Trade Credit Default*

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November 15, 2021

Abstract

Recent micro evidence shows that default on trade credit repayments is substantial. What is the role of trade credit default in the transmission of macroeconomic shocks? We build a heterogeneous-firms quantitative model where an intermediate input is purchased by final-goods producers partly on trade credit before observing the realisation of their productivity. A bad productivity shock may ex-post induce final good producers to skip payment to suppliers or, alternatively, liquidate via bankruptcy. Aggregate trade credit delinquency and liquidation are taken into account by input suppliers; the individual liquidation risk is priced in by lenders supplying bank credit. The response of trade-credit delinquency and bankruptcy, via their effect on intermediate input supplier's markups, provides an amplification mechanism of aggregate shocks. We consider productivity, financial and volatility shocks. In a calibrated version of the model, the surge in trade credit default that follows a negative shock accounts for a large portion of the fall in output and employment, and feeds into further firm liquidation and delinquency. For instance, trade-credit default accounts for about one third of the impact of a volatility shock.

Keywords: trade credit, default, delinquency and bankruptcy, heterogeneous firms, amplification of macroeconomic shocks, markups

JEL No. D21,D25,E32, E44, G33

*Earlier versions of the paper had been circulated under the different titles *Trade credit delinquency, bankruptcy, and the propagation of aggregate shocks* and *The role of trade credit and bankruptcy in business fluctuations*. This paper has been presented at the AEA 2019 Conference in Atlanta, the National University of Seoul 2019, III Workshop Spanish Macroeconomics Network 2020, Universität Konstanz 2021 and the London Macro CfM LSE 2021, and we much appreciate the feedback from audiences there. Special thanks to Dean Corbae and Gabriel Michalache for their helpful comments on an early version, and to Stephen Terry for an enlightening conversation. Aizat Token provided superb research assistance. The authors would like to acknowledge financial support by Nazarbayev University through the "Faculty-development competitive research grants program" number SHSS2018002. Emails: x.mateos-planas@qmul.ac.uk; giulio.seccia@nu.edu.kz

1 Introduction

Motivation: Default on trade credit payments is sizable.[1] Jacobson and von Schedvin (2015) report that trade creditors experience significant losses due to failed payments. Amberg, Jacobson, and von Schedvin (2020) provide evidence that firms charge premia in transactions involving trade credit. Furthermore, trade credit default is a channel of transmission of financial disruptions from firm to firm. Jacobson and von Schedvin (2015) provide evidence since it imposes losses on other firms, and interacts with bankruptcy and liquidation. For the U.S., based on inter-firm credit sales data, Costello (2020) reports a very substantial fraction of receivables that are past due, and establishes the importance of this form of trade credit deterioration in the transmission of liquidity shocks. While this evidence suggests that trade credit default might be consequential at the micro level, not much is known about its actual quantitative importance in the transmission of macroeconomic fluctuations. In effect, trade credit default has been thus far absent in the macroeconomics literature.[2]

What we do/Question: In this paper we set out to investigate the determinants of trade credit default jointly with bankruptcy and liquidation, and its significance for macroeconomic variables. The specific objective is to assess the contribution of trade-credit default to fluctuations in GDP and employment, and its association with firm bankruptcy risk and external financial conditions of firms. Using an equilibrium dynamic model of heterogeneous firms with endogenous credit risk, we find that trade credit default, by distorting the markup charged by intermediate input suppliers, amplifies the impact of aggregate shocks substantially. This is true of shocks to total factor productivity, financial shocks, and volatility shocks, instances of the driving factors considered in the rapidly growing literature on aggregate fluctuations with firms.[3]

Model: We build a quantitative general-equilibrium heterogeneous-firms model which contains the essentials of the relevant mechanism of amplification, including aspects highlighted in the firm-level empirical work referred to at the start. The model consists of four types of agents: a representative intermediate-good supplier that uses labor to produce an intermediate input; heterogeneous final-good producers that use the input from intermediate suppliers to produce a final good; households who act as consumers, shareholders, bond holders and workers; lenders/banks that take deposits from households and lend to final-goods firms. Some proportion of inputs are purchased on within-period trade credit, meaning that these inputs are delivered at the beginning, and due to be paid at the end only after productivity shocks to final-goods firms are realised. The fraction of inputs sold on trade credit will be exogenous in this model.[4] Final-goods firms may also

[1]Trade credit is a significant source of firm short-term finance, probably the most important source of short-term credit in the U.S. (e.g., Petersen and Rajan (1997)).

[2]Including, for instance, the vast body of work emphasising the role of financial frictions in fluctuations since the seminal Bernanke, Gertler, and Gilchrist (1999) and Kiyotaki and Moore (1997b).

[3]Without attempting to be comprehensive, it includes Jermann and Quadrini (2012), Khan and Thomas (2013) Bloom (2009) and Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018), and more recently Khan, Senga, and Thomas (2016), Arellano, Bai, and Kehoe (2019) and Ottonello and Winberry (2020).

[4]Our focus here is on default rather than on the determination of trade credit itself since it poses yet unresolved

hold non-contingent bank debt. Final-goods firms in operation cannot issue new shares so dividends must be non-negative; we also assume non-positive dividends if there is bankruptcy or trade-credit delinquency. Trade credit debt is junior to bank credit debt.

Bankruptcy by a final-goods firm means liquidation of the firm and exit, and implies default on trade credit. A firm can also default on trade credit while honoring their other financial commitments in what we call delinquency. Delinquency implies a loss of future output to the firm over a random number of periods. Final-goods firms are subject to idiosyncratic productivity shocks. These shocks are observed only after inputs are purchased. Because of this timing, final-goods producers might be unable to honor all of their financial obligations (i.e., they become liquidity constrained). The aggregation of default on trade-credit payments results in a trade-credit loss rate that is factored in the intermediate-producer's pricing of inputs. On the other hand, an individual firm's bankruptcy risk is reflected in the lending rates offered by lenders.

Mechanism: This model contains the core macroeconomic interactions between final-goods producers, input producers, and lenders. Trade-credit default, stemming from both delinquency and bankruptcy of some final-goods firms, imposes losses directly on intermediate-input firms which necessarily require a rise in the markup of the price of inputs over the unit labour wage cost. This markup effect calls for some mix of lower wages and higher input price, the former discouraging labour supply and the supply of inputs, and the latter imposing higher operating costs to the other final-goods firms which are conducive to a reduced demand for inputs and hence employment and to further bankruptcies and defaults. Thus, if correlated with the macroeconomic shocks driving fluctuations, trade-credit default may amplify recessions and fuel further bankruptcies. It is to be noted that, behind the scenes, bankruptcy risk is also fostering trade-credit delinquency as it makes bank liquidity constraints more stringent. On the other hand, delinquency may mitigate bankruptcies as it provides an alternative option for firms to deal with tight financial constraints.

Calibration: We provide some analytical characterisation regarding final-goods firm's outcomes within each of the possible options of full repayment, delinquency and bankruptcy. The equilibrium implications will have to be studied numerically. To this end, parameter values are chosen so the model, in the stationary equilibrium, matches a number of targets based on U.S. firm-level Compustat data and Flow of Funds data regarding bankruptcy, firm indebtedness, operating profits, trade credit, and trade-credit default. The calibrated model is able to deliver a good approximation to the data targets. Given the terse structure model and the rich interactions it contains, its ability to match closely observations is quite informative. The calibrated model also does reasonably well regarding some non-targeted variables. For insight into the workings of the model, we report the policy functions, the stationary distribution, and the patterns of defaults that emerge.

Results: To address the central question of this paper, we study the dynamic response of the economy to various types of one-time aggregate shocks. We consider a shift in the

challenges. Trade credit determination is the subject of our own ongoing work Mateos-Planas, Seccia, and Yavuzoglu (2021) but in a partial equilibrium context.

distribution of individual firm productivity, a total factor productivity shock, a financial shock, and a volatility shock. Besides the main case of flexible price adjustment and free entry in the benchmark model, to investigate the question further we also consider a case of the model with rigid wages and number of firms along the transition. The main finding is that there is generally a sizable role of trade-credit default in amplifying the impact response of employment and output, as well as defaults, to those shocks. The mechanism works, as already outlined above, through a widening of the markup in the form of lower wages and labour supply or higher cost of inputs and a lower demand for inputs and labour. For instance, trade credit default accounts for nearly one third of the impact of the volatility shock in the flexible case benchmark, and for the entire fall in aggregates in the rigid case.

Contribution: Most closely related to the present paper are recent analyses of firm bankruptcy risk in the quantitative macroeconomics literature, and in particular Arellano, Bai, and Kehoe (2019), Khan, Senga, and Thomas (2016) or, more recently, Ottonello and Winberry (2020), and Corbae and D'Erasmo (2021). The questions these papers address concern firm bankruptcy and exit, and also the implications of different types of shocks. Our analysis deals with similar variables but, in addition, introduces trade credit and the decision regarding defaulting on this trade credit. Our modelling strategy, while distinct, shares various aspects with the papers cited since it belongs in the same class of general equilibrium models with idiosyncratic shocks and heterogeneous firms. At a more specific level, similarly to Arellano, Bai, and Kehoe (2019) we introduce an information friction whereby choices about inputs are made before the realisation of the firm's productivity shocks. Hence supplying the input is a risky endeavor since idiosyncratic shocks occur between the time of its production and the receipt of payments.[5] The transmission of financial and volatility shocks we analyse contains the mechanisms uncovered in Khan, Senga, and Thomas (2016) and Arellano, Bai, and Kehoe (2019), respectively, but adds the new considerations coming from trade credit default which, incidentally, brings a novel channel for the macroeconomic relevance of bankruptcy studied in these papers. While our model sticks with the simple representation of bankruptcy as liquidation, Corbae and D'Erasmo (2021)'s focus is on the finer elements of the actual bankruptcy code.[6]

A different strand of literature consists of the work on credit chains since Kiyotaki and Moore (1997a) which showed how shocks propagate through chains of firms borrowing and lending to each other.[7] The idea is extended to network structures in Altinoglu (2021) and Reischer (2019). The focus there is on the propagation of individual-firm effects through networks, while the economic and financial changes experienced by firms are themselves exogenous or mechanistic within essentially static economic models.[8] In any event, those works are only vaguely related to our paper since they are not about the determination

[5] Interestingly, Arellano, Bai, and Kehoe (2019) also have to deal with the possibility that due payments may not be feasible ex-post but, unlike us, this is a mere technicality of no practical significance.

[6] Finally, like Arellano, Bai, and Kehoe (2019), but unlike Khan, Senga, and Thomas (2016) and Corbae and D'Erasmo (2021), there is no choice of investment in capital, and our model is best interpreted as one where capital is uniform and constant across firms.

[7] For empirical work on the existence of these links see Boissay (2006) and Boissay and Gropp (2013).

[8] To the best of our understanding, there trade credit is exogenous in the same sense as it is in our model.

trade-credit default and the endogenous responses to aggregate shocks. All said, network effects may well be an important channel complementary to the amplification effects we study, and further work should follow that up.

The next Section 2 sets out the model's main elements and assumptions. Section 3 expresses more formally the decision problems and the conditions of the recursive equilibrium, and presents some useful characterisation results and the key mechanisms. Section 4 calibrates the model and discusses properties of the stationary equilibrium. The main results about the dynamic response to shocks are in Section 5. Finally, Section 6 concludes.

2 Model

The model consists of four types of agents: producers of intermediate inputs, producers of final goods, financial intermediaries, and households. Firms are all competitive and inputs and goods are homogeneous. Final-goods firms face financial constraints on issuance of shares, and experience idiosyncratic shocks. They pay for some inputs with a delay, only after shocks are realised. Since we assume there is no commitment to repayments on this within-period trade credit, these firms may fail to pay suppliers of inputs. Final-goods firms can also borrow from banks, and may declare bankruptcy and liquidate.

In this section we introduce notation and describe the main elements of the model. We will wait until the next section to set out the formal details and equilibrium conditions.

2.1 Input producers

There is a continuum of intermediate producers with mass 1. The intermediate input x is produced from labour n on a one-for-one basis:

$$x = n.$$

The price of the input is p. Payments from retail customers are received with a delay within the period. A given proportion of sales τ is on trade credit and a proportion of those, given by the trade-credit default, or loss, rate θ, will not receive payment. The remaining fraction of sales $1 - \tau$ are on cash and receive payment for sure. A retail firm may liquidate but still carry out production in the current period, in which case cash sales always receive payment and there may be some recovery. If the firm liquidates and stops operation in that period, the cost of cash input purchases will be paid by the shareholders. We are assuming that the representative intermediate input producer cannot see the individual firms' types and takes as given the aggregate trade-credit default, or loss, rate θ that pools the default risk across all the individual retailers.

The cash-flow to the representative intermediate input firm includes the costs of labour at the wage rate w, and it becomes $px - \theta\tau px - wx$. In this description, trade credit does not command an explicit interest rate or discount price (as it would be if the firm could issue financial contracts).

There is free entry in the input producing sector. The rate of default on trade credit sales θ is forecast at entry. Free entry thus implies zero profits given the information available at the beginning of the period:

$$px - \theta\tau px - wx = 0.$$

The markup of the input price over the wage rate, w/p, is thus determined by θ, and will provide the main channel for the effect of trade-credit default on macroeconomic variables.

2.2 Final-goods producers

Output from a final-goods firm, y, depends on aggregate productivity z, idiosyncratic productivity ϵ, and the amount purchased of intermediate input x as follows:

$$y = ze^{\epsilon}F(x).$$

The idiosyncratic productivity is stochastic and follows a Markov chain with transition probabilities $\psi_{\epsilon}(\epsilon'|\epsilon)$ over a support $\mathcal{E}$.

There is free entry of new firms and an entry cost ξ^E which must be paid by issuing shares. After paying this cost, the firm draws a realisation of the initial idiosyncratic shock ϵ_{-1} from an initial distribution given by its stationary distribution $\overline{\psi}_{\epsilon}(\epsilon_{-1})$.

There is a fixed cost of operating the firm c_F in every period. This cost may include, for example, the replacement of capital.

In any given period, the firm chooses the amount of intermediate input x before the realisation of the shocks. After the shocks are observed, the firm can issue one-period debt b at a discount price q, decide whether to repay input suppliers by choosing $d^x \in \{0, 1\}$, and whether to declare bankruptcy on bank debt, $d^b \in \{0, 1\}$. Delinquency on input payments $d^x = 1$ implies a loss of a proportion of output $\tilde{\nu} > 0$ in future periods. The delinquency indicator is $\nu \in \{0, \tilde{\nu}\}$. If $\nu > 0$ the probability of forgiveness and having the penalty cleared, conditional on not incurring further delinquency, is λ. Bankruptcy $d^b = 1$ leads to liquidation and exit. Under bankruptcy or delinquency, claimants (i.e., creditors or trade-credit suppliers) receive the residual value of the firm; r^b goes to banks and r^x goes to trade-credit suppliers. Because of the fixed cost c_F and the cash input payments $(1 - \tau)px$, the residual value may be negative in which case the firm does not pay the fixed cost and fails to operate but still pays the cash inputs.

The firm maximises the expected discounted value of dividends. The discount rate is ρ, which will be determined in equilibrium. A firm faces the constraint that it cannot issue new shares so dividends cannot be negative during the life of an operating firm.[9] However

[9] Here we follow Arellano, Bai, and Kehoe (2019) and Khan, Senga, and Thomas (2016).

negative dividends may occur for firms that are liquidating and, additionally, fail to operate since these firms must still bear the cost of cash inputs. A firm also faces a non-positive dividends constraint when choosing to liquidate or become delinquent. Finally, debt has seniority over trade credit sales. Hence we have the following financing constraints:

Assumption 1: Firms cannot pay negative dividends, except when liquidating and generating zero current revenues.

Assumption 2: Firms cannot pay positive dividends when defaulting or becoming delinquent. (Therefore claimants receive the residual.)

Assumption 3: Bank debt is senior to trade credit liabilities.

2.3 Lenders

Lenders issue one-period loans to final goods firms. They have the same information that is available to firms, so there is a contract for each type of loan in terms of size and characteristics of the firm. Competition drives the surplus for lenders on all loan types to zero. In this way, the discount prices of debt q reflects the default risk— implied by firms' decision d^b and recovery r^b — over the market stochastic discount rate ρ. These lenders fund their loans by selling securities to households.

2.4 Households

Households own the firms and the labour force. There is a representative household who can borrow and lend freely at the discount rate Q. The stochastic subjective discount rate will determine the firms' and banks' discount rate, ρ. Households supply labour to input producers and consume optimally.

3 Recursive equilibrium

The aggregate state at the beginning of a period, S, includes the distribution $N \times \mu$ of firms over characteristics (ϵ_{-1}, b, ν) at the start of the period, and the claims held by the representative household, A:

$$S = (N \times \mu, A).$$

This distribution of firms consists of a probability measure μ scaled by the mass of firms N. More formally, μ belongs in the set of probability measures over a measurable space consisting of the set of elements of the individual final-goods firm's state, and the product of the corresponding Borel algebras.

The input price and wage rate can therefore be written as $p(S)$ and $w(S)$. The final-goods firm's individual state before shocks are realised consists of (ϵ_{-1}, b, ν). After shocks are realised, in the second part of the period, the individual state becomes (ϵ, b, ν, x), which includes the level of input x chosen in the first part of the period. The price of claims can be written $Q(S)$, and the discount rate of the firm can be written $\rho(S)$. The discount price of bank debt is a function $q^{ND}(b', \epsilon, \nu|S)$ if there is currently no delinquency, and $q^{x}(b', \epsilon|S)$ if there is delinquency.

The law of motion for the aggregate state is $S' = H(S)$, with its two components denoted $N' \times \mu' = H^{\mu}(S)$ and $A' = H^{A}(S)$. As a fraction of N, the level of entry of new firms can be written $m(S)$.

An equilibrium satisfies a set of conditions: Decision rules for borrowing and repayments maximise final-goods firms' objective given debt prices and input price; the price of inputs is such that intermediate producers make zero profits given aggregate trade-credit loss and wages; the wage and mass of final-goods firms is such that labour market clears and there is free entry; prices of loans satisfy zero profit for lenders given the decision rules of final-goods firms; exit flows reflect optimal decisions; households choose consumption and supply labour optimally; the distribution of firm's types and their is consistent with the above.

In the rest of this section, we spell out the elements necessary to make this definition operational using the definitions and notation introduced so far here and in Section 2. Section A.1 in the Appendix contains the formal statement.

3.1 Final-goods firms

We describe the decision problem of existing firms, the conditions for entry, the motion of the distribution, and the determination of the aggregate trade-credit loss, which is the variable focus of this paper, and some other macro aggregates.

3.1.1 Decision problem

There are two stages to the firm's problem. Denote by $V(\epsilon, b, \nu, x|S)$ the value function in the second stage, after the realisation of the shock, and by $W(\epsilon_{-1}, b, \nu|S)$ the value in the first stage before the current shock are observed.

