3.5 Escape clauses

Why does moving from a peg to a feedback rule make such a difference? Imagine that inflation is higher at date t by one log unit relative to our solution. Then, the central bank will raise the interest rate on reserves by ϕ leading to an increase in expected inflation between t and $t+1$ of ϕ (the logic is the same for the other rules). But this in turn leads

[16]Carvalho, Nechio and Tristao (2021) argue that as long as the output gap is used to proxy for x_t, what is left that drives policy has a small enough variance that the bias will be small.

the central bank to raise i^v_{t+1} by ϕ^2, which raises expected inflation between $t+1$ and $t+2$ by that amount. The process continues so inflation keeps on rising exponentially and the feedback rule imposes inflation in T periods to be larger by ϕ^T.

Inflation on target is the unique possible solution because the terminal condition ruled out these deviations by imposing that the random variable $\mathbb{E}_t\left(\pi_{t+T} - \pi^*_{t+T}\right)$ belongs to $O(\ln(\phi))$. That is, if expected inflation deviates from target, those deviations cannot grow faster than at the rate $\ln(\phi)$. The larger is ϕ, the weaker is this condition.

But where did that condition come from in the first place? We discuss it now.

3.5.1 The elusive terminal condition

The terminal condition is not an optimality condition, the way that transversality conditions are. Those apply to the real value of savings, whereas the condition needed here is on a purely nominal variable, the price level. Additionally, optimal behavior imposes no money illusion in the optimality conditions for savers or in the transversality condition.

Furthermore, there is no sense in which the economy blows up if this condition does not hold. In the neoclassical economy, the unit of account may be exploding, but agents with no money illusion would be indifferent as real outcomes continue to be finite. In the New Keynesian model, real outcomes would explode, but assuming that prices would remain sticky as inflation shoots to infinity is absurd.

Some authors argue that explosive paths for inflation are implausible. Perhaps people would never believe them. More formally, if people's expectations of inflation deviations from target in the future are constrained to stay locally bounded, then $\mathbb{E}_t\left(\pi_{t+T} - \pi^*_{t+T}\right)$ is $O(0)$. This would satisfy the terminal condition for $\phi > 1$.[17]

3.5.2 Escape clauses as anchors

Escape clauses are valid and realistic justifications for the terminal condition. The idea is that the central bank commits to the feedback rule *only* if inflation does not go on an explosive path. If inflation exceeds a pre-announced threshold, the central bank switches to a different policy approach. Realistically, if inflation start rising without bound, no central bank would stick to following blindly a Taylor rule that tells it to raise policy rates more and more, even as it sees inflation rising faster and faster.

[17] Cochrane (2011) makes a scathing critique of these arguments.

If the approach dictated by the escape clause pins down the price level at the date of the switch, then it provides the terminal condition for the feedback rule. Formally, the central bank follows the feedback rule only while inflation relative to target is within some interval $[\varepsilon^L, \varepsilon^H]$. If, at some date T, inflation π_T relative to target π^* falls outside this interval, then it switches to a different policy at $T+1$. Take as given that this other policy is able to determine uniquely π_{T+1} with some given effectiveness $\varepsilon(s)$. It could, for instance, set a real payment on reserves as we already saw, or involve any of the other approaches this paper discussed in subsequent sections.[18]

Going back to the solution for inflation with a Taylor rule, by iterating the Fisher equation from $t \geq 1$ up until a finite date T, we reach:

$$\pi_t = \left(\frac{1-\phi^{t-T}}{\phi-1}\right)(r(s)-x(s)) + \phi^{t-T-1}(\pi^*(s)+\varepsilon(s)) \,. \tag{29}$$

The right-hand side is uniquely pinned down, thanks to the switch in regime, so inflation on the left-hand side is uniquely pinned down as well.

If the regime switch happens very rarely, then T is very large. The effectiveness of this rule is still approximately given by the formula in equation (25), even if the regime to which it switches has poor effectiveness. As long as the feedback rule itself is effective, in keeping inflation close to target and away from the switching bounds, then the switch may almost never happen.

3.5.3 Escape clauses as off-equilibrium threats

A different way of writing the regime switch ensures that it never happens. In this case, it works as an off-equilibrium threat.[19]

Say that monetary policy is still committed to a feedback rule while the difference of inflation from its target stays in a bounded interval. If inflation at date T is outside of it, there is still a switch in policy that would uniquely pin down inflation, but now this switch only happens next period, at $T+1$. Moreover, now the new regime pins down inflation to some level well inside the interval, and in particular to a level such that

[18] The classic analysis is Obstfeld and Rogoff (1983), and see also Taylor (1996) and Christiano and Rostagno (2001).

[19] Much of this work builds on Bassetto (2005), and includes Atkeson, Chari and Kehoe (2010)'s sophisticated equilibria, Christiano and Takahashi (2018)'s strategy equilibria and Loisel (2021)'s implementability criteria.

$\pi_{T+1} < \pi^* + \varepsilon^H - r_T$.[20]

The Fisher equation (17) at date T together with the regime switch pins down $i^v_T = \pi_{T+1} - \pi^* + r_T < \varepsilon^H$. At the same time, the Taylor rule at T implies that since π_T was larger than π^H, and given that the Taylor rule coefficient is larger than one, $i^v_T > \pi^H$. This is a contradiction.

The only way to avoid the contradiction is for inflation to never leave the bounded interval, and the switch to never happen. If the width of the interval is large enough such that the size of the exogenous shocks would never send the economy outside the interval, then the explosions that lead to indeterminacy with a Taylor rule are ruled out. As the feedback rule implies that inflation explodes at rate ϕ, then one of the bounds will be reached for sure in finite time for any inflation path that does not satisfy the elusive terminal condition. Thus, the condition holds.

Just like in the previous case, the central bank is making the promise that it will not stick to the Taylor rule if inflation enters one of the explosive paths that violate the terminal condition. But now, the escape clause is inconsistent with equilibrium, and so it is assumed that rational agents would never expect it to be used. This requires commitment by the central bank.

3.5.4 Monetary pillars for the regime switch

With a feedback rule, the size of reserves is irrelevant for inflation, as long as it is large enough to keep their demand satiated. Yet, if the escape clause is triggered, the central bank can shrink V_t to the point where a convenience yield for reserves emerges. Just like we discussed for banknotes in section 2.1.2, the Fisher equation now includes the quantity of reserves. Fixing that quantity to some exogenous path uniquely pins down inflation. We will carefully explain the intuition for how this leads to determinacy in section 5, but for now note that this could be equally done by setting a quantity for banknotes.

In this case, the central bank is said to rely on a monetary pillar. This is understood as a commitment to switch to a monetary approach to pin down inflation if the interest rate approach leads to an exploding path for inflation. The ECB made this commitment explicit until 2021.

The pillar also serves as an alert to central banks that have successfully used feedback rules to control inflation for decades and think this is enough. Even though T may be large, it is finite. Even in the case of off-equilibrium threats, they must be credible. Having

[20] This may require nominal rigidities to make the policy consistent by having deviations from it be costly.

a monetary anchor as an escape clause behind the interest rate rule is crucial. Many central banks have such monetary anchors, often in the form of gold reserves, or holdings of foreign currency, even if they are rarely used.

3.5.5 The 1970s experience and the monetary pillar

Inflation was high during the 1970s across most advanced economies. One explanation for why this happened is that central banks did not satisfy the Taylor principle in setting interest rates (Clarida, Gali and Gertler, 2000, Coibion and Gorodnichenko, 2011). A complementary explanation is that policymakers at the time settled for a higher inflation target π^* (Meltzer, 2010), mis-estimated the state of the economy or mis-understood what drove it r_t (Orphanides, 2003, Romer and Romer, 2002), and neglected the measurement and management of private-sector expectations (Reis, 2021). Each of these, and all combined, could have contributed to inflation exploding as a result of either indeterminacy or the lack of a terminal condition anchoring expectations. The conquest of US inflation by Paul Volcker after 1979 came with a brief switch to monetarism (explained in a later section), precisely what an escape clause would dictate.

An exception to the dismal inflation performance during the 1970s was West Germany. The Bundesbank, both set interest rates following a feedback rule that satisfied the Taylor principle but also had targets for monetary growth that made explicit the potential switch to a monetary strategy if inflation ever got too far from target (Clarida and Gertler, 1997). This experience had an important impact on the design of the ECB twenty years later as following a two-pillar strategy. As described in Rostagno et al. (2021), from the 1998 initial strategy to the 2003 review, the ECB emphasized the flexibility of potentially switching to monetarism as a pillar that would stabilize expectations.

4 Unconventional interest rate policies

Starting in 1999, the Bank of Japan found itself unable to use feedback interest rate rules to control inflation. The Federal Reserve, the ECB, and other major central banks faced similar struggles following the great financial crisis. The main reason was that feedback rules prescribed setting a very low policy rate, well below zero.

