Energy Prices and Household Heterogeneity: Monetary Policy in a Gas-TANK*

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Abstract

How does household heterogeneity affect the transmission of an energy price shock? What are the implications for monetary policy? We develop a small open economy TANK model that features labor and an energy import good as production inputs (Gas-TANK). Given complementarities in production inputs, higher energy prices reduce the labor share of total income. Due to borrowing constraints, this translates into a drop in aggregate demand. Higher price flexibility insures firm profits from adverse energy price shocks, further depressing labor income and demand. We illustrate how the transmission of shocks in a RANK versus a TANK depends on the degree of complementarity between energy and labor in production and the degree of price rigidities. Optimal monetary policy is less contractionary in a TANK and can even be expansionary when credit constraints are severe. Finally, the contractionary effect of an energy price shock on demand cannot be generalized to alternate supply shocks, as the specific nature of the supply shock affects how resources are redistributed in the economy.

JEL Codes: F41, E52, E31, E21, E23

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1 Introduction

In early 2022, energy prices increased to historically high levels as Russia's invasion of Ukraine increased the risk of disruptions to the energy trade. From the standpoint of an energy importer, the developments in global energy prices represent a deterioration in the terms of trade. This implies a contraction in income flowing to domestic production inputs, including labor income. If households face limits in their access to financial markets, the contraction in income can translate into a drop in aggregate demand. That is, a supply shock can have demand side effects.

We highlight the demand side effects of this supply shock with a two-agent New Keynesian (TANK) model where agents differ in their sources of income and ability to smooth consumption. We use this setting to show that the implications for aggregate demand and inflation depend on how the cost of the energy price shock is distributed between the labor share and profit share of total income. The model features two types of households: *constrained worker households*, who consume out of their labor income and have no access to financial markets, and *unconstrained households*, who earn firm profits and have free access to financial markets.[1] Our small, open-economy model also features labor and imported energy as complementary inputs in production. We assume a constant elasticity of substitution (CES) production technology with low elasticity of substitution between labor and energy, which allows the labor share of total income to fall as energy prices increase.

We show that the impact of energy prices on demand depends critically on the substitutability of production inputs and household heterogeneity. This is because the degree of substitutability among production inputs determines the response of workers' income to the shock. Due to borrowing constraints, this affects aggregate demand. Compared to the representative household in a RANK (representative agent New Keynesian) model, the constrained worker household will experience a stronger consumption response to the real income squeeze following an energy price shock because of its inability to smooth consumption by borrowing.[2] The channels we highlight are absent in the standard RANK model, which assumes all households are the same and that they can borrow to smooth consumption in the presence of adverse shocks. We illustrate this mechanism in a small, stylized model and embed it in medium scale model that is amenable for studying optimal policy.

The magnitude of these channels depends on the degree of price rigidity and the elasticity of substitution between energy and labor. Assuming production inputs are sufficiently difficult to substitute or that prices are sufficiently flexible,[3] an energy price shock has a negative impact on aggregate demand. This supply shock therefore has a self-correcting effect, as the consequent contraction in aggregate demand dampens inflationary pressures.

Is the demand contraction that follows a rise in energy prices a common feature of supply disturbances? We consider the dynamics of a productivity shock in our TANK model.[4] Both an increase in

[1] Heterogeneous-agent New Keynesian (HANK) models are not analytically tractable, as they typically feature a wealth distribution that responds endogenously to aggregate shocks. Their complexity makes it difficult to identify the mechanisms at work. Debortoli and Galí (2017) show that a two-agent New Keynesian (TANK) model is able to capture fundamental properties of heterogeneous-agent models. Such models admit analytical solutions and can be extended to match the implications of HANK models, in terms of consistency with micro data and predictions for the macroeconomic effects of policy (Blanchard and Gali, 2007; Bilbiie, 2008; Cantore and Freund, 2021).

[2] In other words, while an energy price shock is a supply shock in a RANK model, it has elements of both a supply and demand shock in our TANK model.

[3] The aforementioned contraction in aggregate demand can be moderated by the behavior of markups. Given price rigidities, an increase in energy prices reduces firms' markups. This redistributes income in favor of constrained worker households, hence increasing aggregate demand. Instead, with higher price flexibility, firms are able to pass the cost of the more expensive energy to workers by raising prices.