In the second stage, the decision needs to evaluate the value from 3 different courses of action: $V^{ND}(\epsilon, b, \nu, x|S)$ if both inputs and debts receive payment; $V^{x}(\epsilon, b, \nu, x|S)$ if debt receives payment but trade-credit inputs do not; $V^{b}(\epsilon, b, \nu, x|S)$ if neither trade-credit inputs nor debts receive payment. The value functions and outcomes in each case are determined as follows.

Repayment. When honoring all payments, the firm chooses borrowing b' to solve

$$V^{ND}(\epsilon, b, \nu, x|S) = \max_{b'}\{(1-\nu)ze^{\epsilon}F(x) - c_F - p(S)x + q^{ND}(b', \epsilon, \nu|S)b' - b + \\ \rho(S)[\mathcal{I}_{\nu>0}(\lambda W(\epsilon, b', \nu' = 0|S') + (1-\lambda)W(\epsilon, b', \nu' = \tilde{\nu}|S')) + \mathcal{I}_{\nu=0}W(\epsilon, b', \nu' = 0|S')]\}, \quad (1)$$

with $S' = H(S)$, which gives as solution $b' = g^{ND}(\epsilon, b, \nu, x|S)$, and the value of dividends

$$\pi^{ND}(\epsilon, b, \nu, x|S) = (1-\nu)ze^{\epsilon}F(x) - c_F - p(S)x + \\ q^{ND}(g^{ND}(\epsilon, b, \nu, x|S), \epsilon, \nu|S)g^{ND}(\epsilon, b, \nu, x|S, z) - b. \quad (2)$$

Delinquency. When repudiating payments for trade-credit input supplies, the firm determines borrowing b' and the supplier's recovery r^x according to

$$V^x(\epsilon, b, \nu, x|S) = \max_{b', r^x \geq 0} \Big\{(1-\nu)ze^{\epsilon}F(x) - c_F - (1-\tau)p(S)x - b + q^x(b', \epsilon|S)b' - r^x \\ + \rho(S)W(\epsilon, b', \nu' = \tilde{\nu}|S')\Big\}, \quad (3)$$

subject to dividends being zero by Assumptions 1 and 2 so $(1-\nu)ze^{\epsilon}F(x) - c_F - (1-\tau)p(S)x - b + q^x(b', \epsilon|S)b' - r^x = 0$. The solution gives borrowing $b' = g^x(\epsilon, b, \nu, x|S)$, and the residual recovered $r^x = r^x(\epsilon, b, \nu, x|S)$.

Liquidation. Finally, declaring bankruptcy means liquidation. There are two possible situations. The first is when cash-in-hand, $(1-\nu)ze^{\epsilon}F(x) - c_F - (1-\tau)px$, is non-negative, in which case the value of the firm is zero and the residual is the recovered value by debt creditors r^b. The second case is when cash-in-hand is negative, meaning that the firm's output would not cover the fixed cost and the cost of cash inputs, and the firm must therefore cease production altogether, with no recovery, while the firm must still cover the incurred cost on the cash inputs. We represent the firm's failure to produce by the indicator $d^f(\epsilon, b, \nu, x|S) = 1$. When, otherwise, cash in hand is positive $d^f(\epsilon, b, \nu, x|S) = 0$. In sum

$$V^b(\epsilon, b, \nu, x|S) = \begin{cases} 0 & \text{if } (1-\nu)ze^{\epsilon}F(x) - c_F - (1-\tau)p(S)x \geq 0 \text{ (i.e,} d^f(\epsilon, b, \nu, x|S) = 0) \\ -(1-\tau)p(S)x & \text{otherwise (i.e,} d^f(\epsilon, b, \nu, x|S) = 1) \end{cases} \quad (4)$$

This results in the residual value

$$r^b(\epsilon, b, \nu, x|S) = \max\{(1-\nu)ze^{\epsilon}F(x) - c_F - (1-\tau)p(S)x, 0\}$$

being recovered by bank debt creditors, while trade-credit suppliers recover nothing, an implication of the assumed seniority of bank debt. Dividends in this case π^b coincide with the value of the firm

$$\pi^b(\epsilon, b, \nu, x|S) = V^b(\epsilon, b, \nu, x|S) \quad (5)$$

and can therefore be negative when the firm fails to operate in the period of liquidation.

Naturally, the option of liquidation and exit is open also to firms without debts, that is with $b = 0$, but who cannot meet the fixed cost and cash input payments.

Repayment choice. The optimal choice among the three options above solves

$$V(\epsilon, b, \nu, x|S) = \max\left\{V^{ND}(\epsilon, b, \nu, x|S), V^{x}(\epsilon, b, \nu, x|S), V^{b}(\epsilon, b, \nu, x|S,)\right\} \tag{6}$$

and gives decision rules $d^x(\epsilon, b, \nu, x|S)$ and $d^b(\epsilon, b, \nu, x|S)$.

The demand for input. We now turn to the first stage within the period, before the realisation of the shocks. There the optimal choice of input solves

$$W(\epsilon_{-1}, b, \nu|S) = \max_{x} \sum_{\epsilon} \psi_{\epsilon}(\epsilon|\epsilon_{-1}) V(\epsilon, b, \nu, x|S), \tag{7}$$

yielding decision $x = x(\epsilon_{-1}, b, \nu|S)$.

3.1.2 Entry

The value of a new entrant $W^E(S)$ is the expectation of $W(\cdot)$ over the unconditional distribution on the starting ϵ_{-1} and b, μ^E:

$$W^E(S) = \int W(\epsilon_{-1}, b, \nu = 0|S) \mathrm{d}\mu^E(\epsilon_{-1}, b). \tag{8}$$

Here the probability distribution of entrants is

$$\mu^E(\epsilon_{-1}, b) = \begin{cases} \overline{\psi}_{\epsilon}(\epsilon_{-1}) & \text{for } b = 0 \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

reflecting the assumption of zero initial debt financing.[10]

The free-entry condition is

$$W^E(S) \leq \xi^E, \tag{10}$$

with strict inequality only when there is zero entry $m(S) = 0$.

3.1.3 Distribution

The probability measure μ is defined over the ex-ante firm types (ϵ_{-1}, b, ν). It evolves according to $N' \times \mu' = H^{\mu}(S)$ where, as defined earlier, $S = (N \times \mu, A)$. We define the transition probabilities for existing firms Prob$(\epsilon, \mathcal{B}', \nu'; \epsilon_{-1}, b, \nu \mid S)$, where $\mathcal{B}$ is a set containing ele-

[10] Ottonello and Winberry (2020), for instance, also make this assumption. The fraction of the entry cost financed by debt could be made positive. In this case, the debt issued to cover the fraction of the entry cost $\alpha^E \xi^E$ is given by $b = b^E : q^E(b^E) = \alpha^E \xi^E$. Since the productivity type is unknown before entry, competitive lenders price this debt by pooling across types according to the function $q^E(b')$ such that $q^E(b) = \sum_{\epsilon_{-1}} \overline{\psi}_{\epsilon}(\epsilon_{-1}) q^{ND}(b, \epsilon_{-1}, \nu = 0)$. Here we have decided to keep $\alpha^E = 0$ for simplicity.

ments b, and for entrants $\text{Prob}^E(\epsilon, \mathcal{B}', \nu'; \epsilon_{-1}, b \mid S)$. The transition probabilities are given by the firms' optimal decisions and the process for the delinquency flag. For existing firms

$$\text{Prob}(\epsilon, \mathcal{B}', \nu'; \epsilon_{-1}, b, \nu \mid S) = \begin{cases} \psi_\epsilon(\epsilon \mid \epsilon_{-1}) & \text{if } \; g^{ND}(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S) \in \mathcal{B}' \\ & \quad\; d^b(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S) = 0 \\ & \quad\; d^x(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S) = 0 \\ & \quad\; \nu' = 0, \;\; \nu = 0 \\ \lambda\psi_\epsilon(\epsilon \mid \epsilon_{-1}) & \text{if } \; g^{ND}(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S) \in \mathcal{B}' \\ & \quad\; d^b(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S) = 0 \\ & \quad\; d^x(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S) = 0 \\ & \quad\; \nu' = 0, \;\; \nu = \tilde{\nu} \\ (1-\lambda)\psi_\epsilon(\epsilon \mid \epsilon_{-1}) & \text{if } \; g^{ND}(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S) \in \mathcal{B}' \\ & \quad\; d^b(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S) = 0 \\ & \quad\; d^x(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S) = 0 \\ & \quad\; \nu' = \tilde{\nu}, \;\; \nu = \tilde{\nu} \\ \psi_\epsilon(\epsilon \mid \epsilon_{-1}) & \text{if } \; g^x(\epsilon, b, x(\epsilon_{-1}, b, \nu \mid S) \mid S) \in \mathcal{B}' \\ & \quad\; d^b(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S) = 0 \\ & \quad\; d^x(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S) = 1 \\ & \quad\; \nu' = \tilde{\nu} \\ 0 & \quad\;\; \text{otherwise} \end{cases} \tag{11}$$

For new entrants, for whom $\nu = 0$,

$$\text{Prob}^E(\epsilon, \mathcal{B}', \nu'; \epsilon_{-1}, b \mid S) = \begin{cases} \psi_\epsilon(\epsilon \mid \epsilon_{-1}) & \text{if } \; g^{ND}(\epsilon, b, 0, x(\epsilon_{-1}, b, 0 \mid S) \mid S) \in \mathcal{B}' \\ & \quad\; d^b(\epsilon, b, 0, x(\epsilon_{-1}, b, 0 \mid S) \mid S) = 0 \\ & \quad\; d^x(\epsilon, b, 0, x(\epsilon_{-1}, b, 0 \mid S) \mid S) = 0 \\ & \quad\; \nu' = 0 \\ \psi_\epsilon(\epsilon \mid \epsilon_{-1}) & \text{if } \; g^x(\epsilon, b, x(\epsilon_{-1}, b, 0 \mid S) \mid S) \in \mathcal{B}' \\ & \quad\; d^b(\epsilon, b, 0, x(\epsilon_{-1}, b, 0 \mid S) \mid S) = 0 \\ & \quad\; d^x(\epsilon, b, 0, x(\epsilon_{-1}, b, 0 \mid S) \mid S) = 1 \\ & \quad\; \nu' = \tilde{\nu} \\ 0 & \quad\;\; \text{otherwise} \end{cases} \tag{12}$$

The motion for the mass of existing firms counts in the mass of current firms surviving into next period, thereby

$$\begin{aligned} N' = N \times \int \sum_{\epsilon, \nu'} \text{Prob}(\epsilon, R^+, \nu'; \epsilon_{-1}, b, \nu \mid S) \mathrm{d}\mu(\epsilon_{-1}, b, \nu) \\ + Nm(S) \int \sum_{\epsilon, \nu'} \text{Prob}^E(\epsilon, R^+, \nu'; \epsilon_{-1}, b \mid S) \mathrm{d}\mu^E(\epsilon_{-1}, b) \end{aligned} \tag{13}$$

The transition function is

$$H^{\mu}(\epsilon, \mathcal{B}', \nu' \mid S) \equiv N \times \int \text{Prob}(\epsilon, \mathcal{B}', \nu'; \epsilon_{-1}, b, \nu, \mid S) \text{d}\mu(\epsilon_{-1}, b, \nu) + Nm(S) \int \text{Prob}^{E}(\epsilon, \mathcal{B}', \nu'; \epsilon_{-1}, b \mid S) \text{d}\mu^{E}(\epsilon_{-1}, b), \quad (14)$$

where $\mu^{E}(\epsilon_{-1}, b)$ is the probability distribution of productivity and debt for new entrants.

Now, since the distribution evolves as $N' \times \mu' = H^{\mu}(S)$, the probability measure follows

$$\mu' = H^{\mu}(S)/N'. \quad (15)$$

For calculating outcomes affected by new firms, it will be convenient to define the post-entry probability distribution over firm's types as $\hat{\mu}$. It accounts for the proportion $m(S)$ of new firms entering the market relative to the mass of firms N, as well as the firms in the probability measure μ already existing at the start of the period. Therefore $(N+Nm(S))\hat{\mu}$ is the total scaled measure of firms. Given $m(S)$, N and μ and μ^{E}, the post-entry probability measure obtains as

$$\hat{\mu}(\epsilon_{-1}, b, \nu) = \frac{\mu(\epsilon_{-1}, b, \nu) \times N + \mu^{E}(\epsilon_{-1}, b)\mathcal{I}_{\nu=0} \times Nm(S)}{N + Nm(S)}. \quad (16)$$

3.1.4 Trade-credit loss rate and other aggregates

Here we specify endogenous variables that obtain from aggregating individual firms' decision rules.

Producers of intermediate inputs take as given the expected aggregate default rate on trade credit, or lost fraction of sales on trade credit, θ given the initial state S. The trade-credit loss rate results from aggregating up the individual firm's delinquency decisions $d^{x}(\epsilon, b, \nu, x|S)$ and bankruptcy decisions $d^{b}(\epsilon, b, \nu, x|S)$, given that their choice of inputs is determined by $x = x(\epsilon_{-1}, b, \nu|S)$. The loss rate also depends on the recovery from the delinquent firms' cash in hand left after repaying debts and cash inputs, which we have defined as $r^{x}(\epsilon, b, x|S, z)$. Specifically,

$$\theta(S) = \Big[\int \sum_{\epsilon} \psi_{\epsilon}(\epsilon \mid \epsilon_{-1}) \Big(d^{x}(\cdot)(\tau p(S)x(\cdot) - r^{x}(\cdot)) + d^{b}(\cdot)\tau p(S)x(\cdot) \Big) \hat{\mu}(\text{d}\epsilon_{-1} \times \text{d}b \times \text{d}\nu) \Big] \Big/ \Big[\int \sum_{\epsilon} \psi_{\epsilon}(\epsilon \mid \epsilon_{-1}) \tau p(S)x(\cdot)\hat{\mu}(\text{d}\epsilon_{-1} \times \text{d}b \times \text{d}\nu) \Big] \quad (17)$$

where, for convenience, we are using the shorthand notation $r^{x}(\cdot) \equiv r^{x}(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S)|S)$, $d^{x}(\cdot) \equiv d^{x}(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S)|S)$, $d^{b}(\cdot) \equiv d^{b}(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S)|S)$, and $x(\cdot) \equiv x(\epsilon_{-1}, b, \nu|S)$.

Given the firms' dividend policies from (2) and (5), the aggregate dividend received by the household/shareholder is given by

$$\Pi(S) = \sum_{\epsilon} \psi_\epsilon(\epsilon \mid \epsilon_{-1}) \Big[\int \mathcal{I}_{d^b(\dots)=0, d^x(\dots)=0} \pi^{ND}(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S) \mathrm{d}\hat{\mu}(\epsilon_{-1}, b, \nu) + \int \mathcal{I}_{d^b(\dots)=1} \pi^{d}(\epsilon, b, x(\epsilon_{-1}, b, \nu \mid S) \mid S) \mathrm{d}\hat{\mu}(\epsilon_{-1}, b, \nu) \Big] (N + m(S)N) - \xi^E m(S) N \quad (18)$$

3.2 Lenders and debt prices

Lenders use firm's decision rules and shock transition probabilities to make projections about the probability of default. They also take into account that they recover the residual value of the firm in case of default.

The price of debt when there is no delinquency today can be written

$$q^{ND}(b', \epsilon, \nu|S) = Q(S)(1 - \Lambda^{ND}(b', \epsilon, \nu|S')), \quad (19)$$

where the components in S' are given by $N' \times \mu' = H^{\mu}(S)$ and $A' = H^A(S)$, and $\Lambda^{ND}(\cdot)$ denotes the forecast of default losses or expected default for a given future aggregate state, which depends on the default decision $d^b(\cdot)$ and the recovery $r^b(\cdot)$ expressed as a rate. We define this expected recovery rate as

$$rec^b(\epsilon', b', \nu' \mid S') \equiv \frac{r^b(\epsilon', b', \nu', x(\epsilon, b', \nu'|S') \mid S')}{b'}.$$

Therefore the expected default $\Lambda^{ND}(\cdot)$ in Eq. (19) can be written

$$\Lambda^{ND}(b', \epsilon, \nu|S') \equiv \mathcal{I}_{\nu>0} \sum_{\epsilon'} \psi_\epsilon(\epsilon'|\epsilon) \Big\{ (1-\lambda) d^b(\epsilon', b', \tilde{\nu}, x(\epsilon, b', \tilde{\nu}|S')|S')(1 - rec^b(\epsilon', b', \tilde{\nu} \mid S')) + \lambda d^b(\epsilon', b', 0, x(\epsilon, b', 0|S')|S')(1 - rec^b(\epsilon', b', 0 \mid S')) \Big\} + \mathcal{I}_{\nu=0} \sum_{\epsilon'} \psi_\epsilon(\epsilon'|\epsilon) \Big\{ d^b(\epsilon', b', 0, x(\epsilon, b', 0|S')|S')(1 - rec^b(\epsilon', b', 0 \mid S')) \Big\}$$

The price of debt when there is delinquency today can be written

$$q^x(b', \epsilon|S) = Q(S)(1 - \Lambda^x(b', \epsilon|S')), \quad (20)$$

where $S' = H(S)$ and Λ^x is the conditional expected default loss

$$\Lambda^x(b', \epsilon|S') = \sum_{\epsilon'} \psi_\epsilon(\epsilon'|\epsilon) \Big\{ d^b(\epsilon', b', \tilde{\nu}, x(\epsilon, b', \tilde{\nu}|S')|S')(1 - rec^b(\epsilon', b', \tilde{\nu} \mid S')) \Big\}$$

with the components in S' again given by $S' = H(S)$.

3.3 Input producers

Given $\theta(S)$, $\eta(S)$, and $w(S)$, zero profits in the production of inputs means that the ratio of price of inputs to the wage rate satisfies

$$\frac{p(S)}{w(S)} = \frac{1}{1-\theta(S)\tau}. \tag{21}$$

This markup depends positively on the trade-credit default loss rate θ. This will be a key relationship for the macroeconomic role of trade-credit default.

3.4 Consumers

At the beginning of a period, the state for the representative consumer is (a, S), where $S = (z_{-1}, N \times \mu, A)$, and a is the individual's risk-free asset.[11] Utility within a period is a function of consumption c and hours worked l, $u(c, l)$. The savings decision $a'(a \mid S)$ and labour supply $l(a \mid S)$ solve

$$U(a \mid S) = \max_{\{a',l\}} \{u(c, l) + \beta U(a' \mid S')\} \tag{22}$$

subject to

$$c + Q(S)a' = w(S)l + a + \Pi(S),$$

where the components of S' obey $S' = H(S, z)$. The consumption decision $c(a \mid S, z)$ then follows easily.

As standard, the first-order condition for the saving decision implies

$$u_c(c, l)Q(S) = \beta u_c(c(a' \mid S'), l(a' \mid S')),$$

and for labour supply

$$u_c(c(a \mid S), l)w(S) + u_l(c(a \mid S), l) = 0.$$

Firms discount future values expected before the realisation of future shocks. The appropriate rate is given by the subjective discount rate based on a risk-free portfolio. From the consumption first-order condition, this means

$$\rho(S) = Q(S) \tag{23}$$

[11] Even with aggregate shocks, contingent securities play no role here and we can think of a single bond. In Arellano, Bai, and Kehoe (2019) contingent securities are used in the context of a small open economy to provide full consumption insurance.