Central banks throughout the 2010s continued to rely on the forces of no arbitrage, and to use interest rates as the policy tool. However, they now did it using different strategies, which were labeled unconventional in spite of their persistent use (Bernanke, 2020). As much, or even more than before, they relied on expectations of private agents, often well into the future.

This section discusses these alternative interest rate policies. We show that they can determine inflation, while at the same time arguing that in general they are inferior to feedback rules for the interest paid on reserves, justifying their unconventional label.

4.1 Bounds on interest rates

Banknotes have a net nominal return ξ that is below but close to 0, since they pay no interest but have storage costs and risk of theft. As banks would want to substitute all of their reserves for banknotes if interest rates went below ξ, banknotes impose an effective lower bound (ELB) on the payment of interest on reserves. The policy rule is now $i_t^v = \max\{\phi\pi_t + x_t, \xi\}$ (Benhabib, Schmitt-Grohe and Uribe, 2001).

To study the impact of this bound, we focus on what happens from period 1 onwards so there is no uncertainty. The left panel of figure 1 plots the dynamics of the system that results from combining the new policy rule with the log-linear Fisher equation (17). As was the case earlier, if inflation is to the right of the target, π^*, then it grows without bound, violating the terminal condition, and triggering the escape clause. This is not an equilibrium.

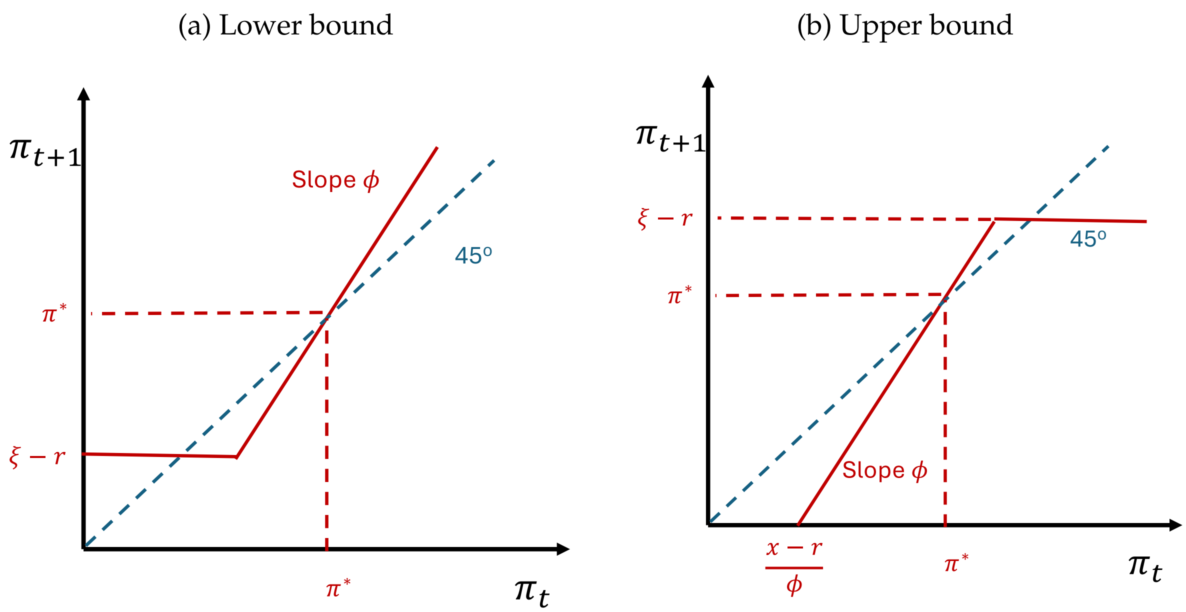
What is new, however, is that once inflation is equal to $\xi - r(s)$, it stays there forever. This is a global steady state equilibrium of the difference equation: a deflation trap. If π_1 is below the target, inflation will fall. Instead of exploding, it will now converge to the deflation trap. This is true for any initial π_1, so inflation is again indeterminate: any initial value between $\xi - r(s)$ and $\pi^*(s)$ is consistent with an equilibrium.[21]

The same problem arises in the other direction. Sometimes, under pressure from the Treasury to keep the interest rate on government bonds low, central banks do not raise interest rates above a certain level (Reis, 2009). This upper bound can be captured as a policy rule $i_t^v = \min\{\phi\pi_t + x_t, \xi\}$, where with abuse of notation we use ξ to denote the upper bound. The right panel of figure 1 shows this case. There is also a second global equilibrium steady state, but it is now above the inflation target. Thus, there is indeterminacy of inflation, which may be well above the desired target.

Setting indeterminacy aside, the existence of two steady states implies that, in the

[21] Christiano, Eichenbaum and Johannsen (2018) show that an e-stability restriction on the set of equilibria delivers uniqueness. In the other direction, when expectations are backward-looking, there may be no steady state and the economy spirals into ever higher deflation.

Figure 1: Phase diagrams for inflation



presence of shocks to the state of the economy, to the inflation target, or to policy mistakes, there will be two stochastic solutions fluctuating around these steady states. Then, if there is a sunspot that triggers a change between them, equilibrium inflation will alternate between being close to target or being close to the deflation trap (Mertens and Ravn, 2014, Aruoba, Cuba-Borda and Schorfheide, 2017). Depending on the exogenous distribution of this sunspot, the effectiveness of policy can be arbitrarily poor.

In general equilibrium, if prices are flexible, nothing changes. The interaction of nominal rigidities with the effective lower bound does make a difference, that has been explored in large strand of the literature (Eggertsson and Woodford, 2003). While this is not the place to survey it, we focus on how it affects the dynamics of inflation.

Once again, there is a permanent-deflation equilibrium since nominal rigidities do not bind at a steady state. It is also the case that different initial prices p_0 come with different paths for inflation converging in finite time to that deflationary steady state (Werning, 2011, Cochrane, 2017). However, now small changes in how the nominal rigidities are modeled, including whether prices are sticky as in Calvo (1983) or as in Rotemberg (1982), or in how the sunspots that coordinate the equilibria are introduced, or even in what numerical methods are used to solve the model in a log-linear approximation or globally, seem to matter significantly for the properties of the equilibrium.[22]

4.2 Unconventional times

In principle, one can eliminate the deflation or high-inflation equilibrium with an escape clause. However, with an inflation target of 2%, deflation is never too far, and escapes would be too frequent, making the feedback rule not useful.

To pursue its feedback rule, the central bank would like to remove the bounds. Eliminating the upper bound is usually done by having *operational* independence to set interest rates as it sees fit, without any interference from the Treasury. The Treasury-Fed accord of 1952 is the classic example.

Eliminating the lower bound is harder, but there are measures to lower the log return on banknotes ξ, perhaps all the way to minus infinity. Some suggestions in the literature on how to lower ξ are to eliminate banknotes, charge a tax on them, or default on the commitment to exchange currency and reserves one-for-one (Goodfriend, 2016, Rogoff, 2017, Agarwal and Kimball, 2019).

If, after trying these, the central bank finds itself unable to escape the bound, it can still use interest rates and the power of arbitrage to steer inflation. For the remainder of this section, we model these unconventional times in the following simple way: at date 0, we assume the interest rate is at its bound ξ, which we set to zero. The Fisher equation (17) becomes:

$$r_0 = -\mathbb{E}_0(\pi_1)\,. \tag{30}$$

In the long run, from date 1 onwards, the limit does not bind and the central bank can go back to using feedback rules to hit its inflation target $\pi^*(s)$.

4.3 Forward guidance

From equation (30), the price level at date 0 is: $p_0 = r_0 + \mathbb{E}(p_1(s))$. If the central bank chooses to hit its inflation target, then $p_1(s) = p_1^*(s)$. Since $r_0 + \mathbb{E}(p_1^*(s)) \neq p_0^*$, then p_0 may be very far from p_0^*. Having lost its tool, the central bank cannot deliver inflation on target.

Forward guidance is an unconventional interest-rate tool to try to remedy this situation. Say the central bank announces that it will keep $i_t^v = 0$ for several periods, until

[22] See Fernández-Villaverde et al. (2015), Boneva, Braun and Waki (2016), and Christiano, Eichenbaum and Johannsen (2018) among others.

date T. As a result, in period 1, the price level now is $p_1(s) = Tr(s) + p^*_{T+1}(s)$. The central bank has delayed achieving its target until after period T, when finally it will get to $p_{T+1} = p^*_{T+1}(s)$. In exchange, by manipulating $\mathbb{E}(p_1(s))$, it can bring p_0 closer to its target p^*_0.