[4] An energy price shock has also traditionally been modeled as a technology shock, or a shock that affects the productive capacity of the economy (Bruno and Sachs (1985), see Kilian (2008) for references).

energy prices and an adverse productivity shock raise firms' marginal costs, leading to an increase in inflation. While the supply-side impact is the same, energy prices and productivity shocks yield opposing effects on the demand side. An adverse productivity shock leads to a fall in markups, as firms must hire more labor for the same amount of output. This increases constrained worker households' income, which boosts aggregate demand. However, an energy price shock in our model lowers constrained worker households' income and leads to a fall in economic activity. We conclude that no generalization can be made about the effects of supply shocks on aggregate demand, as the nature of the shock crucially affects the way resources are redistributed in the economy.

Next, we consider a normative question: what is the optimal response of monetary policy to an energy price shock in our model and how does it depend on the degree of household heterogeneity? In contrast to a RANK economy, energy price shocks in the TANK economy have both supply and demand side effects. On the one hand, higher energy prices place upward pressure on inflation, which calls for a monetary policy tightening. On the other hand, it restricts aggregate demand, which instead calls for a monetary loosening. In our baseline calibration, we find that in both the RANK and the TANK models, optimal monetary policy is contractionary in order to counteract the inflationary effect of the shock. However, in the TANK model, the negative impact of higher energy prices on aggregate demand mitigates inflationary pressures. An energy price shock therefore has a milder inflationary effect in the medium term, which requires a milder increase in the interest rate.[5] Finally, we explore conditions under which optimal policy may actually be expansionary in the presence of an adverse supply shock. We find that this is true when the share of financially constrained households increases.[6]

1.1 Related Literature

This paper contributes to a literature that emphasizes the demand side effects of an energy price shock.[7] While such shocks have traditionally been modeled as aggregate supply shocks or as technology shocks in domestic production, these approaches cannot explain large fluctuations in real output. More recent approaches place the main transmission channel on the demand side of the economy. That is, energy price shocks affect the economy primarily through their effect on consumer expenditures and firm investment expenditures. Hamilton (2008) provides empirical evidence to show that energy price shocks mainly affect the economy through a disruption in consumers' and firms' spending on non-energy goods and services. There is also evidence that firms perceive energy price shocks as shocks to product demand rather than shocks to the cost of production (Lee and Ni, 2002) . Among policymakers, an increase in energy prices is also thought to slow economic growth primarily through its effects on consumer spending (Natal, 2012).[8] This paper formalizes this intuition by allowing energy prices to affect aggregate demand through a heterogeneous impact on households depending on their sources of income and access to borrowing. To our knowledge, this is the first paper to explore this transmission

[5]Recent work by Guerrieri et al. (2022a) and Caballero and Simsek (2022) also provides conditions under which optimal monetary policy is less contractionary in response to supply shocks.

[6]Higher price flexibility also warrants more expansionary policy. As we emphasize throughout this paper, the demand effect of higher energy prices crucially depends on the evolution of firms' markups. If firms are able to increase prices to preserve markups, the costs of the energy price shock will be passed to workers, who will experience a more severe reduction in their income. Assuming a higher degree of price flexibility, constrained households experience a more pronounced drop in their income relative to unconstrained households, as reflected by the income gap. This leads to a deeper contraction in aggregate demand, which warrants looser monetary policy in the TANK model relative to its RANK counterpart.

[7]The mechanism in our model relies on complementarities between production inputs. Supply shocks with demand side effects can also be found in models with complementarities between consumption goods and distribution services (Corsetti et al., 2008) and complementarities among sectors (Guerrieri et al., 2022b; Cesa-Bianchi and Ferrero, 2021).

[8]See Kilian (2008) for a discussion of this literature. Share of energy in consumption basket: X, versus production expenditures: Y.

channel and to study the optimal monetary policy response when accounting for such demand side effects of an energy price shock.