3.5 Aggregation and market clearing

Aggregate consistency requires individual assets coincide with the aggregate:

$$a = A. \tag{24}$$

Given the household's policy functions $a'(a \mid S, z)$ from (22), the transition function for the aggregate portfolio A' is

$$H^A(S) = a'(A \mid S). \tag{25}$$

Similarly, aggregate labour supply and consumption are given by

$$L(S) = l(A \mid S) \text{ and } C(S) = c(a \mid S) \tag{26}$$

Equilibrium requires clearing in the market for labour, final output and assets. By Walras' Law, we only need to consider the first two. Clearing in the labour market means[12]

$$L(S) = N \times \int x(\epsilon_{-1}, b, \nu, \mid S) \mathrm{d}\mu(\epsilon_{-1}, b, \nu) + Nm(S) \int x(\epsilon_{-1}, b, \nu = 0 \mid S) \mathrm{d}\mu^E(\epsilon_{-1}, b). \tag{27}$$

For final goods, the condition is

$$\begin{aligned} C(S) + \xi^E Nm(S) &= (N + Nm(S)) \\ &\times \sum_{\epsilon} \left[\int \psi_\epsilon(\epsilon \mid \epsilon_{-1})((1-\nu)z\epsilon F(x(\epsilon_{-1}, b, \nu|S)) - c_F)(1 - d^d(\cdot)d^f(\cdot))\mathrm{d}\hat{\mu}(\epsilon_{-1}, b, \nu) \right] \end{aligned} \tag{28}$$

where $d^f(\cdot) \equiv d^f(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S))$ and $d^d(\cdot) \equiv d^d(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S))$. The two terms on the right correspond to existing and new entrants, respectively. The possibility of firm failure is captured by the failure indicator $d^f(\cdot) \equiv d^f(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S))$ which is 1 when the firm declares bankruptcy and cannot cover the fixed cost and payments for cash inputs.

3.6 Characterisation

Here we discuss some properties of the model, outline aspects of the computation of the stationary equilibrium, and highlight the nature of key interactions between variables of the model.

3.6.1 Final-goods firms' decisions

The outcomes for final-goods firms under each of the three end-of-period options of repaying, delinquency and liquidation laid out in Section 3.1.1 can be described simply. These

[12] We could also write it in terms of $\hat{\mu}$, that is $l(S) = (N + Nm) \int x(.)\mathrm{d}\hat{\mu}(.)$, but the present renders more clearly the role of entry $m(S)$ in market clearing.

results will also be helpful in informing some aspects of the computation. This characterisation relies on the fact that a firm will borrow at most as much as needed to meet its financial commitments, not more. Borrowing to pay dividends is not optimal.[13]

In this model, the price of debt and therefore the resources raised via borrowing, may be discontinuous in the level of debt chosen. The reason is that at the level of debt where the firm may become delinquent at the end of the period with some positive probability, the marginal cost of hiring inputs drops and the firm's chosen amount of x may jump. The probability of bankruptcy may drop since operating profits will increase, and the price of debt would also drop at that level of debt as a consequence. The price of debt might jump however if the cost of delinquency were large enough. So the price of debt is in general discontinuous with an indeterminate sign. One consequence is that the firm's choice of debt b' may raise more resources than necessary to meet its liquidity needs, and this residual has to be apportioned to creditors accordingly. The characterisation that follows takes this possible discontinuity into account.[14]

Consider first the case that the firm chooses to fully repay creditors and suppliers. In this case, dividends are positive if and only if cash in hand is positive except for the possible discontinuity in the price. (In the other two cases dividends will be zero by A1 and A2.) Borrowing is positive only if cash in hand is negative.

Proposition 1 *(No-liquidation no-delinquency case) Suppose the firm chooses option ND in state $(\epsilon, b, \nu, x|S)$.*

1. *If cash in hand is positive or zero, that is $(1-\nu)ze^{\epsilon}F(x) - c_F - px - b \geq 0$, then borrowing is zero $g^{ND}(\epsilon, b, \nu, x|S) = 0$ and dividends $\pi^{ND}(\epsilon, b, \nu, x|S)$ are positive and equal to the value of the cash in hand.*

2. *If cash in hand is negative, that is $(1-\nu)z\epsilon F(x) - c_F - px - b < 0$, and borrowing cannot meet this gap*
$$\max_{b'} b' q^{ND}(b', \epsilon, \nu|S) < px + b - (1-\nu)ze^{\epsilon}F(x) + c_F,$$
the choice set is empty (in practice, $V^{ND} = -\infty$).

3. *Otherwise, if cash in hand is negative, that is $(1-\nu)z\epsilon F(x) - c_F - px - b < 0$, then borrowing $g^{ND}(\epsilon, b, \nu, x|S)$ is positive and given by the smallest value of b' such that*
$$b' q^{ND}(b', \epsilon, \nu|S) \geq px + b - (1-\nu)ze^{\epsilon}F(x) + c_F,$$
and dividends $\pi^{ND}(\epsilon, b, \nu, x|S) = b' q^{ND}(b', \epsilon, \nu|S) - (px + b - (1-\nu)ze^{\epsilon}F(x) + c_F)$ if this amount is positive, or zero zero otherwise.

[13]Strictly speaking, the firm is indifferent, and any arbitrarily small cost of borrowing will render this operation suboptimal. This fact would be clear in the absence of default risk as the firm's discount rate ρ coincides with risk-free discount price of debt Q. The result carries over to the present case with default risk as the net current gain to borrowing and failing to repay in some states is, under full information, offset by the risk-based pricing of debt.

[14]Our computational approach, as explained below, will dispense with the existence of such discontinuities

Consider now the option of becoming delinquent, without bankruptcy. The dividend paid is zero. If cash in hand is negative, then borrowing is positive and the residual repaid is zero; otherwise, borrowing is zero and there is a positive residual repayment so delinquency is partial.

Proposition 2 *(Delinquency no-liquidation case) Suppose the firm chooses option x in state $(\epsilon, b, \nu, x|S)$. Dividend $\pi^x(\epsilon, b, \nu, x|S, z)$ is zero.*

1. *If cash in hand is positive or zero, that is $(1-\nu)ze^{\epsilon}F(x) - b - c_F - (1-\tau)px \geq 0$, then borrowing is zero $g^x(\epsilon, b, \nu, x|S) = 0$ and the residual $r^x(\epsilon, b, \nu, x|S)$ is positive and equal to the value of the cash in hand.*

2. *If cash in hand is negative, that is $(1-\nu)ze^{\epsilon}F(x) - b - c_F - (1-\tau)px < 0$, and borrowing cannot meet this gap*

$$\max_{b'} b'q^x(b', \epsilon, \nu|S) < b - (1-\nu)z\epsilon F(x) + c_F + (1-\tau)px,$$

   the choice set is empty (in practice, $V^x = -\infty$).

3. *Otherwise, if cash in hand is negative, that is $(1-\nu)ze^{\epsilon}F(x) - b - c_F - (1-\tau)px < 0$, then borrowing $g^x(\epsilon, b, \nu, x|S)$ is positive and given by the smallest value b' such that*

$$b'q^x(b', \epsilon, \nu|S) \geq b - (1-\nu)ze^{\epsilon}F(x) + c_F + (1-\tau)px,$$

   and the residual is , $r^x(\epsilon, b, \nu, x|S) = b'q^x(b', \epsilon, \nu|S) - (b - (1-\nu)ze^{\epsilon}F(x) + c_F + (1-\tau)px)$ if this value is positive, and the residual is zero otherwise.

The outcomes under the third option of liquidation were already fully characterised in Section 3.1.1.

3.6.2 Stationary equilibrium

Before studying the dynamics, as a baseline for calibration we will consider a stationary equilibrium which is one where the aggregate state S is constant over time. Therefore, there the endogenous equilibrium functions will be constant functions, and prices and discount rates scalar numbers p, w, Q and ρ, and so will the quantities m, N, l, c, and θ. Note that, with positive liquidation, in this stationary equilibrium it must be the case that entry is strictly positive so $m > 0$.

The algorithm to find the stationary equilibrium exploits a form of block recursivity of this equilibrium. The input price p can be found iteratively as the one that solves the final-goods firm's free-entry condition by using the equilibrium functions for final-goods firms and lenders. Note that these limited set of conditions suffice to fully determine p

though.

because strictly positive entry m means that the zero-profit condition for final-goods firms binds. Given p, then a second block determines aggregates including the distribution, trade-credit loss, and the wage via the input producer's pricing conditions, as well as firm entry, the mass of firms, and labour supply to meet market clearing. Details are in Appendix A.2.

The computation of this model- particularly the joint determination of final-good firms' choices and lenders' debt pricing- presents some practical challenges related to the potential discontinuities already mentioned and to the inherent non-convexities introduced by the binary repayment outcomes. To overcome issues of accuracy and convergence in the iterations, for computational purposes we introduce type-I extreme-value shocks affecting the discrete choices of delinquency and liquidation, and also the level of a firm's demand for input.[15] For good reason, this approach turns out to dispense with the complications noted. Appendix A.3 provides more details.

3.6.3 Economic mechanisms

Consider first the determination of the stationary equilibrium. Consistent with the algorithm outlined above, the input price p mainly adjusts to satisfy the zero-profit entry condition (10), since p determines the profitability of final-goods firms. The flow rate of entrants m helps meet labour market clearing (27) as the mass of firms affects the aggregate level of demand for intermediate inputs x. The wage rate w meets the pricing condition of input producers (21). Trade-credit default θ, determined by firms payment decisions as in (17), therefore impacts the wage-to-input-price markup p/w, calling for lower wage or more expensive inputs. Whereas in the baseline stationary equilibrium θ determines directly w, and only indirectly p, more generally, including situations where temporarily entry may cease or there are wage rigidities, trade-credit default will have a direct impact on the price of inputs p too.

The main focus of this paper is actually on the contribution of trade-credit default to the transmission of fluctuations outside the steady state. Although we have not studied the dynamics of the model yet, it is already possible to outline the mechanisms at work on the basis of the elements of the model presented thus far. The main idea is that an adverse shock in this model can have a direct impact on the estimated loss to trade credit, represented by θ. According to (17), this will mainly come through the bankruptcy and delinquency policy rules, or the distribution of firm types. Note these defaulting rules will be jointly determined with the equilibrium debt prices as they represent the extent of credit constraints coming from bankruptcy risk.

Whatever the specific details, the key macro mechanism is the rise in θ which calls for an increase in the mark up w/p via the equilibrium pricing condition of input producers in Eq. (21). This adjustment can in principle happen through various combinations of increases in the input price p or reductions in the wage w. The former means that final-

[15]This device is becoming increasingly common, especially in related models of sovereign debt.

goods firms face more expensive inputs and will respond by reducing their demand for inputs x, downsizing their production plans; the latter would reduce the supply of labour supply and of inputs, calling for reduced entry of firms. Either way, this will have detrimental consequences for GDP and employment. Which one is more prevalent, the wage or the input price channel, will depend on the specific nature of the shock and, particularly, the extent of short run frictions in the economy.

This model also contains the elements for a feedback effect of the type emphasised in the empirical literature cited earlier in Section 1 whereby increased trade-credit default leads into further defaults and their macroeconomic consequences. Specifically, more expensive production inputs following the rise in trade-credit losses θ may be conducive to more bankruptcies and delinquencies, which will in turn be reflected in tighter credit conditions and costlier inputs, and increased trade-credit losses, and so on.

4 Quantitative benchmark model

In this section we build and analyse the stationary equilibrium under specific functional forms and parameter values. Numerical values for the parameters will be chosen so that the model delivers realistic levels of trade credit and default, among other observable target variables. We will consider properties of this model in terms of targeted and some non-targeted aggregate variables, and the shapes of the distribution and equilibrium functions for some insight into the workings of the model.

4.1 Specification

We specify the technology for final-goods firms as the concave production function

$$F(x) = x^{\gamma}, \text{ with } \gamma \leq 1.$$

The discrete Markov chain for the log of idiosyncratic productivity ϵ is chosen to approximate a continuous first-order autoregressive process with persistence ρ_ϵ and where the innovations η follow an iid Normal distribution with standard deviation σ_η, of the form

$$\epsilon' = \rho_\epsilon \epsilon + \eta', \text{ with } \eta' \sim N(0, \sigma_\eta).$$

We choose a Markov chain with N_ϵ states to make this approximation following the discretization method in Tauchen (1986).

We assume an additively separable period utility function of the form

$$u(c, l) = \frac{c^{1-\sigma}}{1-\sigma} - B\frac{l^{1+\phi}}{1+\phi},$$

where B is the weight on the disutility of work, $1/\phi$ is the Frisch elasticity of labour supply, and σ is the inverse of the intertemporal elasticity of substitution.

The Type I extreme-value shocks to final-goods firms follow Gumbel distributions with a common dispersion parameter σ_ζ.

4.2 Calibration

A model's period corresponds to one year. In the stationary equilibrium, aggregate productivity is a constant parameter that can be normalised to $z = 1$. Several other parameters are assigned from direct observations. The parameters set directly are summarised in Table 1. The discount rate β is in equilibrium equal to discount price and thus chosen to be equivalent to an annual rate of return of 4%. The labour utility parameter ϕ corresponds to a Frisch elasticity of 2, and the intertemporal substitution σ is given a standard value. The curvature of the production function for final goods γ corresponds to the labour share if we think of capital as given and uniform across firms, and we pick a value common in the literature, for instance Corbae and D'Erasmo (2021), Khan and Thomas (2013) or Arellano, Bai, and Kehoe (2019). The parameters of the idiosyncratic productivity process, ρ_ϵ and σ_η, are annual estimates from Compustat panel data on operating income obtained by Corbae and D'Erasmo (2021) which, as they indicate, are in line with estimates on quarterly data in the literature (e.g., Khan and Thomas (2013), Arellano, Bai, and Kehoe (2019), or Cooper and Haltiwanger (2006)). For the discrete approximation, we choose a number of states N_ϵ of 61. The parameter controlling the volatility of the extreme value shocks σ_ζ, while ensuring convergence, is of little consequence in that it will practically render delinquency and liquidation binary outcomes.

Table 1: **Direct Parameters.**

parameter	value	observation
discount	$\beta = 0.9615$	4% annual return
risk aversion	$\sigma = 2.0$	standard
utility labour	$\phi = 0.50$	Frisch elasticity 2.0
curvature final goods	$\gamma = 0.60$	60% labour share
persistence	$\rho_\epsilon = 0.653$	Corbae and D'Erasmo (2021)
volatility innovation	$\sigma_\eta = 0.20$	Corbae and D'Erasmo (2021)
number of productivity states	$N_\epsilon = 61$	
scale of EV shocks	$\sigma_\zeta = 0.05$	

The remaining six parameters will be chosen so the model matches a number of targets. These parameters are the proportion of intermediate input sales on trade credit τ, fixed cost c_F, penalty size for delinquency $\tilde{\nu}$, probability of forgiveness λ, cost of entry ξ^E, and utility weight of work B. Since a number of targeted moments will depend directly on the price of inputs p, it is efficient to control p in order to meet those targets, and then choose deep parameters to be consistent with the chosen p.[16] The steps of the procedure, in outline, are as follows: (i) Set p, and the four deep parameters τ, $\tilde{\nu}$, λ, and c_F; (ii) Solve for the firms-lenders equilibrium outcomes Eq. (1) to (7), (19) and (20); (iii) Find entrants' distribution and the value of entry in Eq. (9) and (8); (iv) Solve for the distribution of firms as in Eq. (13) to (16); (v) Calculate trade-credit default rate in Eq. (17); (vi) Calculate target moments (debts, defaults, etc) and check against data; (vii) Update p and parameters τ, $\tilde{\nu}$, λ and c_F; Back to point (ii) and repeat until best match to data; (viii) Back out ξ^E via free entry condition Eq. (10); (ix) Back out w via input pricing Eq. (21), the number of firms N by labour market clearing Eq. (27), consumption via clearing in final-goods Eq. (28) and, finally, the parameter B to match the target for employment via the household's optimality condition solving Eq.(22). Specifics about the procedure are in the Appendix, Section A.4.

In step (vi) of the calibration, we will be targeting values for six model's moments. The moments in the model are measures of firms' debt level and dispersion, bankruptcy, trade credit losses, the size of trade credit, and employment. For debt we use the average of the ratios of debt to operating income across firms, conditional on operating income being positive. For each, firm debt is given by the state variable b, and operating income is defined as revenues minus variable costs and fixed cost, that is $(1-\nu)ze^{\epsilon}F(x) - px - c_F$. The standard deviation of this ratio across firms then gives the debt dispersion moment. The bankruptcy rate is the proportion of firms who liquidate when holding positive debt liabilities. It is calculated by integrating the bankruptcy policy rule $d^b(\epsilon, b, \nu, x)$ over the distribution of firms ex-post $\mu^{\text{ex-post}}(\epsilon, b, \nu, x)$, that is after their choice of input x and realisation of the shock ϵ, for strictly positive $b > 0$. The trade credit loss is measured as the fraction of intermediate input trade-credit sales that fail to perceive payment, adjusted by the possible recovery in cases of delinquency without liquidation. This corresponds to the variable θ in the model. As the target for the size of trade credit, trade credit to GDP is measured as the value of intermediate inputs sold on trade credit as a proportion of the value of final goods produced. Let $\bar{x}$ the total amount of inputs purchased by final-goods firms, obtained by integrating the input demand policy function $x(.)$ over the distribution of final-goods firms, and let $\bar{y}$ the value of final output, obtained as the integral of firms' output $(1-\nu)e^{\epsilon}F(x)$ over the ex-post distribution of operating firms.[17] The target ratio of trade credit to GDP is therefore $\tau p\bar{x}/\bar{y}$. Finally, labour supply is given by the household's optimal decision l.

The empirical values corresponding to the above targeted moments are derived from aggregate and firm-level data. For debt, operating income and trade credit loss, we use firm-level data from Compustat for the period 1980-2014.[18] The raw data is cleaned in a way similar to Corbae and D'Erasmo (2021).[19]

The variable for debt is *Debt in Current Liabilities*. Operating income is measured by the

[16]In this way, we avoid having to find p as the solution to the zero-profit condition, an implicit non-linear equation, for each trial of ξ^E.

[17]That is excluding those who liquidate and moreover cannot operate due to not being able to cover both the fixed cost and the cost of cash inputs.

[18]Accessed trough Wharton Research Data Service (WRDS) `https://wrds-www.wharton.upenn.edu/`. Data pulled from WRDS Compustat - Capital IQ, North America, Fundamentals Annual.