This strategy involves a trade-off: to get inflation closer to target in the short run, the central bank has to sacrifice inflation being away from target in a potentially long medium run. If it finds itself unconstrained in period 0, so in conventional times, the central bank would never want to do this. But, in unconventional times where it cannot use i^v_0, then since it started by hitting the target exactly in that medium run, by the envelope theorem it will always want to do some forward guidance whenever it hits the lower bound.

In sum, the central bank chooses to keep the interest rate on reserves pegged in the future, even though it is not constrained by the state of the economy to do so. This unconventional announcement of a path for the policy rate is sometimes called Odyssean forward guidance, to distinguish it from the conventional communication of the state of the economy or Delphic forward guidance.[23]

With the New Keynesian model, forward guidance comes with side effects. To save on needless terms, assume that all shocks are zero: $r^n_t = 0$ and $\tilde{y}_t = y_t$. This implies that inflation at $T+1$ is on target and output is zero. Then, in period T, equations (9) and (10) imply that $y_T = \theta\pi^*$ and that $\pi_T = \beta\pi^* + \kappa y_T$, respectively. By keeping the interest rate at zero, the central bank causes a boom that in turn pushes inflation higher. The previous period, output is even higher since $y_{T-1} = y_T + \theta\pi_T$, and likewise inflation will be $\pi_{T-1} = \beta\pi_T + \kappa y_{T-1}$. Continuing until 0, then p^*_0 can be brought closer to target by choosing T adequately, just as before. The combination of the peg with the Calvo Phillips curve makes forward guidance in the distant future a powerful tool to control inflation in the present.

What is new, along this path, is that output is above its natural level. Forward guidance causes a boom from period 1 onwards, and if T is sufficiently large it may cause a boom in period 0 as well.

This result has been called the forward guidance puzzle since it is easily contradicted by empirical estimates of the effects of forward guidance.[24] At the same time, the litera-

[23]Disentangling Delphic from Odyssean forward guidance is empirically challenging even with high-frequency data (Gürkaynak, Sack and Swanson, 2005, Campbell et al., 2017, Andrade and Ferroni, 2021).

[24]The puzzle was identified in Del Negro, Giannoni and Patterson (2023) and Carlstrom, Fuerst and Paustian (2015). For empirical estimates, see Nakamura and Steinsson (2018), Cieslak and Schrimpf (2019) and Lunsford (2020)

ture has found that limits to rationality, incomplete insurance markets that change the IS relation in equation (9), or different models of price rigidity like sticky information that change the Phillips curve in equation (10) can weaken the puzzle or make it go away.[25]

4.4 Quantitative easing

Following the great financial crisis, interest rates in the Euro area stayed near zero for almost one decade. Following the Fed's experience in previous years, the ECB purchased long-term bonds with reserves in 2014. These policies became known as quantitative easing.

To understand how they work in our general model, consider another financial asset $j = o$ that is scarce, so it carries a convenience yield q_t^o. The optimality condition for savers in equation (8) applies to it, as it does to any asset. Importantly, the convenience yield depends on the quantity held by the private sector, just as with banknotes.

Combining the optimality conditions for this new asset and for pure real bonds at date 0, gives the log-linearized arbitrage condition:

$$p_0 = r_0 + \mathbb{E}(p^*(s)) - \eta^o b_0(o) + u_0(o) \tag{31}$$

where η^o is the elasticity of the convenience yield with respect to quantities, and $u_0(o)$ are shocks to it. When the central bank buys long-term bonds with reserves, it increases v_t and reduces $b_t(o)$. Whereas before the central bank had an interest rate to steer inflation given a real state of the world, now it has the convenience yield taking its role. Still, by the same no-arbitrage logic, it is able to ensure determinacy and effectiveness.

This equation also shows the limitations of quantitative easing. The key coefficient η^o is hard to estimate but seems to be small (Krishnamurthy and Vissing-Jorgensen, 2013, Fabo et al., 2021). In the spirit of the Taylor principle, this implies very large bond purchase programs when inflation is below target, in order to bring it back on target.

Moreover, the price of long-term bonds can be quite volatile, for reasons that seem divorced from expectations or risk. These are often called liquidity shocks, and are captured by the $u_0(o)$ term. With large shocks to $u_0(o)$, the central bank would have to adjust its asset purchases to counteract its effect, creating a new source of deviations of inflation

[25] Angeletos and Lian (2018), Gabaix (2020), and García-Schmidt and Woodford (2019) study deviations from perfect-foresight rationality in this context; Del Negro, Giannoni and Patterson (2023) and McKay, Nakamura and Steinsson (2016) explore incomplete insurance against income risks by households, and Carlstrom, Fuerst and Paustian (2015), Kiley (2016), Eggertson and Garga (2019) explore sticky information.

from target. The literature has struggled to find a sizable and persistent impact of liquidity shocks on inflation (Krishnamurthy and Vissing-Jorgensen, 2013, Fabo et al., 2021), perhaps because, as financial markets adjust to central bank actions, the $u_0(o)$ shocks are correlated with monetary policy, x_0 .

In effect, through quantitative easing, the central bank is targeting long-term interest rates. Historically, the source of doing so has been the Treasury rather than the central bank. Especially in the aftermath of wars, when long-term government debt is high, fiscal policy imposes low long-term interest rates. This has side effects on the balance sheet, which we will discuss in section 6.

4.5 Going long

Both forward guidance and quantitative easing focus monetary policy on long-term interest rates. The Bank of Japan went the furthest in this regard by announcing a desired target for the 10-year interest rate, standing ready to buy and sell government bonds of this maturity to hit the target. This policy was called yield curve control.

In theory, if the central bank issued bonds of a fixed maturity that were later paid off with reserves, it could choose how to remunerate these bonds just as it does with reserves. Concretely, if the central bank issues a two-period bill at date 0, and pays an interest rate of $I_0(2)$ on it, then the optimality conditions for savers in equation (3) that applies to this new pure financial asset is:

$$\mathbb{E}\left[\frac{M(s)^2(1+I_0(2))}{\Pi_1(s)\Pi_2(s)}\right] = 1. \tag{32}$$

By choosing a feedback rule for $I_0(2)$ in much the same way as it did for one-period reserves, the central bank can control the price level. The condition for determinacy still requires ϕ to be larger than some threshold, but the threshold is now equal to the sensitivity of long rates to short rates. The effectiveness of this policy involves similar terms but with different weights (McGough, Rudebusch and Williams, 2005, Reis, 2019*b*).

There is an ingenious alternative where the central bank chooses a short-term interest rate, and a sequence of long-term interest rates at different horizons (say by issuing reserves of different maturities). For each of these, there is an optimality condition like equation (32). If there are as many of these as there are states of the world at date 1 then, potentially, inflation can be pinned down.

To see this in action, assume that there are only two states of the world revealed at date 1, s^H and s^L, with probabilities f and $1-f$, respectively. From date 1 onwards, the central bank chooses two separate nominal interest rates $I(s^H)$ and $I(s^L)$ and these pin down inflation from date $t=2$ onwards at two separate levels. If the central bank only chose the short-term interest rate I_0, then this would only pin down expected inflation at date 1, but not its separate values per state: $\Pi_1(s^H)$ and $\Pi_1(s^L)$.

If the central bank also chooses at date 0 a 2-period rate $I_0(2)$ then we now have two versions of the optimality condition for savers in equation (3):

$$(1+I_0))\left(\frac{fM(s_H)}{\Pi_1(s_H)}+\frac{(1-f)M(s_L)}{\Pi_1(s_L)}\right)=1, \tag{33}$$
$$(1+I_0(2))\left(\frac{fM(s_H)}{\Pi_1(s_H)(1+I(s_H))}+\frac{(1-f)M(s_L)}{\Pi_1(s_L)(1+I(s_L))}\right)=1.$$

As long as $I(s^H)\neq I(s^L)$, then these two equations have a unique solution in the two unknowns $\Pi_1(s^H)$ and $\Pi_1(s^L)$ (Adão, Correia and Teles, 2014, Magill and Quinzii, 2014).

Intuitively, if the level of the short-term rate pins down expected inflation, now the long-term rate pins down how it covaries with the intertemporal marginal rate of substitution. This is sometimes called the inflation risk premium. By pinning it down, the central bank achieves the determinacy of inflation across states of the world.

This approach has clear limitations. First, it does not pin down the initial price level P_0. Only the stochastic degree of indeterminacy disappears. Second, counting the number of possible states of the world, and responding with just as many maturities for reserves, is a hopeless task. Third, while the effectiveness of a conventional feedback interest rate rule depends on the real interest rate—the expected value of the stochastic discount factor—the effectiveness of going long requires evaluating the stochastic discount factor in every state. Fourth, slight mis-calibrations of the short and long interest rates would induce savers to take very large positions in one versus the other causing wide fluctuations in the composition of the liabilities of the central bank.

4.6 The cost of credit

Going back to equation (30), the central bank could try to affect r_0 directly in order to steer p_0. An unconventional tool used in the 2010s were credit policies, whereby the Bank of England (through the Funding to Lending scheme) and the European Central Bank (through the Targeted Long-term Refinancing Operations) lent funds to banks at favorable rates under the condition that these funds would then be used to provide loans to firms. The goal was to lower the cost of bank credit.

Since bank credit is an input in production, if its cost falls, so will the marginal costs of production $\mathbb{C}_t(i)$. With flexible prices, this real cost of credit would still be determined with other real variables independently of inflation. It takes a nominal rigidity, like loans being set in sticky nominal amounts, or their rates in sticky nominal units, for this to lead to another transmission channel of monetary policy over inflation. Through credit policy, the central bank can then affect the real costs of credit, marginal costs of production, and through the Phillips curve, the optimal price set by firms (Christiano and Eichenbaum, 1992, 1995, Barth III and Ramey, 2001, Christiano, Trabandt and Walentin, 2010, Fiore and Tristani, 2013).

Beyond firm credit, similar mechanisms could operate through household credit, especially on mortgages that have features set in sticky nominal terms, affecting demand for goods as opposed to supply (Greenwald, 2018, Berger et al., 2021). A third channel through which lending rates can affect credit is if they affect the net worth of borrowers and tighten borrowing constraints (Bernanke, Gertler and Gilchrist, 1999).

While there is strong evidence for a credit channel of monetary policy (Ciccarelli, Maddaloni and Peydró, 2015, Gertler and Karadi, 2015), using credit supply or credit rates as the main strategy to control inflation is rarely used today. When tried in the United Kingdom in the 1950s in the context of the Radcliffe report, it failed (Capie, 2010). Central banks have an influence on lending conditions, but are very far from controlling them. There are large financial shocks in lending markets that would translate into large fluctuations in inflation.

4.7 Non-rational expectations

The final term in equation (30) that the central bank could affect is the expectations operator directly. With rational expectations, this is not possible, but moving away from it creates room for policy to steer "animal spirits". More generally, all of the unconventional policies require affecting expectations in financial markets. Even in conventional times, the feedback rule requires that people do not start expecting that inflation in an arbitrary far-away future will grow (or fall) at an explosive rate, so it relies heavily on rational expectations into the infinite future. This section discusses the relaxation of rational expectations.

The literature on non-rational expectations is too rich to cover here and has already been reviewed by Woodford (2013). Instead, we just describe three approaches that have been used to study the control of inflation.

The first are learning models that assume that expectations are formed by agents that behave like statisticians using past data to form their beliefs. A learning rule gives a mapping from past outcomes to current expectations. In turn, recall equation (22), from combining the Fisher equation with the interest rate rule, which mapped expectations into outcomes (this is sometimes called a temporary equilibrium). Combining the two gives the learning equilibrium.

The most popular such model is least-squares learning, where agents use least-squares regressions on past outcomes to form their beliefs (Eusepi and Preston, 2018). Taking the limit, as the sample for these regressions goes to infinity, delivers what is known as the learnable equilibrium. The literature focuses on the e-stability principle, that establishes that learning converges to the non-explosive rational expectations equilibrium if certain stability conditions hold. In our simple model with constant r and x after period 1, one can show that $\phi > 1$ makes this learnable and e-stable (Evans and Honkapohja, 2001, Bullard and Mitra, 2002, McCallum, 2003).

Another popular class of non-rational expectations models are models of e-duction. Their central idea is that agents go through a mental process whereby they iterate on what expectations to have, and what their implications are for equilibrium inflation, until the two converge. This convergence need not happen at the fixed point of rational expectations, nor does it have to happen over time, like with learning, but rather occurs in agents' minds. For instance, with reflective expectations, at each stage of inference, agents update their expectations to close the gap to the expectations that are model consistent. In that case, it turns out that in the limit, as the rounds of reflection go to infinity, only the non-explosive rational expectations equilibrium is selected.[26]

Third, there are models of discounting the future through limited foresight (Gabaix, 2020) or the past through imperfect memory (Angeletos and Lian, 2023). Both imply that, either looking forward or backwards, current inflation depends less on far-away expectations. Because of that, both can deliver determinacy of inflation without escape clauses and with conditions on ϕ that are less strict than the Taylor principle. Sometimes, limits to rationality or information are enough to select one of the multiple equilibria that arise even with an interest rate peg.

[26]See García-Schmidt and Woodford (2019), building on the calculation equilibrium of Evans and Ramey (1992) for the reflective case, and Farhi and Werning (2019) for k-level thinking.

More generally, once one entertains non-rational expectations, then measuring expectations becomes important as an independent source of data and shocks. As much, or more, than measures of the output gap or natural rates of interest, these data on inflation expectations become part of the state of the economy, x_t, that an effective policy rule should include to keep inflation near its target (Reis, 2022).

5 Steering inflation using the money supply

Monetarism has had many proponents in the past. Yet, it has rarely been voluntarily adopted by advanced-economy central banks. Sometimes they have been forced to follow it by Treasuries, usually with dismal consequences. This section discusses the active use of H_t to control inflation (while interest on reserves and income shortfalls are passive) and the economic force through which it controls inflation.

5.1 The economic force that drives inflation

The optimality condition for saving in banknotes comes from combining equation (3) with equation (11) to get:

$$\mathbb{E}_t \left[\mathbb{M}_{t+1} \left(\frac{P_t}{P_{t+1}} \right) + \left(\frac{H_t}{P_t Y_t} \right)^{-1/\eta^h} U_t \right] = 1. \tag{34}$$

This is sometimes called a money demand equation: it links the banknotes that people want to hold, captured by the convenience yield in the second term on the left-hand side, to the opportunity cost of holding those banknotes, captured by the interest rate foregone on pure nominal bonds in the first term on the left-hand side.

Recall that reserves paid an interest rate that the central bank could choose, and they gave no additional convenience because central banks abundantly supply them. Because banknotes pay no interest but give a convenience yield they bring a different economic force to drive the price level.

It works as follows: all else equal, a higher P_t today lowers the real balance of banknotes in the hands of the private sector. This higher scarcity raises their convenience yield via the second term in the equation. At the same time, a higher P_t raises the first term in the equation: it lowers expected inflation between the present and the next period, so it lowers the shadow nominal interest rate. With both terms higher, only if P_t falls will their expected sum be one again.

This mechanism can be explained using a standard demand-supply story. When the price level is higher, the supply of banknotes in real terms is lower (the second term). At the same time, because expected inflation is lower, the opportunity cost of holding banknotes is smaller, so the demand for them is higher (the first term). With lower supply and higher demand for banknotes, the price level must fall. This re-equilibrates the market by both increasing the supply, and by lowering demand through a higher nominal interest rate.

This Marshallian-sounding supply and demand story is appealing but it can also be misleading. After all, we saw that nothing about demand and supply pins down the price level in section 1. The previous paragraph uses a sleight of hand: the price level is *not* the price of the banknotes. Changes in the price level bring the market to equilibrium by both affecting the opportunity cost of currency *and* by directly changing the quantity of real banknotes that is held.

5.2 Money growth rules

The log-linearized version of equation (34), from combining equations (8) and (11), is:

$$h_t - p_t = c_t - \eta^h (r_t + \mathbb{E}_t \, \pi_{t+1}) + \eta^h u_t^h. \tag{35}$$

The classic monetarist policy rule is: $h_t = \pi^*(s)t + x_t$, so the central bank grows the supply of currency at the desired inflation target. This rule depends on exogenous variables, so it is not a feedback rule.

5.2.1 Determinacy

Replacing the policy rule into equation (35) and iterating forward gives a unique solution for inflation. In the long run, for $t \geq 2$, the central bank exactly achieves the target for inflation, while beforehand, inflation is on target if x_0 and $x(s)$ are picked adequately.

As before, this result depends on a coefficient condition and on a terminal condition. The condition is now that $\eta^h > 0$. In other words, it is simply that the demand curve of banknotes slopes down.[27] This is not a condition on the policy followed by the central

[27]Different micro-foundations for money imply different forms of the money demand function, and as such potentially slightly different conditions for determinacy of the price level. Still, the basic result and economic intuition remains, see Carlstrom and Fuerst (2003).

bank, nor is it a requirement for feedback as before. Rather, it is a statement of the sign of the economic force that we just described.

As for the terminal condition, it is that $\lim_{T\to\infty} M(s)^T (h_T - p_T) = 0$. This is a version of the no Ponzi scheme condition for banknotes, as in equation (14). Again, it is a feature of the economic environment, not of the policy regime. There is no need for an escape clause. In fact, monetarism is a possible escape clause, in the sense discussed in section 3.5. Instead of shrinking the supply of reserves, the central bank can switch to targeting the supply of banknotes, and it will achieve determinacy.