[19]We have borrowed some of their Stata code. We drop observations with 0 or missing assets, sales and

variable *Income Before Interest*, which is consistent with our definition in the model.[20] The ratio of debt to operating income is then calculated for firms with strictly positive operating income, and we remove the top 0.05% ratios to avoid outliers. These operations result in an average debt-to-income ratio of about 1.20, with a standard deviation of 6.99.[21]

For the empirical counterpart to the trade-credit loss rate in the model, we need firm-level measures of amounts of trade credit extended and of the part of it that is deemed lost or uncollectable. For the former, we use the Compustat variable *Accounts Receivable - Trade* which represents amounts on open account owed by customers for goods and services. For the loss, the Compustat variable *Receivables Estimated Doubtful* is the amount of all current accounts receivable estimated to be uncollectable.[22] The ratio of receivables doubtful over the sum of all receivables, doubtful or not, gives an empirical target for the loss rate on trade credit of 6.6%. This rate is not far from the estimate in Jacobson and von Schedvin (2015) of 8% for Sweden based on direct administrative data.

The target for the ratio of trade credit to GDP is constructed from U.S. aggregate data for the period since 1950. Given that in the model trade credit corresponds to accounts receivable, we accordingly use quarterly trade receivables for corporate and non-corporate business from the Federal Reserve's Z.1 Financial Accounts of the United States. The quarterly series for GDP is from U.S. Bureau of Economic Analysis.[23] We find that the trade-credit-to-GDP ratio has stayed consistently around 0.18 throughout, with no apparent trend.[24]

We choose the target for the bankruptcy rate based on the related literature. Corbae and D'Erasmo (2021) consider a rate close to 1% based on Compustat, and also indicate a 2 per cent based of their measure of distance to default. On the other hand, Ottonello and Winberry (2020) report a 3% default rate based on business survey data. All in all, we take the mid-value target of 2%. Finally, the target for employment is unity, a simple normalisation.

The values of the parameters are obtained from a procedure of minimisation of the sum of squared deviations between model and targets.[25] The calibrated parameters values and targeted variables are summarised in Table 2. While there is no exact correspondence between parameters and targets, the utility parameter B and the trade-credit parameter

property, plant and equipment, and exclude financial firms with SIC codes between 6000 and 6999, utility firms with SIC codes between 4900 and 4999, and firms with SIC codes greater than 9000. Observations are deleted if they do not have a positive book value of assets or if gross capital stock or sales are zero, negative, or missing.

[20]These variables are labelled `dlc` and `ebitda` in Compustat.

[21]Of the 222,388 observations of debt, we drop the two observations with a negative entry. The number of observations with positive operating income is 166,032; after trimming, this leaves 165,948 observations for the ratio of debt to operating income.

[22]The labels in Compustat are `rectr` and `recd`, respectively.

[23]Board of Governors of the Federal Reserve System (US), Nonfinancial Corporate Business; Trade Receivables; Asset, Level TRABSNNCB and TRABSNNB. U.S. Bureau of Economic Analysis, Gross Domestic Product [GDP]. All retrieved from FRED, Federal Reserve Bank of St. Louis; `https://fred.stlouisfed.org/series/`.

[24]Specifically, the mean ratio has been 0.182, 0.185, 0.186, 0.186, 0.186, 0.191 since 1960, 1970, 1980, 1990, 2000 and 2010, respectively.

[25]The algorithm is based on the Software BOBYQA, authored by M. J. D. Powell, to minimize sum of squares with bound constraints by combing trust region method and Levenberg-Marquardt method.

τ intuitively enough affect primarily the targets labour supply and trade credit over GDP. The relationship between the rest of parameters and targets is less straightforward.

Table 2: **Calibrated Parameters**

parameter	calibrated value	target variables
fixed cost c_F	0.698	fraction in bankruptcy
penalty size $\tilde{\nu}$	0.155	debt to operating inc
penalty forgiveness λ	0.396	st dev debt to operating inc
entry cost ξ^E	10.519	trade-credit loss rate
input sales on trade credit τ	0.291	trade credit to GDP
weight of work in utility B	2.410	labour supply

This calibration implies that about 29% of sales of input are on trade credit. The delinquency penalty parameters imply for the firm a future loss of about 15% of output over, on average, a period of two and a half years. The total operating fixed costs incurred by final-goods firms amounts to about 19% of the value of aggregate final output, above but not far from the replacement cost of capital of 16% implied by the capital-to-output ratio of 2.3 and depreciation rate of 6.9% in, for instance, Khan, Senga, and Thomas (2016).

Table 3 presents the results from the above parameters. The second column contains the implications of the model for the target moments. The match to the data is quite close, especially considering the highly parsimonious model and the complex interactions that complicate identification.[26]

Table 3: **Moments in model and data**

Moments	Data	Model
fraction in bankruptcy	0.020	0.014
debt to operating inc	1.15	1.19
st dev debt to operating inc	7.39	7.00
trade-credit loss rate	0.066	0.071
trade credit to GDP	0.18	0.18

For some sense of external validity, about 78% of firms have strictly positive operating income in the model which compares with about 75% in Compustat data. The endogenous exit rate is about 2.5%, which sits in the range between the exit rate of 1.20% in Corbae and D'Erasmo (2021) and the 5.2% in Khan, Senga, and Thomas (2016).[27]

[26] Labour supply, not shown, matches its normalisation target exactly.
[27] The latter paper targets a total 10% exit rate, but 4.8% consists of exogenous departures.

4.3 Policy functions

We describe the properties of the main equilibrium objects of this economy. We begin with objects pertaining to the first part of a period, before the productivity shocks occur. Figure 1 displays first the distribution of final-goods firms without a delinquency flag $\nu = 0$ over the initial individual states of previous individual productivity and debt (ϵ_{-1}, b). The mass of firms is spread over the domain of productivity values as expected, and this mass declines smoothly in the debt position. The bottom panel displays the corresponding optimal choice of intermediate input x. In terms of sign, the amount of input x is an increasing function of initial productivity, thereby inheriting the properties of the first best. The demand for x is generally increasing in the value of debt due, and more markedly so at low levels of productivity. Larger debt obligations make it optimal to increase production and operating income to meet repayments.[28]

Consider now decisions at the end of the period, after the realisation of the new shock ϵ. Figure 2 shows firm's outcomes in the case of no defaulting ND in the space of input and debt (x, b), for a given level of the new productivity ϵ. The top graph displays the borrowing function, with positive values where debt is large enough or small enough, U-shaped, with zero debt in a middle region of x, one which would become wider with larger productivity. The flat area beyond the edges is where debt collapses and corresponds to states where the option of not defaulting ND is unfeasible. The dividends policy is also non-monotonic in x, and positive in the inner region where borrowing is zero. These patterns are valid across all realisations of productivity ϵ.

Associated with the decision to be delinquent on trade-credit payments, the pattern of borrowing described by the function g^x (not shown) will be qualitatively similar to the one for the no-default case g^{ND} shown in Figure 2, and the shape of the recovery function under delinquency r^x is also similar to the shape of the dividends function π^{ND} already shown.

Figure 3 displays the bankruptcy and delinquency rules in the space of input and debt (x, b), for a low-productivity realisation. Bankruptcy occurs when debt b is sufficiently large, and when the level of the input x is too large or too small. Delinquency happens when x is sufficiently large and the range of debt positions is broader the larger x. Delinquency happens before bankruptcy in terms of debt. There is a hint here that delinquency relieves pressures to having to liquidate.[29]

The above decision rules on repayment are major determinants of the pricing of debt. Figure 4 displays the price of debt in the space (e^{ϵ}, b') for the cases of no-default, q^{ND}, and delinquency, q^x. As expected, debt prices decline in debt borrowed and increase in productivity, and are lower in the event of the firm being currently delinquent. Delinquent firms thus face tougher conditions in the market for credit and may find it harder to roll

[28] An exception to this pattern occurs at very large-debt low-productivity positions where higher debt makes it very likely that the firm will liquidate, with positive losses to the firm that depend directly on the value of inputs purchased. These situations are well outside the range of the stationary distribution.

[29] Notice these decisions are virtually binary, a confirmation that the assumed extreme-value shocks on the discrete options are largely inconsequential in practice.

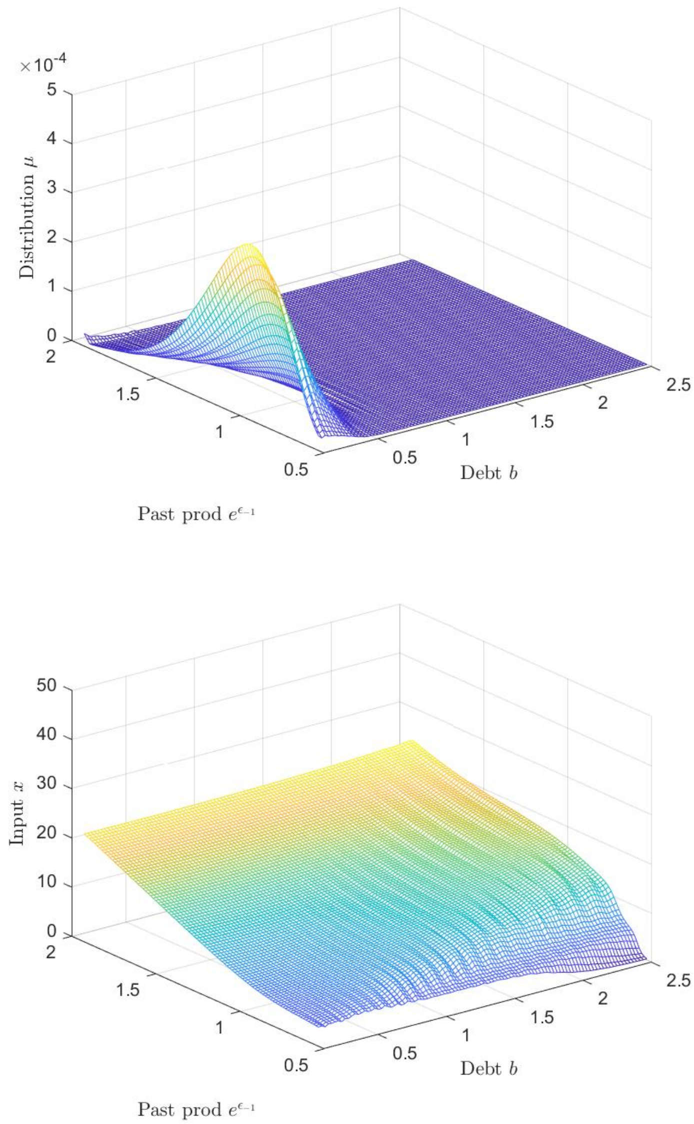

Figure 1: Before shocks: Distribution μ (top) and the demand for inputs x (bottom) as functions of past productivity, $e^{\epsilon_{-1}}$, and debt, b, for firms without a delinquency flag $\nu = 0$. Productivity is normalised to average 1, so debt units are the proportion of output of a final-good firm using the market clearing average level of inputs (equal to 1) and of average productivity.

over debt. That might push them into liquidation. On the other hand, for the same reason, by reducing their liabilities, via delinquency a firm might be less likely to liquidate.

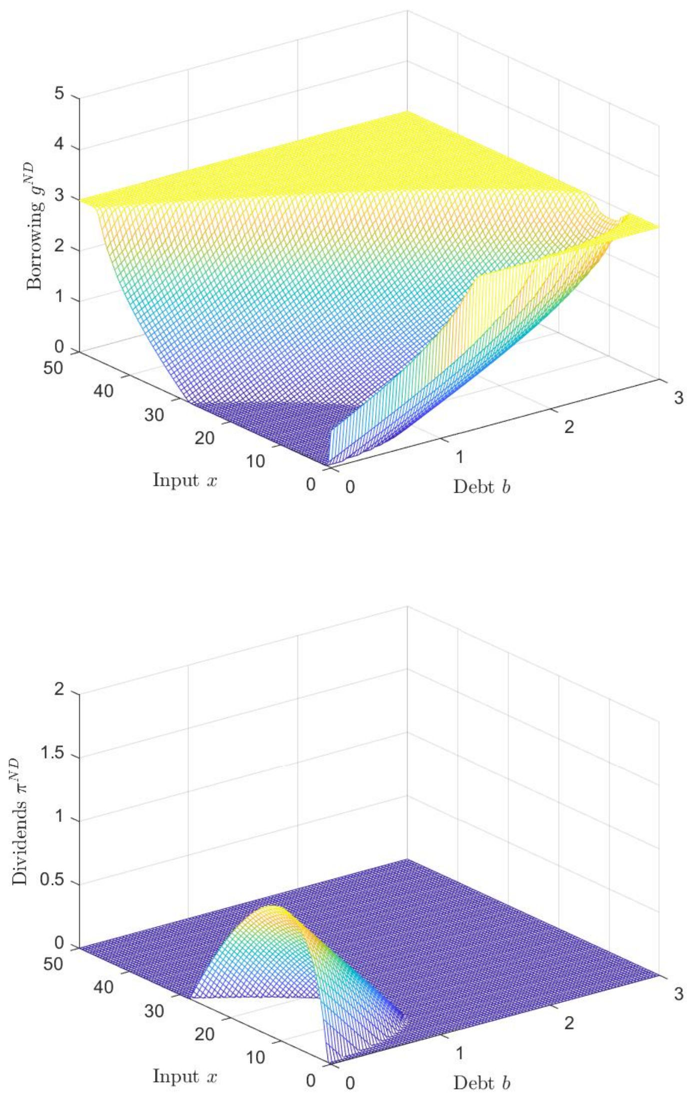

Figure 2: No default case: Policy functions for borrowing, g^{ND}, and dividends, π^{ND}, as functions of debt, b, and inputs purchased, x. Firms with productivity, e^{ϵ}, 12% above the mean, and without a delinquency flag $\nu = 0$.

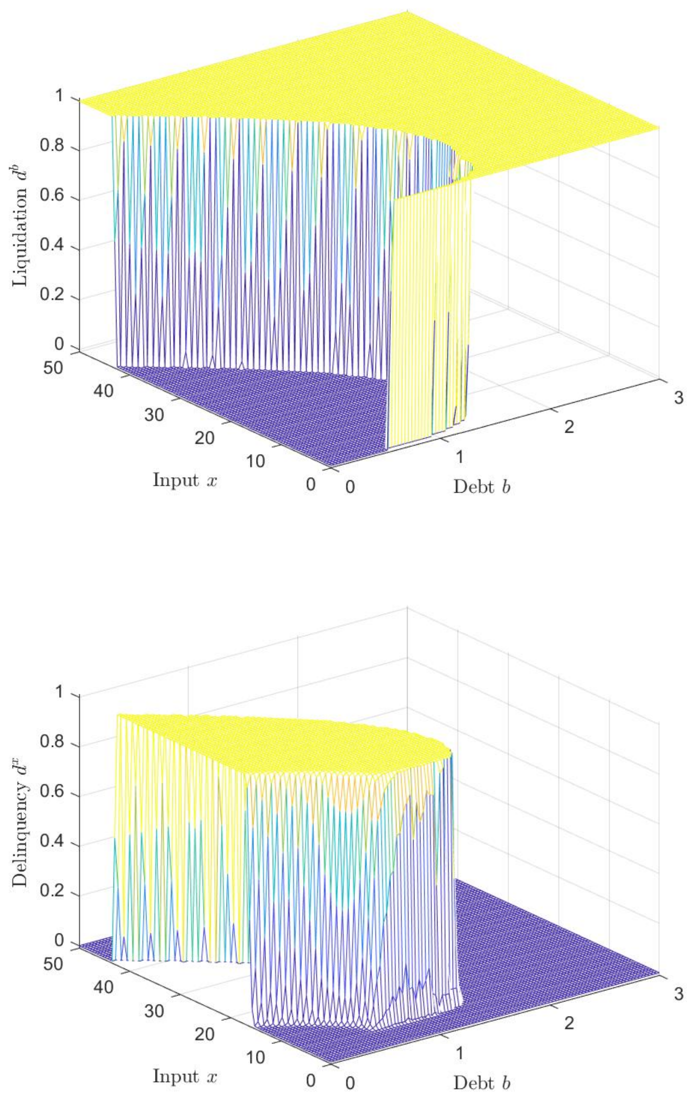

Figure 3: Default rules: Policy functions for bankruptcy, d^b, and delinquency, d^x, as functions of debt, b, and inputs purchased, x. Firms with productivity, e^ϵ, 15% below the mean, and without a delinquency flag $\nu = 0$.

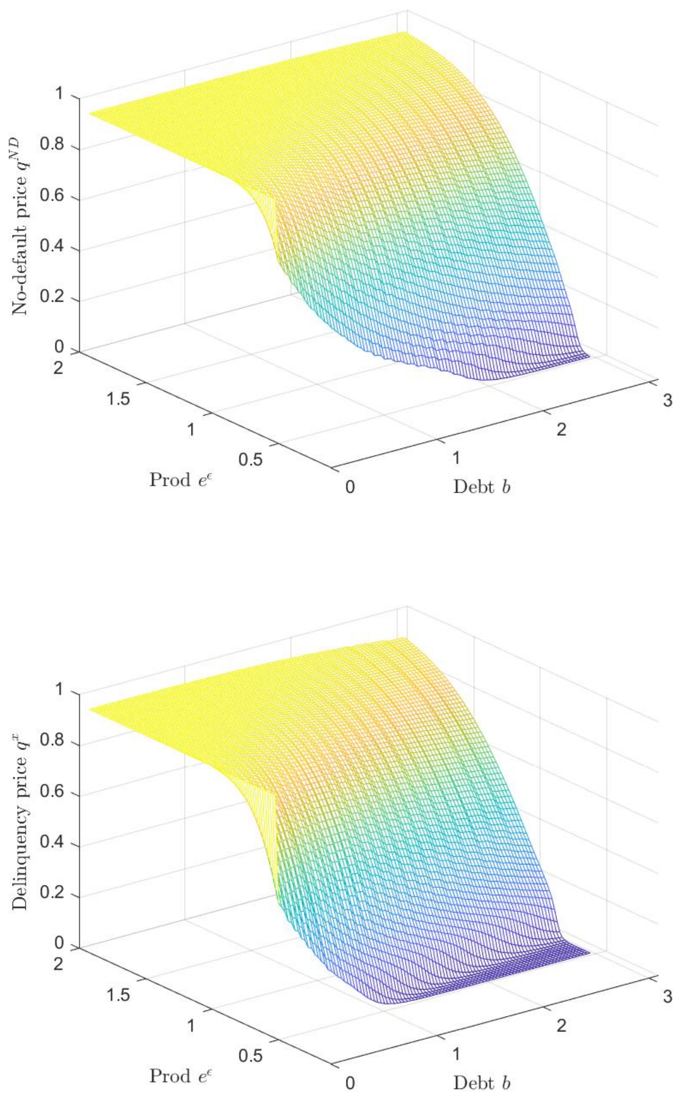

Figure 4: Debt prices: discount prices in no-default case, q^{ND}, and in trade-credit delinquency case, q^{Dx}, as functions of new debt, b', and productivity, e^{ϵ}, for firms without a delinquency flag $\nu = 0$.

4.4 The profile of delinquency and bankruptcy

In this subsection we explore patterns that emerge within the stationary distribution. Compared to firms who do not default in any way, measures of debt, conditional on debt being positive, are higher for delinquent firms and for firms that declare bankruptcy. Delinquent firms have larger debts than firms that liquidate. The figures are in Table 4.