5.2.2 Effectiveness

In the short run, the monetarist rule leads to:

$$p_0 = \frac{1}{1+\eta^h}\left(x_0 + \eta^h r_0 - \eta^h u_0^h - c_0\right) + \frac{\eta^h}{1+\eta^h}\,\mathbb{E}(x(s) + \eta^h r(s) - \eta^h u^h(s) - c(s)) + \eta^h\,\mathbb{E}(\pi^*(s)). \tag{36}$$

Whether the central bank achieves the desired price level depends on how it chooses x_0 and $x(s)$. Higher supply of banknotes, in the present or the future, unambiguously raises inflation.

Just like with interest rate rules, the central bank has to keep track of the state of the economy and adjust policy in response. That involves estimating both the real interest rate and the level of consumption.

Even harder, the central bank also needs estimates of u_t^h. This shock to the demand for central bank currency arises from the difference between the banknotes the central bank prints and the money that people find useful. Because there are close substitutes to currency produced by the private market, including foreign currency, the u_t^h are large and volatile. For instance, u_t^h arises from changes in the availability of ATMs, in the social norms of what shopkeepers will accept as payment, or in the prevalence of crime that drives the demand for the anonymity of banknotes. The volatility of money demand, confirmed by study after study, severely undermines the effectiveness of monetarism at controlling inflation (Poole, 1970).

The effectiveness of monetarism in the short run is also not necessarily compensated by effectiveness in the long run. The result that the central bank exactly hits the target in the long run relies on our assumption that, from period 1 onwards, consumption growth, real interest rates, and the demand shocks would all be constant. Otherwise, fluctuations in any of these would lead to deviations of inflation from target, just like they did in the short run. When it comes to the shocks to the demand for banknotes, there is constant financial innovation on means of payment with long-run consequences that will prevent the central bank from hitting the inflation target.

5.2.3 General equilibrium breakdowns of the classical dichotomy

With nominal rigidities, the changes in money and inflation come with changes in real interest rates and output. Again, nominal rigidities bring into play an aggregate demand channel. As households hold more money, this raises aggregate demand, which leads to an increase in production and prices by firms. Sticky prices spread the short-term volatility of inflation that is due to financial shocks into volatility of output.

Monetarism points to alternatives to sticky prices in breaking down the classical dichotomy. If banknotes are used in transactions, their outstanding stock will facilitate trade. For instance, real money balances may enter the utility function (separable from consumption) and so affect the discount factor $\mathbb{M}(s)$. There are several well-justified models of these interactions (Lucas and Stokey, 1987, Lagos and Wright, 2005). The supply of banknotes may also affect marginal costs and so the production of goods, for instance by affecting the creation of bank deposits and bank lending (Brunnermeier and Sannikov, 2016). These additional monetarist channels come with further shocks that could raise inflation volatility further.

5.2.4 The evidence

In the early 1980s, the US and the UK both briefly adopted money growth rules. Nominal interest rates were very volatile, as were expected annual inflation rates. An empirical regularity emerged—Goodhart's law—stating that once the central bank started using a policy rule for one measure of h_t, the corresponding u_t^h shocks would turn to be even larger than anticipated before. Monetarism is still a useful strategy in scenarios where the central banks lacks credibility in the escape clause of its interest rate rule, so that volatile short-term inflation is tolerable in return for stable long-run inflation.

5.3 Seignorage

Recall that the seignorage earned from printing banknotes is $S_t^H = (H_t - H_{t-1})/P_t$. Seignorage and inflation are tightly linked. A sudden increase in the supply of banknotes, all else equal, raises seignorage. However, as we just saw in the previous section (equation (36)), it also raises the price level, which lowers seignorage. Moreover, an increase in expected inflation comes with higher nominal interest rates, which lowers the demand for currency (equation (35)) and lowers seignorage.

If the net worth of the central bank is constant, and it only holds short safe bonds as assets, then its dividends to the fiscal authorities are equal to this seignorage minus expenses $D_t = S_t^H - E_t$. The central bank may be committed to delivering an exogenous dividend, just like a government fiscal agency that has a target for tax revenues, or a state-owned company providing a public service with a target for profits. Historically, this was common, as central banks have been asked for centuries to provide fiscal resources for the sovereign. Only in the past few decades did inflation targeting replace seignorage as the primary task for the central bank.

5.3.1 Limits to seignorage

In a steady state, seignorage is given by:

$$\bar{S}^H = \left(\frac{\bar{H}}{\bar{P}}\right)\left(1 - \frac{1}{\bar{\Pi}}\right) \tag{37}$$

The first term on the right-hand side falls with inflation, since higher inflation comes with lower holdings of banknotes. The second term rises with inflation. Overall, higher steady-state inflation may raise or lower seignorage: it depends on how elastic the demand function for banknotes is.

History suggests that seignorage at first increases with inflation, and then falls. Estimating the peak of this Laffer curve is an understudied topic. But, in practice, once central banks are turned into fiscal agents, this tends to lead to run-away inflation as the central bank is asked to deliver more resources than the upper bound on feasible seignorage allows. Moreover, the peak of the Laffer curve likely moves around and is flat so that small changes in seignorage close to its peak come with large changes in inflation.

5.3.2 Seignorage policy rules

A seignorage policy rule will print banknotes to ensure that seignorage equals an exogenous amount: $s_t^h = x_t$. Log-linearizing the definition of seignorage and replacing for banknotes using the demand equation (35) gives a difference equation between inflation and seignorage. Its version in the long-run is:

$$\pi_t + \eta^h \, \mathbb{E}_{t-1}(\pi_t) = \eta^h \bar{\Pi} \, \mathbb{E}_t(\pi_{t+1}) + (\bar{\Pi} - 1)x(s) - (\bar{\Pi} - 1)(c(s) - \eta^h r(s) + \eta^h u^h(s)) \quad (38)$$

The determinacy of the price level is again ensured by the monetarist forces. Iterating this equation forward, it is enough that $\eta^h \bar{\Pi} > 1$, which again depends on the features of the economy.

With a seignorage policy rule, both the interest rate and the stock of banknotes follow endogenously, from the Fisher equation and the demand for banknotes, respectively. If the central bank is mandated to hand in a dividend, then the exogenous $x(s)$ that follows will likely deliver an inflation that is far from the target.

If, instead, the central bank was free from government pressure, then in principle it could adjust its target for seignorage to control inflation. While we are unaware of any independent central bank pursuing such an explicit seignorage policy rule, fiscal authorities do trade off their desires for dividends on the one hand, and for inflation on target on the other hand. The actual $x(s)$ may then trade off the seignorage policy rule that delivers the desired dividends with the one that is most effective at delivering inflation on target.

As the equation above shows, using seignorage as a strategy to control inflation is likely to perform poorly. As with all monetarist theories, policy will struggle to keep up with the volatility of the shocks to the demand for banknotes $u^h(s)$. Moreover, calibrating the response to these via $x(s)$, depends on pinning down the steady-state level of real banknotes held, which is likewise hard given the extent of innovation in payment systems.

5.3.3 Experience with seignorage

Seignorage policy rules are not a theoretical curiosity, but a common occurrence, especially one that is imposed on the central bank by the Treasury (Sargent and Wallace, 1984). In Latin America in the 1980s and 1990s, the extent to which these fiscal pressures fluctuated from year to year can explain some of the movements in actual inflation rates (Kehoe and Nicolini, 2022). Further back in history, many hyperinflations were associated with seignorage policy rules (Cagan, 1956), and their ends with fiscal reforms that lowered the government's demand for fiscal revenue from the central bank (Sargent, 1982).

5.4 Different monies: banknotes, reserves, and more

Both reserves and banknotes are sometimes referred to as money. Yet, they provided different approaches to control inflation. Substantially, the two differences between these two monies were whether they paid interest (currency no, reserves yes) and whether they gave a convenience yield (currency yes, reserves no). This section relaxes these differences and, in doing so, further expands on the link between the two approaches to inflation.

Finally, most people make most transactions using neither currency nor reserves, but with a third type of money, deposits at private banks. We discuss its link to inflation.

5.4.1 Reproducing monetarism with an interest rate rule

We can rewrite the equilibrium in the market for banknotes in equation (35) as:

$$i_t = \frac{p_t}{\eta^h} + \frac{c_t + u_t^h - h_t}{\eta^h}. \tag{39}$$

This is mathematically equivalent to a Wicksellian interest rate feedback rule. Since $1/\eta^h > 0$, it satisfies the determinacy condition. But while in section 3 this was a policy rule, here it emerges as an equilibrium condition.

The link to interest rates under a monetarist approach arises because the nominal interest rate i_t adjusts endogenously so that the market for banknotes clears. Canzoneri, Henderson and Rogoff (1983) blur this distinction by specifying a feedback rule for currency that depends on the nominal interest rate: $h_t = x_t + \phi i_t$. In this case, the central bank can limit the volatility of the nominal interest rate. In fact, it can even peg it to follow a pre-determined path, while inflation remains determinate.

5.4.2 Non-interest paying reserves

Before 2008, reserves paid no interest in the United States. Yet, even though nominal interest rates were well above zero, banks still voluntarily chose to hold (small) amounts of reserve balances at the central bank. These allowed the banks to settle unexpected transactions with other banks at the end of each day, especially when people ran to the banks to withdraw their deposits. In other words, reserves paid a convenience yield.

The central bank then engaged in open market operations whereby it would change the supply of reserves to hit a target for the nominal interest rate. Just as with banknotes, in equation (39), the central bank would target i_t by varying reserves and their convenience yield. The level of i_t would then follow a feedback rule, just as in section 3.

Given that level of the interest rate and the level of inflation, the central bank would passively accommodate the demand for banknotes following equation (35). The final piece of the puzzle was to stand ready to exchange reserves for banknotes one-to-one at all times as people preferred one or the other. Effectively, the central bank would only control the sum of banknotes and reserves (the monetary base).

The control of inflation was a hybrid of the monetarist and the no-arbitrage approaches. On the one hand, it was a monetarist principle applied to the narrow market for reserves that allowed it to control the interest rate. But, on the other hand this interest rate then followed a feedback rule that through the force of no arbitrage allowed it to control inflation.

Since the great financial crisis, the market for reserves has been satiated, eliminating the monetarist force. Interest has been paid on these reserves, with this policy rate used to directly shift the safe interest rate (Ihrig, Meade and Weinbach, 2015).

5.4.3 Scarce interest-paying reserves

In principle, central banks could pursue a novel approach, where they keep reserves scarce, but pay interest on them. This may be particularly relevant in a future where households can have digital deposits at the central bank just like banks do today. The benefits from using currency for payments will extend to reserves, and the demand for them may be at a much higher level than in the past.

The optimality condition for savers in equation (3) applied to reserves, when they pay interest, combined with the same optimality condition for pure nominal bonds and ignoring u_t^v for simplicity, gives in log-linear terms:

$$v_t - p_t - c_t = \eta^v (i_t - i_t^v). \tag{40}$$

Whereas before, with satiated reserves, the interest rate on reserves was equal to the interest rate on bonds, now the gap between them $i_t - i_t^v$ gives the opportunity cost of holding reserves that households trade off against their convenience benefit.

The central bank can now choose both v_t and i_t^v (Diba and Loisel, 2021). In particular, consider the case where it follows a Wicksellian rule, whereby the interest on reserves responds to p_t with a coefficient ϕ. In that case, the price level is determinate as long as

$\phi > -1/\eta^v$. This includes the case where $\phi = 0$, that is where there is a pure interest rate peg. The logic is that of the monetarist approach. With two policy tools, the central bank can potentially get closer to tracking the variables it must offset to keep inflation close to its target.

5.4.4 Bank deposits as money

Most households use their bank deposits to engage in transactions. The version of the optimality condition for savers in equation (3) that would hold for the demand for bank deposits h_t^d relates them to their opportunity cost, the gap $i_t - i_t^d$, where the interest rate paid on deposits is i_t^d.

The central bank does not control either h_t^d or i_t^d. Both are determined by the equilibrium in the banking sector. However, banks also deposits reserves at the central bank and can invest in financial assets (Piazzesi, Rogers and Schneider, 2022). Optimality in their portfolio choice leads to a log-linearized relation of the form $i_t - i_t^d = \ell(i_t - i_t^v)$. In section 3, implicitly there were competitive frictionless banks absorbed into the private sector, so that in equilibrium $\ell = 1$, leading to $i_t = i_t^v$. With market power of banks, or financial frictions, $\ell < 1$. Combining these two equations:

$$h_t^d - p_t = c_t - \eta^d \ell(i_t - i_t^v). \tag{41}$$

If the production of deposits by banks was exogenous with respect to the price level, then by choosing the interest on reserves, the central bank could again control inflation. Even though the policy tool is the interest rate, the economic logic is the monetarist one, as the key equation is the demand curve above.

If, instead, the quantity of reserves affects the amount of deposits—a money-multiplier process—then we are back at the previous case, of scarce-interest paying reserves, where the central bank has two tools, v_t and i_t^v, with which to improve the effectiveness of inflation control.

Either way, while central banks' digital currencies, more realistic banking sectors, or scarce reserves all affect the dynamics of inflation, the economic logic and the policy approach by which the central bank can control it are unchanged.

5.5 Pegs

For many emerging and developing economies, the most common approach today to pin down inflation is to peg their currency to another country's currency. This was also the case for most of the advanced world during the Bretton Woods regime, where the peg was to the US dollar between 1944 and 1976 (Bordo, 2017). Until 1971, the US dollar in turn was convertible to gold, following an even older tradition that started in the 1870s of pegging the currency to gold. This is a type of monetarist approach that does not involve banknotes or rules for their supply. Instead, the central bank is following a rule of exchanging domestic reserves for either a commodity or a foreign money.

5.5.1 Commodity pegs

Combining the equality of the marginal rate of substitution of good i with good 0, $\rho_t(i)$, to their relative prices, $p_t(i) - p_t(0)$, and the definition of the price index, p_t, in equations (5)-(6) gives:

$$p_t = \sum_{i=0}^{I} \omega_i p_t(i) = p_t(0) + \sum_{i=1}^{I} \omega_i \rho_t(i). \tag{42}$$

Recall that the parameters ω_i are non-negative and sum to one, reflecting the weights of each good in the price index.

The central bank can announce it will denominate reserves in the units of good 0. Since it can issue reserves in unlimited amounts, the central bank can enforce this denomination by buying good 0 with the reserves and holding it. Seignorage is no longer distributed as dividends to the fiscal authority. This way the central bank can always buy and sell good 0 with reserves to keep their relative price at one forever.

This uniquely determines inflation. From equation (42), having defined that $p_t(0) = 1$, the price level p_t is unique. No expectations of the future or terminal conditions are involved, because the central bank is relying on pegging the value of reserves relative to a commodity.

With this strict peg, changes in relative prices would lead the price level to deviate from target. The central bank could adjust the peg to estimates of relative-price movements using a rule $p_t(0) = p_t^* - \sum_{i=1}^{I} \omega_i \hat{\rho}_t(i)$. The effectiveness would then be: $\varepsilon_t = \sum_{i=1}^{I} \omega_i (\rho_t(i) - \hat{\rho}_t(i))$.

Changes in the supply of good 0, or in the public's taste for it, become sources of deviations of inflation from target. Moreover, if good 0 is a complement with others in consumption, then the impact on relative prices across all goods $\rho_t(i)$ can be large. The ideal commodity to peg the price level to has to: be storable, have a stable supply, and not be complementary or substitutable with many other goods.

Gold or other precious metals meet these criteria and this is why they have often been used with this approach. Still, relative-price movements are large enough that commodity pegs have tended to generate large ε_t (Bordo, 2005).

5.5.2 Exchange rate pegs

Today, it is more common to peg to a foreign currency. This is especially the case in small open economies, which import goods from other countries, often denominated in a dominant foreign currency. A currency board consists of using the same strategy as in a commodity peg, but where reserves are now exchanged for a foreign currency (or a basket of currencies).

The economic logic of how they work is the same. Take the first $J+1$ goods to be domestic, but the remaining $I-J$ goods are foreign. Their price in domestic units is then equal to a foreign price $p_t(j)$ plus the log exchange rate between the domestic and the foreign units of account: e_t. Letting α denote the measure of home bias in consumption, equation (5) for the domestic price level is then equal to:

$$p_t = \alpha \sum_{i=0}^{J} \omega_i p_t(i) + (1-\alpha) \sum_{j=J+1}^{I} \omega_i (p_t(j) + e_t) = \alpha \sum_{i=0}^{J} \omega_i p_t(i) + (1-\alpha)(p_t^f + e_t) \quad (43)$$

where p_t^f is the price index of the imported goods in foreign currency.

The optimality condition between any two domestic and foreign goods in equation (6) is: $\rho_t(i,j) = p_t(i) - p_t(j) - e_t$, where $\rho_t(i,j)$ is the marginal rate of substitution between consumption of domestic good i and foreign good j. It then follows that: $\sum_{j=J+1}^{I} \omega_j \rho_t(i,j) = p_t(i) - p_t^f - e_t$. Replacing for $p_t(i)$ in equation (43) delivers:

$$p_t = e_t + p_t^f + \alpha \sum_{i=0}^{J} \sum_{j=J+1}^{I} \omega_i \omega_j \rho_t(i,j). \quad (44)$$

The second and third term on the right-hand side are exogenous with respect to the price level. An exchange-rate target peg is a choice of e_t. Thus, it uniquely pins down the price level.