Table 4: **Levels of debt** b

If debt positive	0.169
Non delinquent, non bankrupt	0.123
Delinquent, non bankrupt	0.205
Bankrupt	0.146

We also compute the average probabilities of transiting into bankruptcy and into delinquency, for firms who are and who are not currently delinquent. On average, delinquent firms are less likely to be bankrupt in the next period. This must be because delinquent firms have discharged some liabilities.[30] On the other hand, firms currently delinquent are to repeat delinquency next period much more likely than non-delinquent firms do. This must reflect that the larger debts but also a deteriorated productivity inducing the need to skip trade-credit payments. Table 5 displays the figures.

Table 5: **Transition probabilities**

Probability to bankruptcy:		
	non-delinquent firms	0.059
	delinquent firms	0.044
Probability to delinquency:		
	non-delinquent firms	0.071
	delinquent firms	0.181

5 The dynamic response to shocks

We have already discussed in Section 3.6.3 that this model contains mechanisms whereby trade-credit default will affect the determination of employment and output. The main objective of this section is to study whether and how much trade-credit default amplifies the effect of shocks to the economy so we must move beyond stationary situations. Since we do not solve the model with aggregate uncertainty, the focus here will be on unanticipated exogenous deterministic changes in the path of various state variables and parameters of the model, in the spirit of the so called MIT shocks (e.g., Boppart, Krusell, and Mitman (2018), Guerrieri and Lorenzoni (2017)).

[30] On the other hand, as seen above, they tend to carry larger debts which will work towards more bankruptcy. The outcome reflects a specific balance between the two forces.

The model laid out in Section 3 already accounts for the distribution of firms as an aggregate state variable that can be affected by the shocks. However, since we will also consider changes in exogenous parameters, the description of the model needs to be extended slightly so that all the endogenous value and policy functions, and scalars, are suitably indexed by time.[31] One must also account explicitly for the distinct possibility of strictly zero entry in periods of contraction.[32] For ease of exposition, and at little loss for our purpose, we will consider a small open economy along the adjustment path by assuming that the price of bonds is constant and the market clearing condition in consumption need not hold during the transition. This is similar to Arellano, Bai, and Kehoe (2019).

The economy is initially at the baseline stationary equilibrium and, since the shocks are not permanent, will return to that position given enough time. Characterising the transition back to the stationary equilibrium requires solving explicitly for the evolution of, among other variables, the distribution of firms and trade credit default. The procedure to obtain the transition over periods $t = 0, 1, ..., T$ consists of an outer loop in the price of inputs, an inner backward step going from period T to 0 to characterise the firms' and lenders' policy functions and debt price functions, and price of inputs when $m > 0$, and an inner forward step going from period 0 to T to find aggregate quantities and default rates that satisfy the equilibrium conditions, and the price of inputs when $m = 0$. While further details are in Appendix A.5, in outline, the main steps are as follows:

1. Guess a path for input price $\{p_t\}_{t=0}^T$.
2. Guess a path for aggregate entry rate $\{m_t\}_{t=0}^T$.
3. Backward step: Starting at the terminal steady state at $t = T$, proceed backwards to obtain paths for functions $\{g_t, x_t, d_t, r_t, \mu_t^E\}_{t=1}^T$ and $\{q_t\}_{t=1}^T$, by solving the equilibrium between final-goods firms and lenders and the free entry condition, and updated input prices in $\{p_t\}_{t=0}^T$ when $m_t > 0$ via the free entry condition.
4. Forward step: Given the above paths $\{p_t\}_{t=0}^T$ and $\{g_t, x_t, d_t, r_t, \mu_t^E\}_{t=0}^T$, and $\{m_t\}_{t=0}^T$ proceed forwards to obtain the paths $\{\theta_t, \eta_t, w_t, l_t, \mu_{t+1}, N_{t+1}\}_{t=0}^T$, and updated $\{m_t\}_{t=0}^T$ and $\{p_t\}_{t=0}^T$ that satisfy the corresponding equilibrium conditions, and updating p_t via the input markup condition when $m_t = 0$. Back to 3 until convergence.

The paths that result from the shocks are not by and in themselves informative about the role of trade credit default. In order to identify that role, we will conduct a counterfactual exercise where the channel of transmission of trade-credit default to the economy is switched off. Since, as we have learned already, the trade-credit loss rate θ, via its impact on the spread between the price of inputs and the wage cost, acts as the primary such channel, the counterfactual exercise simply holds its value at the stationary equilibrium level in the pricing equation (21). This amounts to asking, what if input producers failed to update the estimated payment risk they face?

[31] Equivalently, we could add those parameters in the description of S. The computation procedure follows the time-indexing option more literally anyway.

[32] In contrast with the equilibrium logic when $m >> 0$, the free-entry condition (10) becomes a strict inequality when $m = 0$, so the wage will now have to fall to clear the labour market, and it is the price of inputs that will then adjust to meet the markup condition (21) rather than the entrants' zero-profit condition.

We will study different separate types of aggregate shocks. The first one is a one-time proportional reduction in the idiosyncratic productivity component of all firms in the economy. The second shock considered will be a reduction in total factor productivity that affects all firms in equal measure. The third shock will consist of an increase in the fixed operating cost of firms, which we think of as a financial shock.[33] Finally, we study increased volatility in the form of a rise in the standard deviation of idiosyncratic firm productivity, which is related to the interest of a rapidly growing literature. It is important to note that we specify the shocks so that, while unanticipated, they become known to all types of firms right at the beginning of the impact period $t = 0$, before idiosyncratic shocks are realised, and thus will cause no losses or gains for input producers.[34]

Furthermore, we also consider two cases in terms of the fulfillment of equilibrium conditions. The first case preserves the full set of equilibrium conditions, including flexible free entry and wage determination. The second case is one where real rigidities in the short run prevent the wage from adjusting and the number of firms remains constant.[35] With an unchanged wage, the price of inputs must now meet the markup condition, implying that the free-entry condition in final goods cannot generally be satisfied and hence some exogenous condition that pins down the mass of firms, like the constant number we assume, is required. On the other hand, the level of employment– possibly implying involuntary unemployment– is determined by the demand for labour from input producers at the unchanged wage.[36]

Table 6 summarises the main findings by displaying, for each shock and case, the percentage of the impact change of employment and final output accounted for by the adjustment of trade-credit default. Entries with 100+ correspond to trade-credit default changing the sign of the impact response. These figures illustrate that generally trade-credit default amplifies the size of fluctuations to an important degree, often a large one, that varies across shocks and the extent of short-run rigidities. The rest of this section will describe the shocks in more detail and analyse what lies behind these findings.

5.1 A reduction in the distribution of idiosyncratic productivity

The shock at time t here consists of a 6% unanticipated reduction in the idiosyncratic productivity of existing firms at the start of the period, ϵ_{t-1}.[37] This change squeezes the individual productivity distribution towards values in the lower tail, with its realisations subsequently evolving according to the Markov chain for the individual firms. New entrants still draw productivity realisations from the stationary productivity distribution.

[33] This is one interpretation in Khan, Senga, and Thomas (2016). These authors are also considering financial shocks in the form of an increased lending spread.

[34] Although there may be some temporary losses or gains for banks absorbed by the consumers/shareholders.

[35] Constant number of firms is assumed in, for instance, Ottonello and Winberry (2020), Arellano, Bai, and Kehoe (2019) and Bordalo, Gennaioli, Shleifer, and Terry (2021). Wage rigidities are more ubiquitous in the literature, e.g., Guerrieri and Lorenzoni (2017).

[36] In this case we are relaxing labour-market clearing and free-entry in final goods, Eq. (10) and (27).

[37] But productivity cannot fall below the lower bound of the grid.

Table 6: **Amplification from trade-credit default**

shock name	shock size	equilibrium case	amplification employment (%)	amplification GDP (%)
productivity distr μ	-6%	flexible	94.4	68.4
		rigid	14.0	8.7
tfp z	-8%	flexible	10.1	9.9
		rigid	14.2	14.5
financial c_F	-10%	flexible	39.8	15.6
		rigid	100+	100+
volatility σ_η	+25%	flexible	36.4	27.6
		rigid	100+	100+

This experiment is therefore deliberately designed to illustrate the amplification effects of trade-credit default through the response of the wage and the input price separately.

5.1.1 Flexible case

We will consider first the case with flexible wages and free entry throughout the transition. It is instructive to discuss first the response of aggregate quantities including output of final goods (GDP), employment and debt, and then the prices and default rates underpinning those changes. The quantities are shown as proportional deviations from their stationary equilibrium values by the solid lines in Figure 5 (ignore the dashed lines for now). The direct impact of the generalised dip of productivity is a reduction in the demand for inputs and production of existing firms. This works in the direction of lower employment and further reduced output. It is to be noted that entry of new firms (not shown) surges markedly given the unrestricted entry conditions, which tempers to an extent the fall in output and employment.[38] Borrowing and debt increase substantially following the shock.

Turning now to prices and default rates, trade-credit default, coming from both delinquency and bankruptcy, increases leading to a reduction in the wage rate. This is associated with a sharp increase in the rate of losses to trade credit. The solid lines in Figure 6 display these variables. The fall in the wage on impact here almost entirely reflects the response of trade-credit default, since the input price (not shown) barely changes. The inputs price p, being consistent with a break-even value of entry, hardly changes as, by design of the experiment, the shock does not affect the value of prospective firms.[39]

The response of labour supply, and hence employment and output, seen above occurs

[38] That entry then collapses in the next period $t = 1$ explains the slow recovery.

[39] Although it reflects some of the increased default risk in the period next to impact because in that period there is strictly zero entry $m = 0$ and the zero-profit condition need not hold.

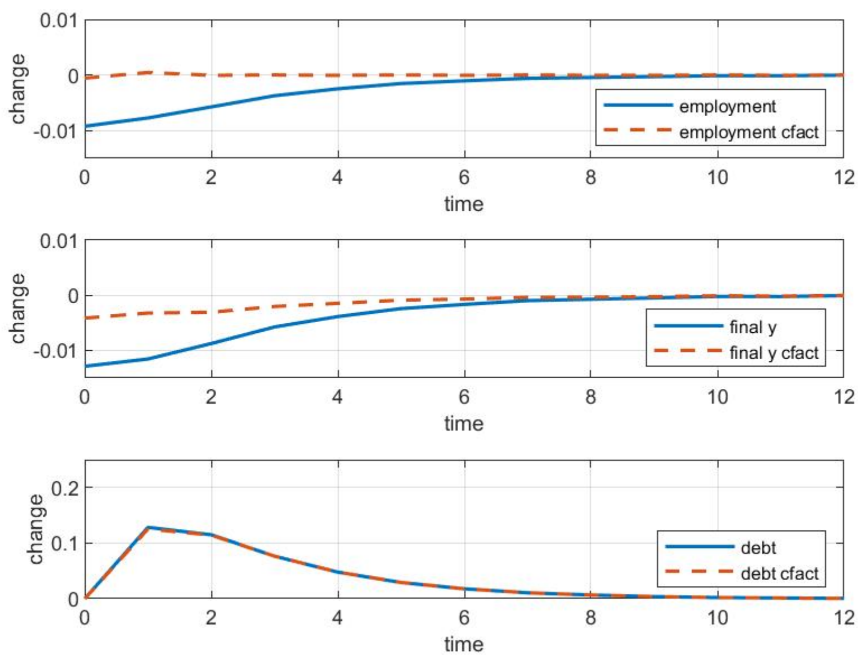

Figure 5: A 6% fall in distribution of productivity, flexible case. Quantities: employment (top), output (middle), debt (bottom). Main experiment (solid lines) and counterfactual (dashed lines).

because of the reduction in the wage that follows from the upswing in trade-credit default. This is an amplification mechanism. In order to assess its quantitative importance we conduct a counterfactual exercise that keeps the trade-credit loss rate θ constant. In this way, as already explained earlier, we control for the feedback effect of trade-credit default on the economy. We look first at the prices and default rates as displayed in Figure 6, where the dashed lines correspond to the counterfactual. By construction, the wage rate remains nearly unchanged in this case. Additionally, compared with the main experiment shown by the solid lines, there is a slightly smaller rise in default rates. Preventing the fall in the wage rate tempers the risk of default so the rise in default and trade-credit losses are smaller, although not by a very large amount.

Turning to quantities, Figure 5 shows that with the nearly constant wage in the counterfactual, labour supply remains almost constant, and there is therefore a much smaller reduction in output, being also underpinned by a sharper boost to firm entry (not shown) to support the market-clearing demand for labour. Quantitatively, trade-credit default amplifies the negative response of output and employment, and accounts for nearly the totality of the reduction in employment and over 2/3 of the reduction in output as reported in Table 6.

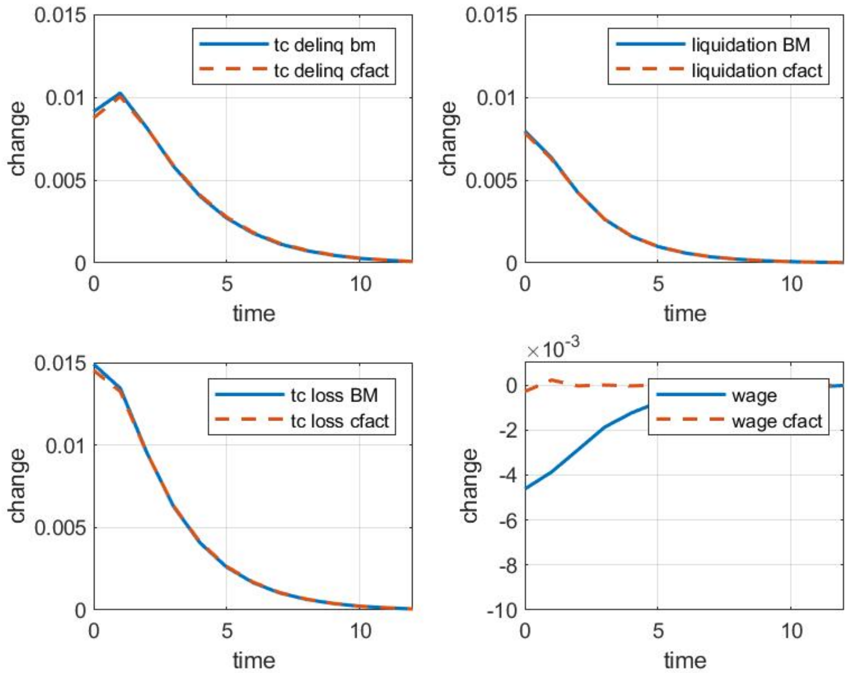

Figure 6: A 6% fall in distribution of productivity, flexible case. Prices and default: delinquency rate (top-left), liquidation rate (top-right), trade-credit loss θ (bottom-left), wage w (bottom-right). Main experiment (solid lines) and counterfactual (dashed lines).

5.1.2 Rigid case

We now consider the same productivity shock in an economy subject to rigidities that limit the adjustment in wages and the change in the number of firms. Constant wages and number of firms relax the equilibrium conditions of clearing in the labour market and zero-profit firm's free entry. The wage being constant, the price of inputs will uniquely reflect the changes in trade-credit default in order to satisfy the input suppliers' pricing condition.

In these circumstances, as displayed by the solid lines in Figure 7, the shock causes an immediate fall in employment and output, and a rise in borrowing and subsequent firm indebtedness. Although qualitatively similar, the response of employment and output is larger than under the flexible-wage free-entry scenario seen before in Figure 5.

Given the unchanged wage, employment is driven by changes in the demand for intermediate inputs, and it is thus important to understand how their price evolves. Figure 8, solid lines, shows the dynamic response of prices and default rates. We see again an increase in both delinquency and bankruptcy rates, implying a rise in trade-credit losses. This

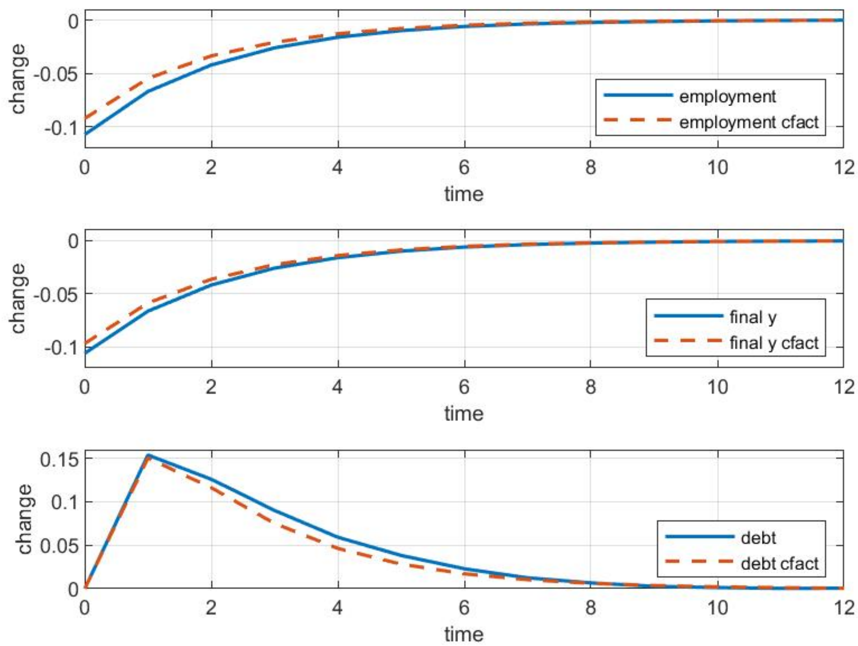

Figure 7: A 6% fall in distribution of productivity, rigid case. Quantities: employment (top), output (middle), debt (bottom). Main experiment (solid lines) and counterfactual (dashed lines).

increased default risk fully accounts for the rise in the price of the intermediate input.

The fact that inputs become more expensive must amplify the impact of the shock on employment and final output. To assess the size of this amplification we consider the counterfactual experiment which here means that the price of the intermediate inputs remains constant. Looking at the implications for default, the dashed lines in Figure 8 show that preventing the increase in the price of inputs tempers the risk of default risk so the rises in default and trade-credit losses are smaller and, in contrast with the results in the flexible case, this effect is quantitatively substantial. Trade-credit default thus fuels more default and bankruptcies as it pushes up the cost of inputs. Turning to quantities, Figure 7, dashed lines, reveals that employment and output decrease to a smaller extent with a constant input price. Quantitatively, as seen in Table 6, the amplification coming from trade-credit default accounts for about 10% of the impact response of final output to the shock, and 14% of the response of employment.

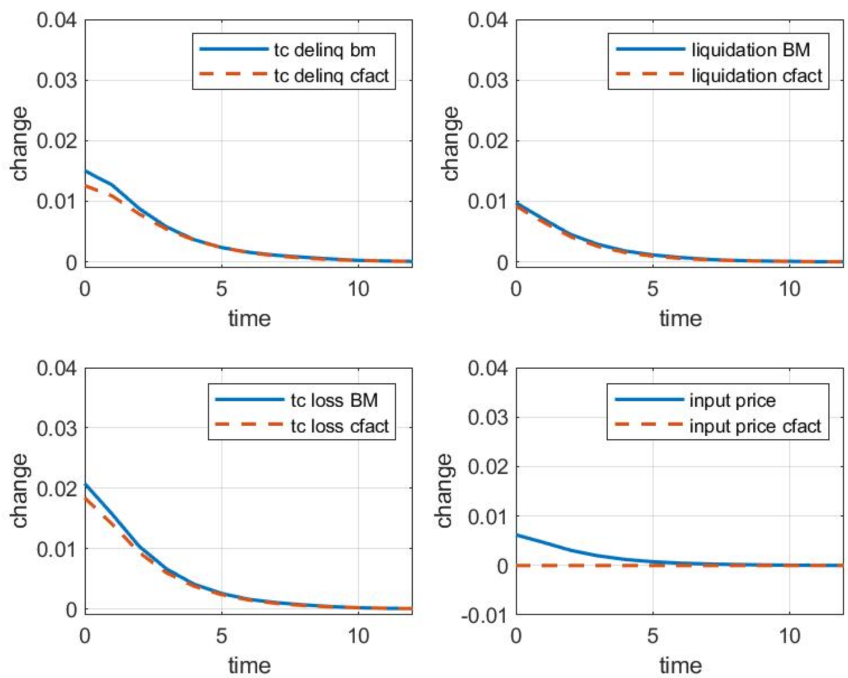

Figure 8: A 6% fall in distribution of productivity, rigid case. Prices and default: delinquency rate (top-left), liquidation rate (top-right), trade-credit loss θ (bottom-left), input price p (bottom-right). Main experiment (solid lines) and counterfactual (dashed lines).

5.2 A shock to total factor productivity

The shock consists of a reduction in aggregate productivity z of final-goods firms in the impact period. It takes the form of a reduction of z_t in period $t = 0$, observed at the start of period 0, before the realisation of the idiosyncratic shocks, so that forecasts about trade-credit risk are fully updated at this point. After the shock, we assume that z_t follows a standard log autoregressive process, $\log z_{t+1} = \rho_z \log z_t$, where $\rho_z < 1$ is its persistence, so it returns to its stationary value of $z = 1$ given time. We specify a 8% reduction, and 0.80 persistence: $z_0 = 0.92$ and $\rho_z = 0.80$.

5.2.1 Flexible case

The quantities, as proportional deviations from their stationary equilibrium values, are shown by the solid lines in Figure 9. The direct impact of the generalised fall in productivity is a reduction in the demand for inputs and production of existing firms, leading to lower employment and further reduced output. Entry of firms m (not shown) collapses to zero on the impact and subsequent four periods, so there will be a net reduction in the number of firms over that subperiod. Borrowing and debt increase substantially following the shock.

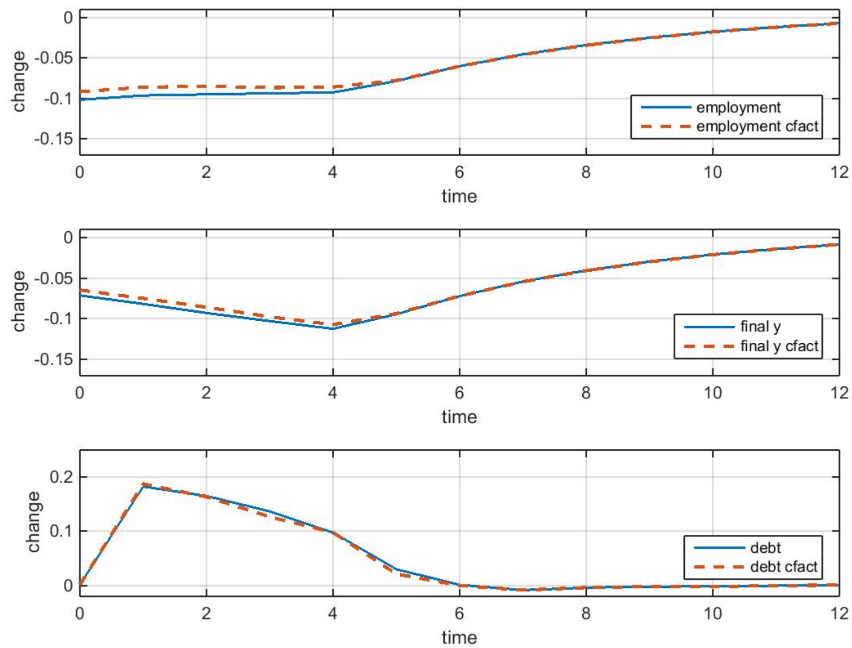

Figure 9: A 8% fall in total factor productivity, flexible case. Quantities: employment (top), output (middle), debt (bottom). Main experiment (solid lines) and counterfactual (dashed lines).

Figure 10, solid lines, displays the response of prices and default rates. Trade-credit default, from both delinquency and bankruptcy, increases. The corresponding sharp increase in the trade-credit loss rate will, by the markup condition, be driving a wider wedge between the price of inputs and the wage. The wage falls quite substantially so, in spite of the increases trade-credit default, the price of the input (not shown) will also be decreasing sharply, albeit by a smaller proportion than the wage. Notice that since the first part of the transition features zero entry, the wage clears the labour market and the price of input is the variable that must adjust to meet the markup condition (21).

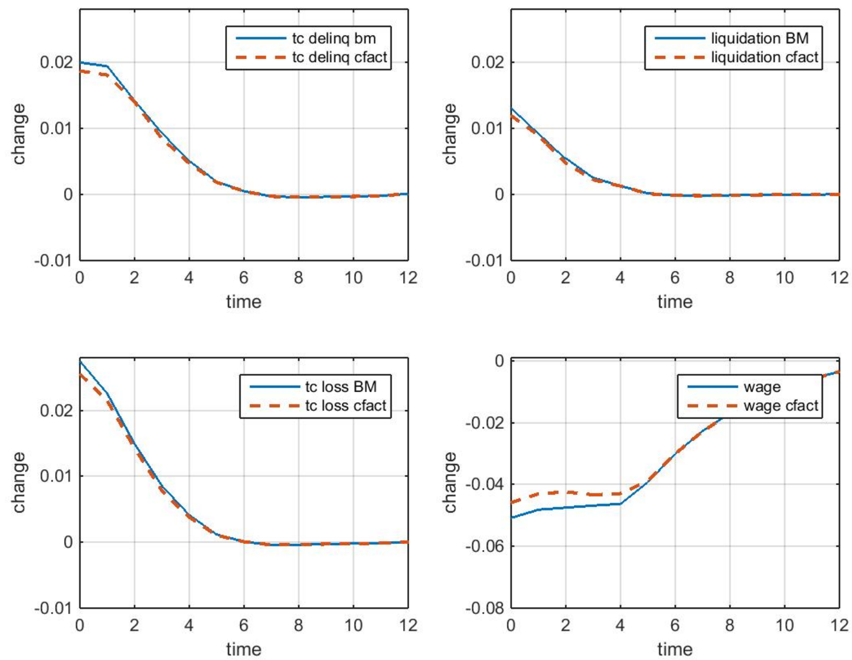

Figure 10: A 8% fall in total factor productivity, flexible case. Prices and default: delinquency rate (top-left), liquidation rate (top-right), trade-credit loss θ (bottom-left), wage w (bottom-right). Main experiment (solid lines) and counterfactual (dashed lines).

What if input producers fail to update their forecast of trade-credit default? Regarding prices, as shown by the dashed lines in Figure 10, this counterfactual scenario implies a smaller fall in the wage and a lower rise in default measures. The latter is an indication of substantial feedback from trade-credit default loss into bankruptcies and further defaults. All in all, there is a larger fall (not sown) in the price of input. These counterfactual changes in both wage and input price show that trade-credit default reduces the wage and raises the price of inputs and thus points at the existence of amplification of trade-credit default. Figure 9, dashed lines, shows the amplification effect on employment and GDP from trade-credit default, in the order of 1/10 as per the figures shown earlier in Table 6.

5.2.2 Rigid case

Consider now the same aggregate productivity shock in the rigid case. The solid lines in Figure 11 show a considerable reduction in both employment and output, larger than in the flexible case. It must be because of a sharper reduction in the demand for inputs and hence labour. This is accompanied by a large rise in borrowing and debt.

Figure 12, solid lines, displays the response of prices and default rates. Trade-credit default, from both delinquency and bankruptcy, increases. The corresponding sharp increase in the trade-credit loss rate drives the increase in the price of input since the wage remains constant in this rigid case. This explains the sharp contraction in employment noted above.

The counterfactual with constant markup and input price, shown by the dashed lines in Figure 12, implies a noticeably smaller rise in defaults and trade-credit losses, an indication of substantial feedback from trade-credit defaults losses into bankruptcies and further defaults. Figure 11, dashed lines, shows there is an amplification effect on employment and GDP from trade-credit default, representing about 15% of the total response, as reported earlier in Table 6.

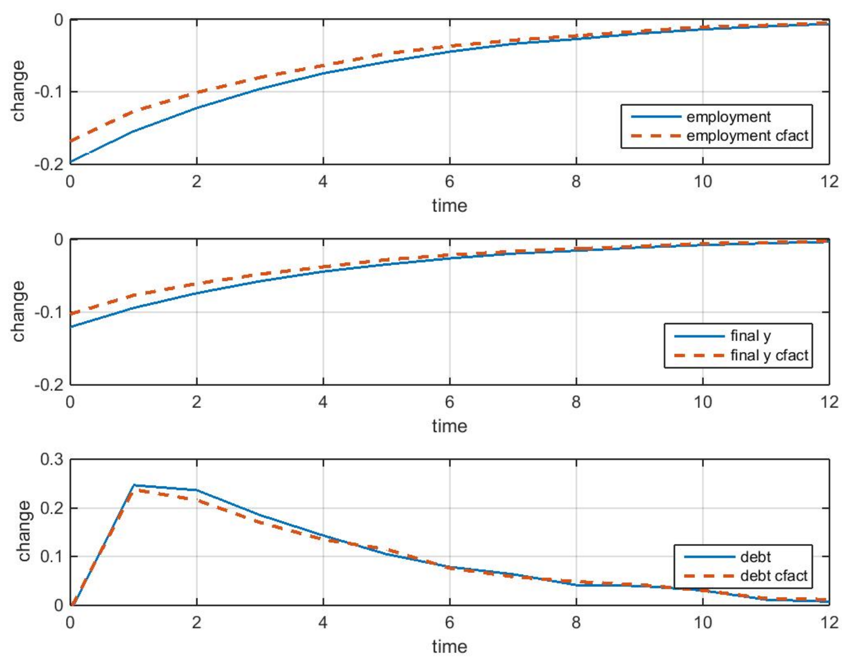

Figure 11: A 8% fall in total factor productivity, rigid case. Quantities: employment (top), output (middle), debt (bottom). Main experiment (solid lines) and counterfactual (dashed lines).

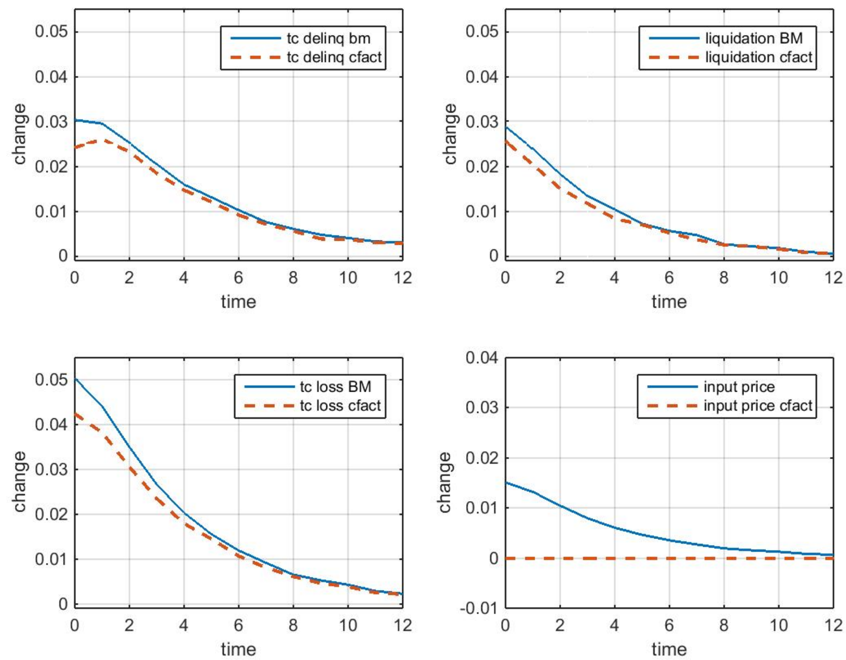

Figure 12: A 8% fall in total factor productivity, rigid case. Prices and default: delinquency rate (top-left), liquidation rate (top-right), trade-credit loss θ (bottom-left), input price p (bottom-right). Main experiment (solid lines) and counterfactual (dashed lines).

5.3 A financial shock

A financial shock consists of an increase in final-goods firms' fixed cost c_F in the impact period. It is represented by an increase of $c_{F,t}$ in period $t = 0$, observed at the start of period 0, before the realisation of the idiosyncratic shocks, so that forecasts about trade-credit risk are fully updated at this point. After the impact at time 0, we assume that $c_{F,t}$ follows a standard autoregressive process in the difference to its steady state value, that is $c_{F,t} - c_F = \rho_{c_F}(c_{F,t-1} - c_F)$, where $\rho_{c_F} < 1$ is its persistence. We specify a 10% increase, and a low persistence of 0.20: $c_{F,t=0} = 1.10c_F$ and $\rho_{c_F} = 0.20$.

Financial shocks have attracted much work in the literature. Our specification shares many aspects with Khan, Senga, and Thomas (2016), including the role of firm exit as an important transmission mechanism.

5.3.1 Flexible case

Consider the case with flexible wage and number of firms. The quantities are shown by the solid lines in Figure 13. On impact, the increased fixed cost of operation causes lower employment and output. This shock does not bear directly on the productivity of firms, and imparts its direct aggregate effect via the default and exit of those firms at the low end of productivity.[40] Entry of firms m (not shown) collapses to zero in the impact period, it is the reduction in the number of firms that explains the impact and propagation on output an employment. Borrowing and debt also increase following the shock.

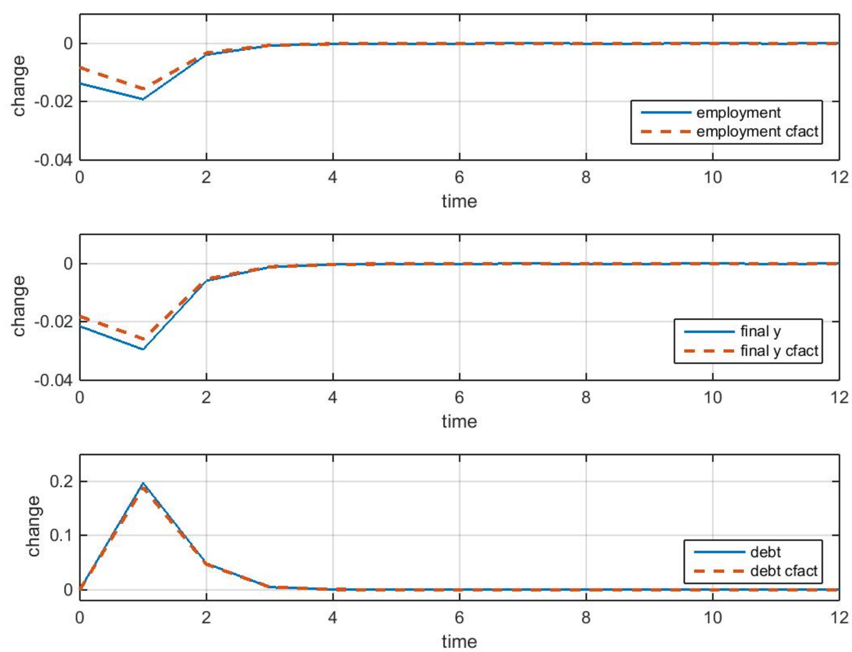

Figure 13: A 10% financial shock, flexible case. Quantities: employment (top), output (middle), debt (bottom). Main experiment (solid lines) and counterfactual (dashed lines).

Figure 14, solid lines, displays the response of the wage and default. Trade-credit default increase by about 1.5 points, driven mainly by more frequent delinquencies. The corresponding increase in the trade-credit loss rate, by the markup equation (21), drives a wider wedge between the price of inputs and the wage. Since the wage falls to clear the labour market on impact, the price of the input (not shown) decreases but by a smaller proportion on impact; subsequently, when positive entry resumes, it will fall further to accommodate the lower value of new entrants.

[40]Generally, c_F has some effect on the amount of input demanded, but then mainly only among the marginal firms.

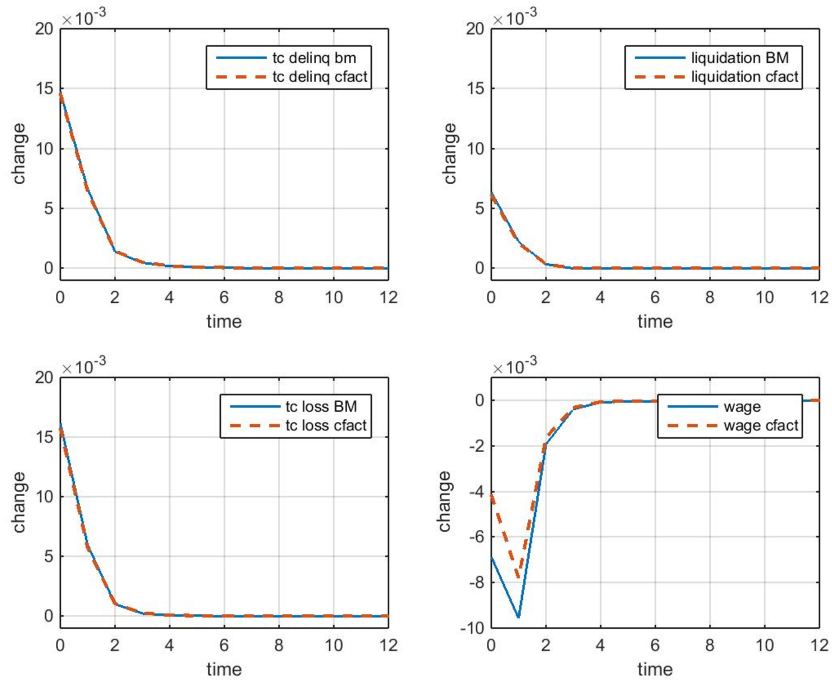

Figure 14: A 10% financial shock, flexible case. Prices and default: delinquency rate (top-left), liquidation rate (top-right), trade-credit loss θ (bottom-left), wage w (bottom-right). Main experiment (solid lines) and counterfactual (dashed lines).

What is the role of trade-credit default? As shown by the dashed lines in Figure 14, the counterfactual constant markup scenario implies a smaller fall in the wage and, accordingly, a larger fall in the price of inputs. Therefore, trade-credit default has a downward effect on the wage and an upward effect on the input price. The former reduces the supply of labour, and the latter reduces the demand for inputs and therefore labour. The difference of the default dashed lines over the solid lines is small, indicating a limited scope for propagation of defaults in this case. Figure 13, dashed lines, shows that there is an amplification effect on employment and GDP from trade-credit default, respectively representing the 40 and 16% of their responses shown earlier in Table 6.