5.5.3 Experience with pegs

The peg of the Hong Kong dollar to the US dollar is perhaps the most famous case of a successful currency peg. In place since October of 1983, the standard deviations of the monthly log nominal exchange rate between the two currencies has been 0.004 over these almost 40 years. And yet, inflation in Hong Kong dollars has been significantly more volatile than inflation in US dollars, peaking at 11% in 1991 and bottoming at -4% in 1999. In part, this happens because choosing e_t to be constant over time implies that changes in $\rho_t(i, j)$, and consequently in the real exchange rate, lead to wide fluctuations in p_t (Obstfeld and Rogoff, 1995, Ilzetzki, Reinhart and Rogoff, 2019).

Currency boards are rarely adopted for two practical reasons. First, central banks often have conflicting goals for the desired price level π_t^* and for the exchange rate e_t. When pursuing one of them has unpleasant consequences for the other one, the currency board is abandoned.[28] Second, currency boards require that the central bank keeps all the foreign currency it buys with its reserves, so it is ready to buy and sell it as needed. In reality, there is pressure on central banks to distribute some of these assets as dividends, or to exchange the foreign currency for domestic government bonds. They then run out of the foreign currency when they need to sell it, and the peg fails.

Without foreign assets to back the reserves, countries that try to maintain exchange rate pegs rely either on choosing the interest rate on reserves to mimic movements on foreign interest rates, or in adjusting the supply of money to control its relative scarcity relative to the foreign currency, or a mix of both. This translates into particular interest rate rules or money growth rules that we have already covered. One way to interpret these pegs is that the value of the exchange rate is seen as a useful indicator of the state of the economy or of the shocks to demand for currency that the central bank aims to track to have a more efficient policy rule. The adoption and abandonment of these pegs follows the usefulness of this indicator as wedges arise between the domestic economy and its foreign counterpart.

6 Steering inflation using income shortfalls

The final set of tools of the central bank are the composition of its assets $A_t(j)$, its spending E_t, and its dividend policy D_t. These policies drive the net income shortfall of the central

[28] See Frankel (2010) for the difficulty of controlling inflation when the central banks cannot commit to a monetary policy strategy.

bank, S_t. This section studies them, while now keeping the interest on reserves and the supply of banknotes passive. This is the most controversial approach to controlling inflation, because its economic force is disputed in theory and has been hard to test. It often goes under the name of the fiscal theory of the price level. It is important to understand it even if only for central banks to know how to avoid it.

6.1 The economic force that drives inflation

The key equation is the no Ponzi scheme condition for the central bank in equation (14). By forward iterating the resource constraint in equation (12), imposing the no-Ponzi condition, and using the definition of net worth we get:

$$\frac{V_0}{P_0} = A_0 - \frac{M(s)}{1 - M(s)} S(s). \tag{45}$$

This provides one equation in one unknown, the price level.[29] The economic force at play is the following: a higher shortfall of the central bank leaves fewer real resources available to back its debt, reducing its real value. As reserves are default-free, they have a fixed value in nominal terms. Given their role as unit of account, the only way for their real value to fall is for the price level to rise.

The price level adjusts because banks choose to hold more or fewer reserves in response to them becoming a Ponzi scheme. When the central bank follows a path for shortfalls that makes it insolvent, then banks run on the central bank's liabilities. This causes these liabilities—reserves—to lose their real value until they are back in line with the central bank's assets and net shortfall so the central bank is solvent. It is the control of the real resources earned by the central bank that gives it control over inflation.[30]

6.2 Shortfall policy rules

Imagine that the central bank adjusts its balance sheet tools to follow a feedback rule for its shortfall:

$$(1 - \beta) s_{t+1} = \phi w_t + x_{t+1}. \tag{46}$$

[29]The original analysis is Woodford (1994) and Sims (1994) and our approach is closer to that in Benigno (2020) or Cochrane (2005).

[30]For criticism of this mechanism, see Niepelt (2004) and Buiter (2017).

If the net worth falls, the central bank may pay less dividends or cut spending to lower its shortfall, in which case $\phi > 0$. The exogenous policy shock x_{t+1} is now an action by the central bank that increases its shortfall.

6.2.1 Determinacy

Combining equation (46) with the law of motion for net worth in equation (15) to replace out shortfalls gives a difference equation for net worth. In turn, the no-Ponzi scheme condition for the central bank's liabilities in equation (14), which is $\lim_{T\to\infty} \beta^T w_T = 0$ in its linear form, provides a terminal condition. Together they give a solution for net worth as long as $\phi < 1 - \beta$.

Recalling that net worth is just the difference between assets and reserves delivers the solution for the price level in the long run:

$$p(s) = \frac{x(s) - r(s)}{1 - \beta - \phi} \left(\frac{\bar{W}}{\bar{V}/\bar{P}} \right) + v(s) - \left(\frac{\bar{A}}{\bar{V}/\bar{P}} \right) a(s) \tag{47}$$

The determinacy condition $\phi < 1 - \beta$ calls for the central bank to not lower its shortfall by too much when its net worth falls. In fact, a reckless central bank that spends more as its net worth falls $\phi < 0$ would be ideal. Its irresponsible behavior forces banks to run on the central bank, exchange their reserves for real goods and assets, and so raise the price level to its new level. Following Leeper (1991), the literature has called feedback rules that satisfy this condition *non-Ricardian policies*.

It seems paradoxical that inflation control results from the central bank actively pursuing insolvency to trigger runs by banks. Any private agent that tries to do this would find that its liabilities become worthless, so it can get no real resources in exchange for the new debt it issues. What makes the central bank special is that its liabilities are the unit of account. It can honor these liabilities in nominal terms, by just issuing more reserves, but also in real terms as long as the price level adjusts. Therefore, when it follows a rule that would lead to a Ponzi scheme for an unchanged price level, the required fall in the real value of reserves requires the price level to rise.

Inflation results from the condition that the central bank must stay solvent. It is a capital gain to the central bank that comes at the expense of a capital loss of the private sector holding the nominal reserves.[31]

[31] If reserves earn a convenience yield that is tied to their safety, then the shortfall policy rules of the central bank can affect inflation now through a monetarist channel, like the one in the previous section, see

6.2.2 The role of dividends

As far as we know, no central bank has ever voluntarily pursued this strategy. Its effectiveness is dubious. A non-Ricardian reckless behavior would get a central banker fired before she got to implement it. Even if not, committing to a precise pattern for future shortfall would be as hard for the central bank as it is for any government agency. Instead, the solvency approach is useful because it sheds light on how the central bank should manage its surpluses s_t to stay away from a reckless shortfall policy.

The shortfalls in equation (13) are the sum of four parts. The first is seignorage from banknotes. A policy rule for seignorage would affect inflation via the supply of currency, so it could not be used to control it via solvency at the same time. The second component are the expenses of the central banks. These are typically too small to be relevant. The third component are the returns from holding assets net of the interest paid on reserves. By choosing the composition of these assets, and so the risk in their returns, the central bank could, hypothetically, guide its net shortfall and so control inflation. However, since financial returns are volatile, most fluctuations in net shortfall are outside the central bank's control.

This leaves the fourth component of shortfalls as their main driver: the dividends to the treasury d_t. Ensuring that the central bank is solvent at all times, or that $\phi > 1 - \beta$, is referred to as the central bank having full fiscal support (Hall and Reis, 2015, Del Negro and Sims, 2015, Benigno, 2020). In practice, this is achieved by requiring the central bank to pay out all of its net income as dividends. In principle, this would imply that w_t is constant, the assumption we made in previous sections. In practice, because net income is calculated differently across central banks, the net worth fluctuates, but the condition we derived shows that what is important is that ϕ is sufficiently large so that the central bank is not running a Ponzi scheme.

6.2.3 Experience

If the central bank has a "narrow" balance sheet, holding almost only short-term bonds, then the term on asset gains is close to zero. Because seignorage from banknotes greatly exceeds expenses, the dividend rule that guarantees solvency has traditionally always implied positive dividends. These would sometimes be larger or smaller, but they have raised little attention.

Bassetto and Cui (2018), Brunnermeier, Merkel and Sannikov (2020).

After the great financial crisis, as central banks in advanced economies held large amounts of long-term safe bonds together with risky bonds, this was no longer the case. Most central banks in the world experienced negative dividends as a result of holding foreign assets whose value fluctuates with the foreign exchange rate (Reis, 2019*a*). Between 2021 and 2024 this led to large losses, and a series of approaches to ensure fiscal support ranging from fiscal transfers from the Treasury (Bank of England), lower future dividends in exchange for setting present negative dividends to zero (the Federal Reserve) or ex-ante provisioning, by paying lower dividends in the past, that allows net worth to now fall to its steady state (European central banks).