5.3.2 Rigid case

Consider now the same aggregate financial shock in the rigid case. The solid lines in Figure 15 show a practically negligible reduction in both employment and output. We must remember that in the present rigid case the number of firms is held constant thus preventing the contraction in the number of firms; moreover, since there is a surge in firm exit, there is replacement by new firms who are on average of higher productivity. The decline in output and employment is very small. This is accompanied by a large rise in borrowing and debt. Figure 16, solid lines, displays the responses of prices and default rates. Trade-credit default, coming from both delinquency and bankruptcy, increases and implies the rise in trade-credit loss. The input price increases as a result.

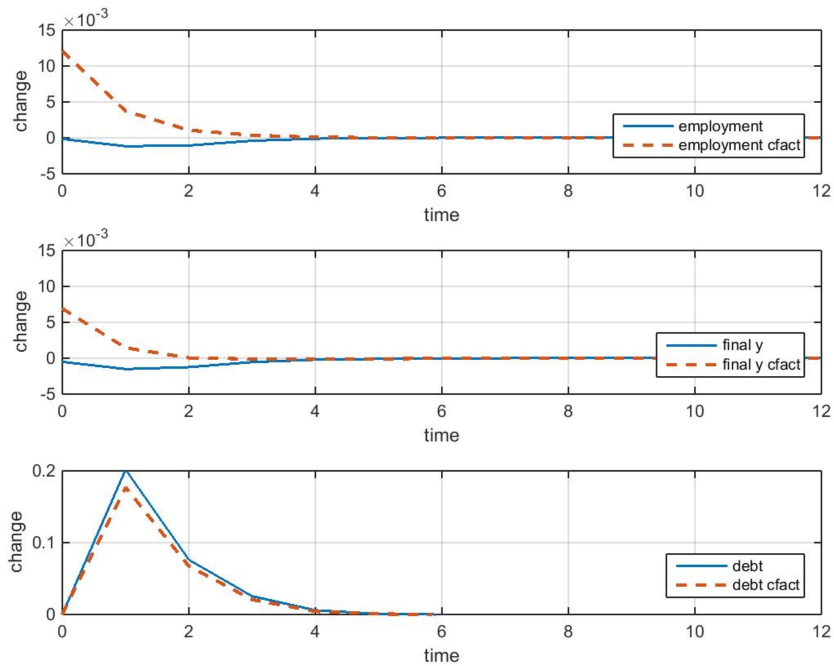

Figure 15: A 10% financial shock, rigid case. Quantities: employment (top), output (middle), debt (bottom). Main experiment (solid lines) and counterfactual (dashed lines).

This rise in the input price must be detrimental to aggregate economic activity. The counterfactual with constant markup and hence input price, shown by the dashed lines in Figure 16, implies a noticeable rise in output and employment, shown in Figure 15, dashed lines. In sum, trade-credit default is the reason why output and employment do not increase following the financial shock in the rigid economy, as indicated by the 100+ amplification markers earlier in Table 6.

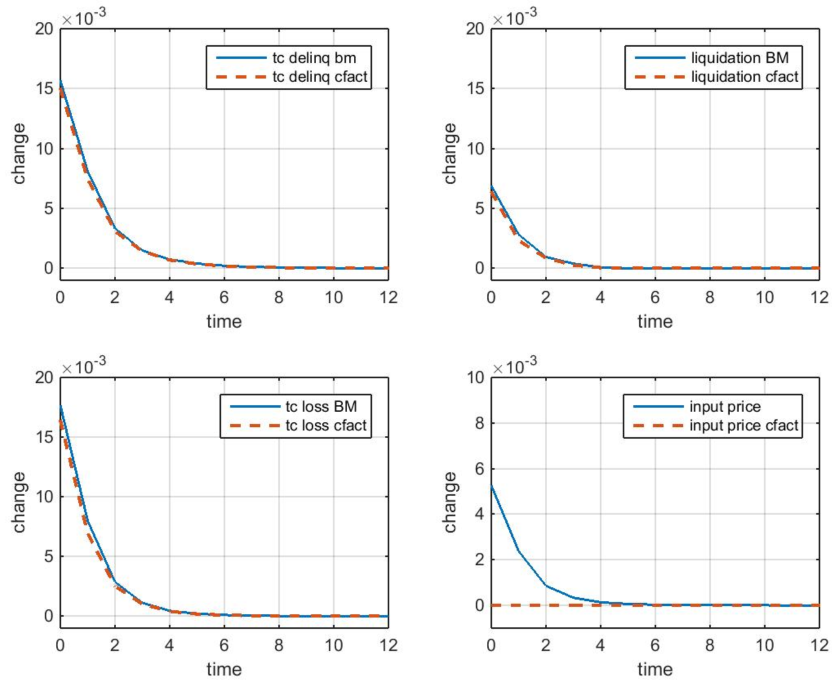

Figure 16: A 10% financial shock, rigid case. Prices and default: delinquency rate (top-left), liquidation rate (top-right), trade-credit loss θ (bottom-left), input price p (bottom-right). Main experiment (solid lines) and counterfactual (dashed lines).

5.4 A volatility shock

Here we consider an increase in the standard deviation of the innovation to the process of individual firm's productivity, σ_η. It is represented by an increase of $\sigma_{\eta,t}$ in period $t = 0$ that is observed at the start of that period, before the realisation of the idiosyncratic shocks. After the impact at time 0, we assume that $\sigma_{\eta,t}$ follows a standard autoregressive process in the difference to its steady-state value, that is $\sigma_{\eta,t} - \sigma_\eta = \rho_{\sigma_\eta}(\sigma_{\eta,t-1} - \sigma_\eta)$, where $\rho_{\sigma_\eta} < 1$ is its persistence. We specify a 25% increase, and a low persistence of 0.20: $\sigma_{\eta,t=0} = 1.25$ and $\rho_{\sigma_\eta} = 0.20$.

Given the series for the standard deviation of the continuous process, to discretise this process, we need to construct a time series for the grid and transition probabilities for ϵ, $\{\mathcal{E}_t, \psi_t\}_{t=0}^T$. We use the idea in Fella, Gallipoli, and Pan (2019) to adapt Tauchen's method to time varying-processes.[41]

[41] They are interested in a life-cyle setting starting from draws over the distribution of innovations. Since we start from the model's stationary distribution, we adapt their fortran module initialisation, check that it reproduces our stationary distribution after two iterations, and in the third iteration we introduce the shock. Then we shift time indexes so the shock corresponds to the model's impact date 0.

This is the type of shock that has received increasing attention since at least Bloom (2009), including Arellano, Bai, and Kehoe (2019) who identify the basic mechanism for the effect of increased volatility. Firms become more cautious in their input hiring because default becomes otherwise a more likely event and, furthermore, credit conditions become tighter. Given the similarities in basic layout of our model with theirs, this mechanism will also be at work here to some degree. We add to the analysis the possible role of trade-credit default.

5.4.1 Flexible case

Consider the case with flexible wage and entry. The quantities are shown by the solid lines in Figure 17. The increased volatility causes lower employment and output, and a rise in debt. This shock does not bear directly on the productivity of final-goods firms on impact, but is saying that firms can expect wider swings going forward. One mechanism at work is that final-goods firms choose to reduce their demand for inputs. In this case, also entry of new firms falls to zero on the impact period.

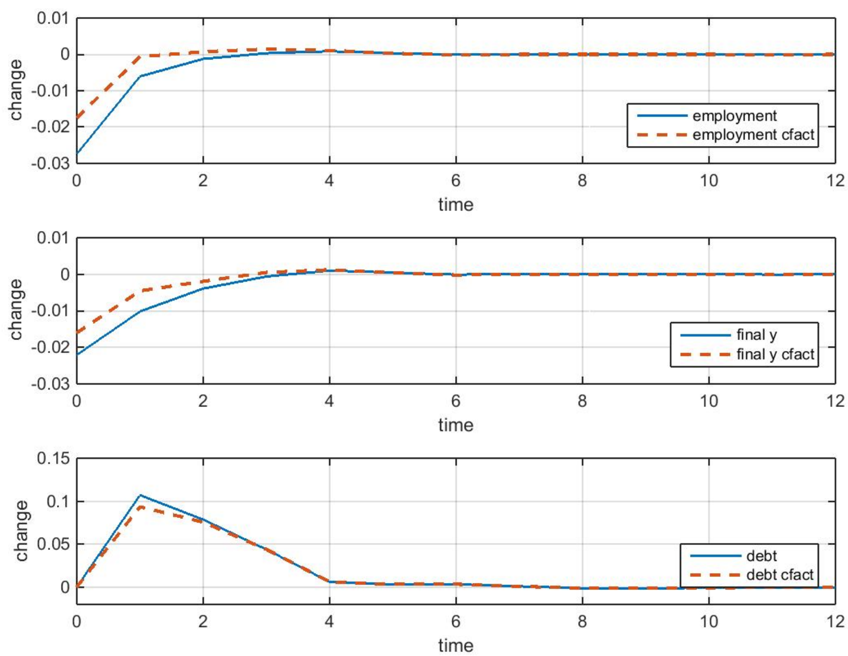

Figure 17: A 25% increase in volatility, flexible case. Quantities: employment (top), output (middle), debt (bottom). Main experiment (solid lines) and counterfactual (dashed lines).

Figure 18, solid lines, displays the response of the wage and defaults. Trade-credit default increases as a result of more frequent delinquencies and bankruptcies, leading to a sharp rise in the trade-credit loss rate. The wage declines to clear the labour market initially on account of the reduced demand for inputs. The price of inputs (not shown) also declines, but to a lesser extent than the wage does given that the intermediate producer must hedge against the increased losses from trade credit, i.e., Eq. (21).

This indicates that trade-credit default must play a role. To assess its quantitative importance we turn to the counterfactual exercise. The dashed lines in Figure 18 show the counterfactual constant-markup scenario. There is some but small feedback from trade-credit losses into further defaults, as shown by the default lines. On the other hand, this counterfactual implies a markedly smaller fall in the wage and, accordingly, a larger fall in the price of inputs (not shown). Trade-credit default exerts a downward effect on the wage and an upward effect on the input price. To quantify the size of this amplification effect, consider Figure 17, dashed lines, which portrays an amplification on employment and GDP from trade-credit default, and one that is quite large, corresponding to the nearly 40 and 30% of the respective responses shown in Table 6 above.

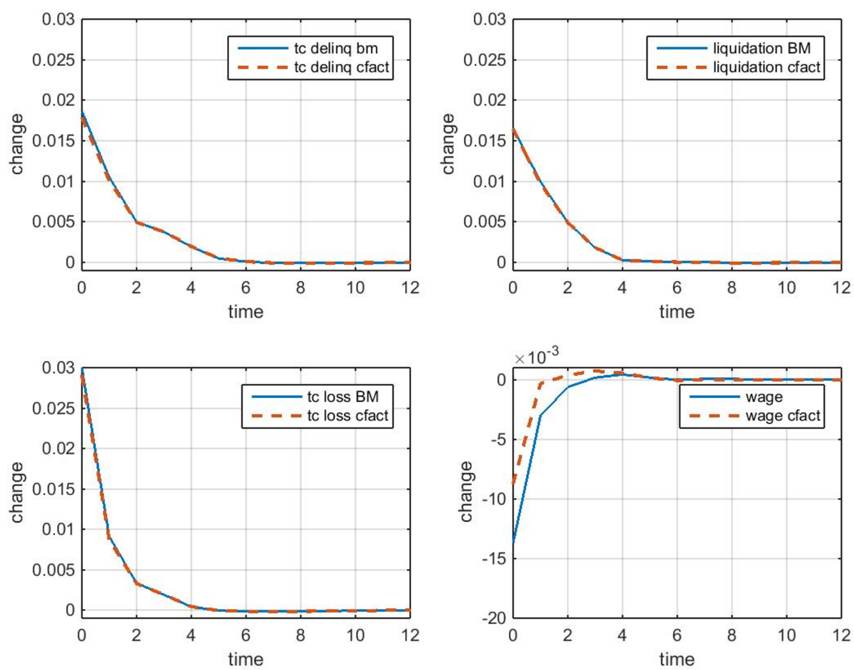

Figure 18: A 25% increase in volatility, flexible case. Prices and default: delinquency rate (top-left), liquidation rate (top-right), trade-credit loss θ (bottom-left), wage w (bottom-right). Main experiment (solid lines) and counterfactual (dashed lines).

5.4.2 Rigid case

Consider now the same volatility shock in the case of constant wage and number of firms. The solid lines in Figure 19 show that while employment clearly falls on impact, GDP decreases by only a small 0.14%. Again firms choose to hire fewer inputs, and that firms are on average more productive explains the muted response of total output. This comes about because the shock leads to almost a doubling of the exit rate of firms which are replaced by new firms that are on average more productive. This shock also causes a rise in borrowing and debt as firms face more liquidity problems. Figure 20, solid lines, displays the response of the input price and default rates. Increased bankruptcy underscores some financial difficulties for firms via the pricing of debt (not shown). Trade-credit default, from both delinquency and bankruptcy, increases and implies the rise in trade-credit loss. The input price increases as a result adding to firms' difficulties.

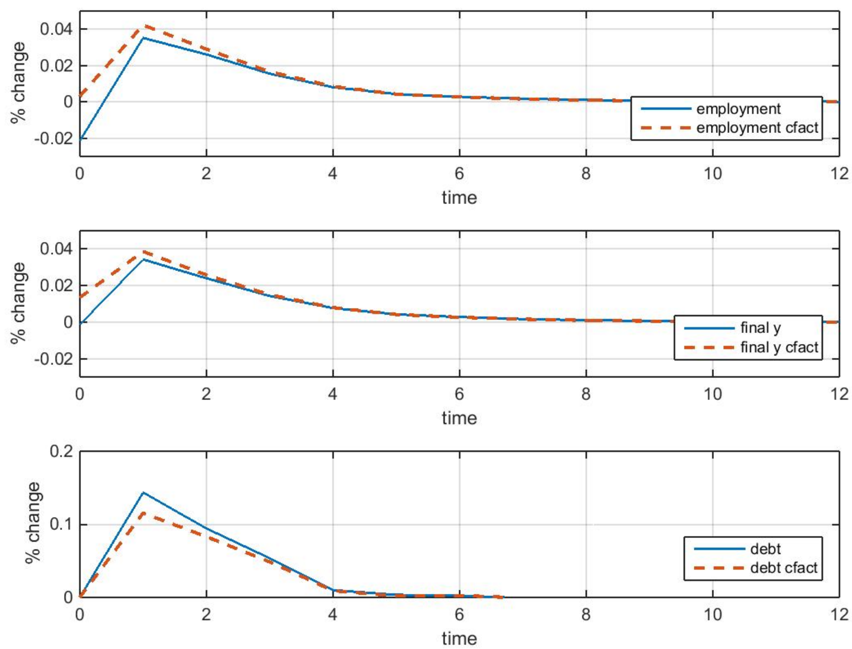

Figure 19: A 25% increase in volatility fixed cost, rigid case. Quantities: employment (top), output (middle), debt (bottom). Main experiment (solid lines) and counterfactual (dashed lines).

How important is this latter channel? Turning to the counterfactual experiment, with constant markup and hence input price, Figure 20, dashed lines, shows that there is some feed back of the trade-credit loss into further defaults. Figure 19, dashed lines, shows a noticeably rise in both output and employment. Trade-credit default is therefore the fundamental reason why output and employment do not increase following the financial shock in the rigid economy, as it was implied earlier in Table 6.

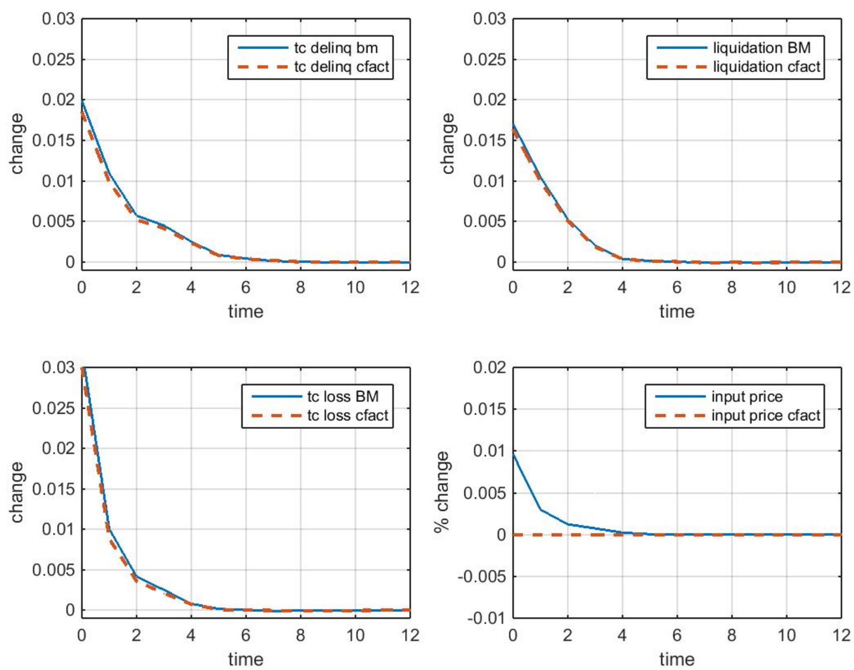

Figure 20: A 25% increase in volatility, rigid case. Prices and default: delinquency rate (top-left), liquidation rate (top-right), trade-credit loss θ (bottom-left), input price p (bottom-right). Main experiment (solid lines) and counterfactual (dashed lines).

6 Concluding remarks

In this paper we ask whether trade-credit default may be an important amplification mechanism of aggregate shocks, and find that it may well be. The fundamental reason is that the aggregate risk of failure in trade-credit payments is priced into the markup of supplied intermediate inputs, depressing wages or increasing the cost of inputs. It is because of the latter than it also can have a feedback effect on the propagation of delinquencies and bankruptcies. This mechanism is appealing and accords with the evidence from recent studies on individual firm data.

Our results rest on a model of heterogeneous firms who purchase intermediate inputs partly on trade credit before observing the realisation of their productivity. This model is a meaningful contribution in its own right as it introduces the trade-credit default channel in a recent generation of heterogeneous-firm models. Besides following from delinquency, in our model trade-credit default also follows from the event of firm bankrupcy/liquidation of the type studied in the existing literature. In this way, the present model uncovers a novel mechanism for a significant role of firm liquidation risk in macroeconomic fluctuations.

Having made the case for the consideration of the trade-credit risk channel in macroeconomic analysis, much work lies ahead. The model is deliberately terse. Immediate extensions will incorporate, for instance, explicit investment choices in capital, more flexible equity financing conditions, and aggregate stochastic shocks. Most interestingly, the presents paper is begging for integrating the endogenous determination of trade credit as part of the intertemporal choices of the firm, a challenge which is part of our ongoing work.