6.2.4 Monetizing the deficit

Historically, with some regularity, the Treasury imposes a dividend process. The central bank is no longer independent to set a rule for its shortfall, whether it is one that ensures fiscal backing or not.

Section 5.3 derived the consequences for inflation if the central bank uses seignorage to satisfy the Treasury's request. The printing of banknotes pins down the price level through monetarist forces, often at high and volatile levels.

The solvency approach shows an alternative. The central bank could refuse to print the banknotes. Insofar as the dividends are exogenous, the resulting shortfalls would be exogenous as well. In that case, $\phi = 0$, and the price level would be pinned down by solvency forces. In effect, the central bank would be issuing reserves to borrow from the private sector and send the resources to the Treasury. Every time it does so, the reserves are worth less because of the inflation it generates.

Whether through seignorage or insolvency, this state of affairs could be referred to as *monetizing the fiscal deficit*. It is the monetary base, currency and reserves, that is adjusting to provide the necessary funding for the Treasury.

6.3 The solvency of other agents and inflation

Even if the solvency of the central bank is ensured, the solvency of other agents can affect the level of inflation.

6.3.1 Fiscal dominance

The bulk of the literature on the fiscal theory of the price level focuses on the intertemporal budget constraint of the government linking the value of public debt to the present value of its primary shortfalls, the deficits (Leeper and Leith, 2016). It makes three strong assumptions.

The first is that the government does not default on its liabilities, so that government bonds are perfect substitutes for reserves. Yet, unlike reserves, government bonds are not the unit of account, and sovereign defaults are frequent (Reinhart and Rogoff, 2009).

Second, it assumes that the central bank dividends are passive and can take any value. In that case, the intertemporal budget constraints of the central bank and the Treasury are no longer two separate constraints, but rather a single consolidated one that states that the sum of reserves and government liabilities (netting out the government bonds held by the central bank) equals the present value of shortfalls of both the government and the central bank (netting out the central bank's dividends). This assumption effectively denies central bank independence over its balance sheet.

The third assumption is that the Treasury solely chooses this shortfall, and any actions of the central bank on its expenses, seignorage or composition of its assets are exactly offset by the Treasury.

Combining these three assumptions, the same logic that allowed the central bank to control inflation is now applied to the Treasury. That is, the solvency of the Treasury becomes tied to the price level. This predicts that exogenous shocks to primary shortfalls are linked to movements in inflation. There is some support for this in the data, especially during the 1970s in the US, when high inflation coincided with large and persistent fiscal deficits (Sims, 2011, Bianchi and Melosi, 2018, Cochrane, 2022*a*).

6.3.2 General equilibrium and nominal rigidities

Nominal rigidities do not alter the underlying logic of the solvency approach (Woodford, 1996, Sims, 2013, Cochrane, 2023). In terms of the basic New Keynesian model, the equilibrium condition that is combined with the IS and Phillips curve in equations (9)–(10) together with the net worth of the central bank in (15) give three equations with three unknowns—the output gap, inflation, and net worth—as a function of nominal interest rates and shortfalls. Instead of a feedback interest rate rule, as in section 3, or a money growth rule, like in section 5, now there are two parts of the policy rule: an exogenous process for the nominal interest rate set by the central bank, and a rule for the central bank's shortfalls as in equation (46). The first part fixes expected inflation, while the second part determines its response to shocks.

As before, nominal rigidities add complementary channels working through output and, especially, real interest rates. Now, after a loss, the jump in inflation to reestablish the solvency of the central bank will lower the real interest rate given a nominal interest rate peg. This stimulates more consumption, which further raises inflation via aggregate demand. In turn, the transfer of wealth from the private sector towards the central bank happens not just via inflation but also because of lower real interest rates over time. Therefore, the dynamics of inflation change relative to when the classical dichotomy holds, becoming more drawn out.

These dynamics feed back into the size of the inflation response because the real value of the central bank's net worth depends on the maturity of its assets. The value of long-dated assets depends on both unexpected inflation and on the path of real interest rates. Quantitative easing strategies that set the maturity of the central bank's assets become a determinant of the persistence of inflation deviations from target (Cochrane, 2001).

6.3.3 Alternatives to break the classical dichotomy

The focus on solvency brings to light different mechanisms that can break the classical dichotomy even with flexible prices. The transfer of wealth to the central bank comes at the expense of banks holding these reserves (Reis, 2016). If banks' net worth constrains their willingness to extend private credit, then this provides a credit channel that complements the ones we already discussed in section 4.

More generally, in an economy where different agents hold different mixes of nominal and real assets, the inflation that is driven by the central bank's solvency requirement will induce redistribution of wealth. With incomplete markets, these will matter for aggregate demand and so for the dynamics of inflation (Auclert, 2019, Kaplan, Nikolakoudis and Violante, 2023).

Incomplete markets raise another channel for breaking the classical dichotomy through the solvency channel. The total stock of reserves outstanding is a relevant variable for inflation because it determines by how much inflation must change to keep the real value of these reserves in line with solvency. Under the fiscal dominance version, the size of the government debt matters as well. In incomplete markets models, the net supply of assets available to agents affects their equilibrium choices. The reason is that, unable to diversify individual risks, agents engage in precautionary savings through these assets, so their relative availability determines the cost and limits of doing so. When inflation results from the central bank's choices regarding its shortfall, this both ex post redistributes wealth across different agents and ex ante affects the expected returns on different savings vehicles. Both affect the desire to consume and produce, and so the real outcomes in the economy (Hagedorn, 2018*a*,*b*).

In the other direction, breaking the classical dichotomy affects the source of shocks to inflation. Changes in output and inflation directly affect the demand for currency and the seignorage revenue of the government. Changes in the path of real interest rates over time affect the capital gains and losses in the central bank's portfolio. And, the effect of monetary policy on real outcomes has an effect on tax collections and government spending and so on the primary shortfalls of the fiscal authority. Through fiscal dominance, these would trigger changes in the demand for dividends from the central bank (Reis, 2019*a*).

7 Conclusion: a unified approach and the 2021-24 inflation disaster

Each of the previous sections emphasized one policy tool that leans on one particular economic force to bring inflation close to target. They all co-existed in the same dynamic general equilibrium model, not as contradictory theories, but as different policy options. The central bank can choose a strategy that relies on arbitrage forces, on money market forces, or on insolvency forces, and within each it can choose one rule to implement it, whether it is an interest rate rule, or a money growth rule, or a net shortfall rule, or one of their many variants.

Importantly, if more than one strategy is active, most likely they will be in conflict with each other; mathematically the economic system for prices becomes over-determined.[32]. We showed how to measure the expected effectiveness for each policy strategy. Our hope is that the academic debate shifts from arguments on which assumptions are perceived as being more convincing to attempts to measure the effectiveness of each strategy. We

[32] A different use of the word active and passive is to describe which of the two institutions, the central bank or the Treasury, is imposing its decisions on the other. If they are playing a game with each other, this will affect how the policy approach is chosen and set. Unfortunately, both definitions of active/passive are used in the literature, generating confusion.

also provided our own take of the history of monetary policy and inflation in post-war advanced economies.

During Bretton Woods, most countries followed a monetarist strategy by pegging to the US dollar. During the 1970s, the US or the UK followed an interest rate strategy without satisfying the conditions for determinacy, both in the responsiveness of the feedback rules and in having a clear pillar as an escape clause. This led in the early 1980s to both countries having brief experiences with money growth rules. At the same time, in countries through Latin America, fiscal authorities imposed a combination of monetarist seignorage rules or net shortfall rules that flirted with insolvency, leading to high and volatile inflation.

The conquest of stable inflation between 1990 and 2010 came from a coherent strategy across the different elements. Central banks used feedback interest rate rules that satisfied the determinacy principle. They adopted inflation targeting regimes to manage expectations, while having escape clauses reliant on monetary anchors. Central bank independence imposed rules on the dividends paid to Treasuries that kept the central bank solvent at all times. These ruled out seignorage or insolvency from driving inflation.

Between 2010 and 2020 this framework was refined through forward guidance, quantitative easing, going long, and communication strategies to overcome the effective lower bound, even if these tools relied heavily on rationality of expectations and were generally less effective.

Between 2021 and 2024, unusual shocks that hit the economy justified an optimal inflation rate well above 2%, but the actual inflation rate was much higher than that. Why this happened is understandably still the subject of debate. Perhaps central banks kept to this same framework but they made mistakes in judging the state of the economy, in anchoring expectations, and in using unconventional interest rate tools that are less effective (Reis, 2023, Eggertsson and Kohn, 2023). Or, perhaps the expansion of the balance sheet of central banks through quantitative easing and the large increase in public debt during the pandemic have made central bank independence untenable and it is concerns about solvency that are driving inflation (Bianchi and Melosi, 2022, Cochrane, 2022*b*). The future will show.

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