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A Appendix

A.1 Definition of equilibrium

An equilibrium consists of the following functions

- For final-goods firms: policy rules $\{g^{ND}, g^x, r^b, r^x, d^b, d^f, d^x, \pi^{ND}, \pi^b, x\}$, and value functions $\{V^{ND}, V^b, V^x, W\}$.
- Loan price functions: q^{ND}, q^x.

- Discount rate function for firms and lenders ρ.
- Risk-free discount price function Q.
- Input price function p.
- Wage function w.
- Aggregate trade-credit default function θ.
- Aggregate dividends Π.
- Policy functions for households: a', l, c.
- Transition function for firms' distribution H^{μ}.
- Transition function for household assets H^{A}.
- Aggregate labour supply L.
- The probability distribution of entrants μ^E (and b^E and q^E).
- The measure μ and mass N of firms.
- Post-entry measure $\hat{\mu}$.
- Rate of entry m.
- Value of entry W^E.

They must satisfy the following conditions:

1. Final-good firms: Given q^{ND}, q^x, ρ, p, and H, final-good firms's outcomes g^{ND}, g^x, r^b, r^x, d^b, d^f, d^x, π^{ND}, π^b, and x, solve (1), (2), (3), (5), (4), (6), (7).
2. Lenders zero profit: Given H, Q, and d^b, r^b, and x, debt prices q^{ND} and q^x satisfy equations (19) and (20).
3. Distribution transition: Given g^{ND}, g^x, d^x, d^b and x, the transition probabilities Prob and Prob^E are determined by (11), (12). Given those and m, N, μ, μ^E, then H^{μ} and $N'\mu'$ follow (14), (13), and (15).
4. Post-entry distribution: Given m, N, μ, μ^E, then $\hat{\mu}$ is given by (16).
5. Distribution of entrants: Given q^{ND}, (9) determines $\mu^E(\epsilon_{-1}, b)$, and b^E q^E.
6. Labour market clearing: The functions m, μ, μ^E, N, L, and x satisfy (27).
7. Trade-credit loss and failure: Given d^x, r^x, x, d^b, d^f, p and $\hat{\mu}$, then θ is given by (17).
8. Input-producers: The functions p, w and θ satisfy (21).
9. Free entry: Given W and μ^E, the function W^E in (8) and p satisfy (10).
10. Discount: Given Q, ρ is determined by (23).
11. Aggregate dividends: Given d^b, d^x, x, π^{ND} and π^b, m, N and $\hat{\mu}$, then Π is given by (18)
12. Consumer: Given H, Q, w and Π, the household's $a'(\cdot), l, c$ solve the problem in (22).
13. Clearing in final goods. Given C, m, x, $\hat{\mu}$ and N, and d^b and d^f, eq. (28) holds.
14. Aggregate consistency: a', H^A, L and C satisfy (24), (25) and (26).

By Walras' law, we omit the condition of clearing in the market for securities.

A.2 Algorithm for the stationary equilibrium

In a stationary equilibrium, the Euler equation associated with (22) implies $Q = \beta$ and then, by (23), we can pin down the equilibrium discount and bond price as

$$\rho = Q = \beta.$$

The labour supply optimality condition associated with (22) reduces to $u_c(c,l)w(S)+u_l(c,l) = 0$. This condition implies, via (26), aggregate labour supply L as a function of c and w which here we denote $l^S(c,w)$.

A stationary equilibrium can be constructed through the following steps:

1. Guess price p.
2. Final-goods firms and lenders: $p \to q, x, g, \pi, r, d, V, W$
   (a) Firms Eq. (1) to (7): $p, q \to x, g, \pi, r, d, V, W$.
   (b) Lenders Eq. (19) and (20): $d, r, x \to q$.
3. Distribution entrants: By (9), 3(i) $q \to q^E \to b^E$, and 3(ii) $b^E \to \mu^E$.
4. Final goods free entry: $W, \mu^E \to W^E$ by Eq. (8).
5. Update p via Eq. (10). Back to 2.
6. Distribution: $d, g, x, \mu^E \to \mu, m$
   (a) Guess μ.
   (b) Find m to match exit by Eq. (13).
   (c) Update μ by transition function Eq. (14), (11), (12), and (15). Back to 6b.
7. Post-entry distribution: $\mu, m, \mu^E \to \hat{\mu}$ by Eq. (16).
8. Trade credit loss: $\hat{\mu}, d, x, r, p \to \theta$ by Eq. (17).
9. Input pricing: $p, \theta, \eta \to w$ by Eq. (21).
10. Market clearing:
    (a) Guess N.
    (b) Consumption: $N, m, x, d, \hat{\mu} \to c$ by Eq. (28).
    (c) Labour supply: c, w, $\to L$ from (22) as given by $l^S(.,.)$ above via (26).
    (d) Labour clearing: $\mu^E, \mu, m, x, L \to N$ by Eq. (27). Back to (b).

Note the first 5 steps fully determine p iteratively. The remaining steps are direct, except for iterations in determining the distribution and market clearing.

A.3 Computation with extreme-value shocks

The main block in the characterisation of an equilibrium is the joint determination of final-goods firms choices and lenders loan pricing, for a given price of the intermediate input p, Firms take as given the loan pricing functions in (19) and (20). Lenders take as given the firms default and delinquency policy rules resulting from (1) through to (7). We define all value functions and policy functions on grids for the states ϵ, b, ν, and x.

Repayment options. The liquidation, delinquency or repayment outcome is a discrete choice variable and we extend the model with extreme-value shocks affecting these choices. In the decision problem of the firm between the three options in (6), we introduce shocks ζ^{ND}, ζ^x and ζ^b, associated with the decisions of repaying, delinquency, and liquidating, respectively. They are collected in a vector ζ. The decision problem is subject to an extra exogenous state ζ and is transformed into

$$\tilde{V}(\epsilon, b, \nu, x, \zeta) = \max \left\{ V^{ND}(\epsilon, b, \nu, x) + \zeta^{ND}, V^x(\epsilon, b, \nu, x) + \zeta^x, V^b(\epsilon, b, \nu, x) + \zeta^b \right\}, \qquad (29)$$

which gives decision rules $\tilde{d}^x(\epsilon, b, \nu, x, \zeta)$ and $\tilde{d}^b(\epsilon, b, \nu, x, \zeta)$. (I omit the aggregate states S for notational simplicity here.) Assume that these shocks follows a Gumbel, or Type-I extreme value, distribution with location and scale parameters μ_ζ and σ_ζ, that is $\zeta^j \sim G(\mu_\zeta, \sigma_\zeta)$, with cdf

$$F(\zeta^j) = \exp\left(-e^{-\frac{\zeta^j - \mu_\zeta}{\sigma_\zeta}}\right),$$

$$\text{mean}(\zeta^j) = \mu_\zeta + \sigma_\zeta\gamma, \text{ with } \gamma = 0.5772, \text{ var}(\zeta^j) = \sigma_\zeta^2\pi^2/6, \text{ mode}(\zeta^j) = \mu_\zeta.$$

Given this specification, the solution to this problem can be characterised in terms of a distribution of ex-ante probabilities among the different options, which are functions of the "fundamental" values V^j's. Specifically:

$$\begin{aligned} d^x(\epsilon, b, \nu, x) &= \frac{e^{V^x(\epsilon,b,\nu,x)/\sigma_\zeta}}{\sum_{j\in\{ND,x,b\}} e^{V^j(\epsilon,b,\nu,x)/\sigma_\zeta}} \\ d^b(\epsilon, b, \nu, x) &= \frac{e^{V^b(\epsilon,b,\nu,x)/\sigma_\zeta}}{\sum_{j\in\{ND,x,b\}} e^{V^j(\epsilon,b,\nu,x)/\sigma_\zeta}} \end{aligned}$$

On the other hand, the expected value

$$V(\epsilon, b, \nu, x) \equiv \mathbb{E}\tilde{V}(\epsilon, b, \nu, x, \zeta) = \mu_\zeta + \sigma_\zeta\gamma + \sigma_\zeta \log \sum_{j\in\{ND,x,b\}} e^{V^j(\epsilon,b,\nu,x)/\sigma_\zeta}.$$

We normalise by a choice of μ_ζ so that $E\max\{\zeta^{ND}, \zeta^x, \zeta^b\} = 0$. It is known that $E\max\{\zeta^{ND}, \zeta^x, \zeta^b\} = \mu_\zeta + \sigma_\zeta\mu_\zeta + \sigma_\zeta \log J$ where $J = 3$ is the number of discrete options. Therefore our normalisation implies $\mu_\zeta = -\sigma_\zeta\mu_\zeta - \sigma_\zeta \log J$. Finally, to deal with issues of computer arithmetics in the evaluation of the exponential function, we perform the calculations under some convenient but innocuous transformations.

Demand for inputs. For a given discrete state, the decision about the quantity of input $x(\epsilon_{-1}, b, \nu)$ solving (7) can be computed on the discrete grid for x. In examples, we have found x to be show discontinuities and unwarranted patterns. Although extreme-value shocks in repayment discrete choices help resolve some of these issues, we have also introduced shocks affecting the choice of x_i on the grid $X = \{x_1, ..., x_{N_x}\}$, given by ζ_i following a Gumbel distribution, which we stack in a vector ζ. The original problem in (7) is reformulated as

$$\tilde{W}(\epsilon_{-1}, b, \nu, \zeta) = \max_i \{RHS^x(x_i \mid \epsilon_{-1}, b, \nu) + \zeta_i\}_{i=1}^{N_x},$$

where we define

$$RHS^x(x_i \mid \epsilon_{-1}, b, \nu) \equiv \sum_\epsilon \psi_\epsilon(\epsilon|\epsilon_{-1}) V(\epsilon, b, \nu, x_i).$$

The solution can be described as an ex-ante probability distribution over the choices given by

$$\text{prob}^x(x \mid \epsilon_{-1}, b, \nu) = \frac{e^{RHS^x(x|\epsilon_{-1},b,\nu)/\sigma_\zeta}}{\sum_{x_i} e^{RHS^x(x_i|\epsilon_{-1},b,\nu)/\sigma_\zeta}}.$$

The maximised value

$$W(\epsilon_{-1}, b, \nu) = E[\tilde{W}(\epsilon_{-1}, b, \nu, \zeta)] = \mu_\zeta + \sigma_\zeta\gamma + \sigma_\zeta \log\left[\sum_{x_i} e^{RHS^x(x_i|\epsilon_{-1},b,\nu)/\sigma_\zeta}\right].$$

Normalising so the expected max of the shocks is zero means $\mu_\zeta + \sigma_\zeta\gamma + \sigma_\zeta \log N_x = 0$.

Borrowing. The borrowing decision in (1) and (3) is computed as a continuous variable. When interior, it is the value that exactly raises the resources qb' required to meet the firm's financial obligations, as in the theoretical characterisation provided in the propositions. However, one consequence of the possible discontinuity in the price of debt is that the value of resources raised by borrowing will also have one discontinuity in b'. In some cases, it will not possible to find the exact zero, which is when the solution corresponds to the discontinuity point. In other cases, the solution will be an exact zero but there might be two roots, and the solution corresponds to the lowest value. Computationally, it will necessary to identify the discontinuity point, the sign of the discontinuity, and the possibility of multiple zeroes. When the solution coincides with the discontinuity value, the firm's choice of debt b' may raise more resources than necessary to meet the needs of liquidity, and this residual has to be apportioned accordingly, as dividends in the no-default case, and as payments to suppliers in the delinquency case. When the solution is a root, it is bracketed and found using simple bisection. When using extreme-value shocks in the firm's decisions, however, the discontinuities in the price of debt are much mitigated and we have found there is only one root.

Debt pricing function. In this model, the price of debt q^{ND} and q^{x} in (19) and (20) and, therefore, the resources raised via borrowing may in general be discontinuous in the level of debt chosen b'. This discontinuity occurs at a b' that appears to coincide with the value leading to the discontinuity in the choice of input tomorrow $x(b', ...)$. This discontinuity could in principle be a jump or a drop: a jump as the threshold implies a drop in the probability of bankruptcy on debt since delinquency means operating profits will increase and release cash for repaying bank debt; a drop as the punishment for delinquency may raise the probability of bankruptcy if the productivity punishment for delinquency is sufficiently large. Therefore the price of debt is in general discontinuous with an indeterminate sign. Nonetheless, the presence of EV shocks appears to remove this complication in practice.

A.4 Internal calibration

The steps of the procedure, in outline, are as follows:

(i) Set p, and the three deep parameters $\tilde{\nu}$, λ and c_F.
(ii) Solve firms-lenders outcomes (step 2 of equilibrium algorithm).
(iii) Entrants' distribution μ^E, with b^E and q^E, and W^E (steps of computation 3(ii), 4, 3(i) and 5). With $\alpha^E = 0$, $b^E = 0$ and step 3(i) and q^E are irrelevant. With $\alpha^E > 0$, loop: guess b^E; find μ^E and q^E (step 3) ; calculate W^E (step 4); update $b^E = \alpha^E W^E/((1-\alpha^E)q^E)$ (steps 3(i) and 5), back to 2nd step here until convergence.
(iv) Distribution μ, m, and post-entry distribution $\hat{\mu}$ (steps 6, 7).
(v) Trade-credit loss rate θ (steps 8).
(vi) Calculate moments (debts, defaults, etc), and check against data.
(vii) Update p and $\tilde{\nu}$, λ and c_F. Back to point (ii).
(viii) Back out ξ^E via step 5.
(ix) Back out w as in step 9, and number of firms N and consumption c via step 10. Find B to imply the target for $L = l_{\text{target}}$ via step 10 of the equilibrium.

A.5 Dynamics

Under flexible wages and free entry, entry m can be zero in some periods. The main steps to finding the equilibrium path are as follows:

1. Guess a long enough time horizon T and consider solving for $t = 0, 1, ..., T$.
2. Guess a path for aggregate discount $\{Q_t\}_{t=0}^T$, and obtain $\{\rho_t\}_{t=0}^T$ by (23).
3. Guess a path for input prices $\{p_t\}_{t=0}^T$.
4. Guess a path for aggregate entry rate $\{m_t\}_{t=0}^T$ and $\{c_t\}_{t=0}^T$.
5. Backward loop: Starting at the terminal steady state at $t = T$, for given $\{\rho_t, Q_t\}_{t=0}^T$, proceed backwards to obtain sequence of input prices $\{p_t\}_{t=0}^T$, and the paths for functions $\{g_t, x_t, d_t, r_t, \mu_t^E\}_{t=1}^T$ and $\{q_t\}_{t=1}^T$:
   - At t where $m_t > 0$, by solving the equilibrium between final-goods firms and lenders and the free entry condition.
   - At t where $m_t \leq 0$, by solving the equilibrium between final-goods firms and lenders, ignoring free-entry, and leaving the input price unchanged.
6. Forward loop: Given the above paths $\{p_t\}_{t=0}^T$ and $\{g_t, x_t, d_t, r_t, \mu_t^E\}_{t=0}^T$, and some $\{m_t, c_t\}_{t=0}^T$ proceed forwards to obtain the paths $\{\theta_t, w_t, l_t, \mu_{t+1}, N_{t+1}\}_{t=0}^T$, and updated $\{c_t, m_t\}_{t=0}^T$ and $\{p_t\}_{t=0}^T$ that satisfy the corresponding equilibrium conditions:
   - At t where $m_t > 0$, by taking p_t as given and updating m_t via labour market clearing.
   - At t where $m_t \leq 0$, by setting $m_t = 0$ and updating p_t via the input markup condition.

   Back to 5.
7. With the updates $\{p_t\}_{t=0}^T$, back to 4.
8. Update $\{\rho_t, Q_t\}_{t=0}^T$ using household's intertemporal optimality and arbitrage. Back to 5 until convergence.
9. Check that T is long enough. Update T and back to 2.

The details of the two loops in steps 5 and 6 above are as follows.

Backward loop:

1. Initialise $q = q_T$ and $W = W_{T+1}$.
2. Let $\rho = \rho_T$ and $Q = Q_T$.
3. Set $t = T$.
4. When $m_t > 0$, find p_t as p that solves free entry (10):
   - Firms Eq. (1) to (7): $p, \rho, q, W \to \tilde{W}$.
   - Free entry Eq. (9), (8), (10): $\tilde{W}, q \to \mu^E, W^E$. Update p and iterate.

   Record time series: $p_t = p$.
   When $m_t \leq 0$, update $p = p_t$. Record time series: $p_t = p$ (although not strictly needed.)
5. Firms' policy functions Eq. (1) to (7): $p, \rho, q, W \to W, x, g, \pi, r, d$.
   Record time series: $W_t = W$, $r_t = r$, $g_t = g$, $\mu_t^E = \mu^E$, $d_t = d$, $x_t = x$.
6. Find debt price functions q_{t-1} as q that meets lenders zero profits at $t - 1$:
   - Let $Q = Q_{t-1}$ and $\rho = \rho_{t-1}$.
   - Lenders Eq. (19) and (20): $Q, d, x, r \to q$.

Record time series: $q_{t-1} = q$
7. If $t > 0$, update $t = t - 1$ and back to step 4.

Forward loop:

1. Initialise $N = N_0$ and $\mu = \mu_0(:)$.
2. Set $t = 0$.
3. Let $g = g_t$, $r = r_t$, $d = d_t$, $x = x_t$, and $p = p_t$, $\mu^E = \mu_t^E$, $c = c_t$, $m = m_t$, $N = N_t$.
4. When $m_t > 0$, loop over m for labour market clearing:
   - Post-entry distribution: $\mu, \mu^E, N, m \to \hat{\mu}$ by Eq. (16).
   - Delinquency: $\hat{\mu}, d, x, r, d, p \to \theta$ by Eq. (17).
   - Input pricing: $p, \theta \to w$ by Eq. (21).
   - Labour supply: c, w, $\to L$ from (22) via (26).
   - Labour clearing: $\hat{\mu}, N, x, L \to m$ by Eq. (27). Back to top.

   Record time series: θ_t, L_t, m_t, w_t.
   When $m_t \leq 0$, no loop required to find p:
   - Set $m_t = m = 0$.
   - Post-entry distribution: $\mu, \mu^E, N, m = 0 \to \hat{\mu}$ by Eq. (16).
   - Labour clearing: $\hat{\mu}, N, x, m = 0 \to L$ by Eq. (27).
   - Labour supply: c, L, $\to w$ from (22) via (26).
   - Delinquency: $\hat{\mu}, d, x, r, d, p \to \theta$ by Eq. (17).
   - Input pricing: $w, \theta \to p$ by Eq. (21).

   Record p_t, and θ_t, L_t, m_t, w_t.
   Record time series: $c_t = c$.
5. Population dynamics: $N, m, \mu, \mu^E, x, d \to N'$ by Eq. (13).
   Record time series: $N_{t+1} = N'$.
6. Distribution dynamics: $m, \mu, \mu^E, g, x, d \to \mu = \mu N_t / N_{t+1}$ by Eq. (14), (11), (12), and (15).
   Record time series: $\mu_{t+1} = \mu$.
7. If $t < T$, update $t = t + 1$, and back to step 3.

The updating of p within point 4 of the forward loop for the case $m_t \leq 0$ may need some damping for convergence.