Recent studies have noted the distributional impact of the energy price shock due to its effect on the consumption baskets of heterogeneous households (Celasun et al., 2022; Bachmann et al., 2022; Battistini et al., 2022; Hobijn and Lagakos, 2005). An increase in energy prices can affect households' purchasing power through higher prices for energy products. Since poorer households spend a relatively large percentage of their income on energy, they receive a larger hit in terms of inflation when energy prices increase. We show that the shock can be regressive through an alternate channel, through a heterogeneous impact on households depending on their income sources and ability to smooth consumption.[9] Moreover, the shock also affects aggregate demand since financially constrained households will reduce purchases of other goods.[10]

Our paper also contributes to a literature that studies the transmission of shocks in a heterogeneous agent model. The interaction of household heterogeneity with nominal rigidities can amplify the contractionary effect of TFP shocks on employment (Furlanetto and Seneca, 2012) and fiscal policy shocks on output (Galí et al., 2007). However, we show that an interaction between household heterogeneity and production complementarity is crucial to generate the contractionary effect of an energy price shock on output. Our assumption of a CES production function with labor and energy allows for changes in energy prices to affect energy costs as a share of total income.[11]

More broadly, this paper builds on the vast literature that studies the implications of household heterogeneity for macroeconomic dynamics (Bilbiie, 2008; Debortoli and Galí, 2017; Kaplan and Violante, 2018; Bilbiie, 2019; Acharya and Dogra, 2020; Bilbiie, 2020; Broer et al., 2020; Bilbiie and Ragot, 2021; Cantore and Freund, 2021; Bilbiie et al., 2022). Ravn and Sterk (2021) also show that a supply shock, namely productivity, can have effects on the demand side due to incomplete markets, sticky prices, and endogenous unemployment risk. The precautionary savings motive is central to their results, which is a different mechanism from ours.

Finally, we contribute to a literature that examines the implications of different monetary policy reactions to energy price shocks. The most closely related papers are Natal (2012) and Montoro (2012), which abstract from household heterogeneity. They show that when energy is a complementary input in production, an endogenous cost-push shock arises from the gap between the natural and efficient level of output. In Montoro (2012), a low elasticity of substitution between labor and energy leads to a tradeoff between stabilizing output and inflation. This tradeoff is generated by the convexity of real marginal costs with respect to the real oil price, which produces a time-varying wedge between the marginal rate of substitution and the marginal productivity of labor. Eliminating the distortions in the steady state makes the wedge less sensitive to the energy price. Similarly, in Natal (2012), the impact of an energy price shock on the oil cost share (and therefore output) in the flexible prices and wages equilibrium is larger when the steady state distortion due to monopolistic competition is larger. Natural (distorted)

[9] In the UK, about X of all energy use takes the form of final consumption (the use of such products by consumers). The remainder involves energy being used in the production of non-energy goods and services (intermediate consumption).

[10] If these costs can be passed onto the final prices, then this affects households' purchasing power directly.

[11] The CES production function is a common feature of models that incorporate energy in the production function. Most recently, Bachmann et al. (2022) show that the losses to the German economy of an embargo on Russian energy imports depend crucially on the degree of substitutability between gas and other inputs. They show that the assumption of Leontief production is inconsistent with empirical evidence and leads to a number of implausible predictions with regard to the evolution of marginal products, prices, and expenditure shares. For example, production would drop one-for-one with energy supply in case of zero substitutability between production inputs. Note that in the case of a Cobb-Douglas production technology (elasticity of substitution equal to 1), energy prices have no impact on the labor share of total factor expenditure. Instead, they would only reduce firms' markups, redistributing income in favor of constrained worker households, which increases aggregate demand.

output falls by more than efficient output, which increases the cost of strictly stabilizing inflation.[12] [13]

The rest of this paper is structured as follows. We discuss our model in Section 2, with an emphasis on the key features: household heterogeneity and product input complementarity. This leads us to Section 3, which shows how these features allow for the demand side effects of an energy price shock. Section 4 presents the baseline calibration and impulse response functions, which illustrates the transmission channels we discuss. We show how the magnitude of the various channels depend on the severity of credit constraints and the degree of substitutability between production inputs. In Section 5, we compare the dynamics of an energy price shock to alternate supply shocks. Finally, we consider optimal monetary policy in Section 6. Section 7 concludes the paper.

2 Key Model Model Features

2.1 Household Heterogeneity

Unconstrained Households A fraction $(1-\lambda)$ of households are financially *unconstrained* (denoted by u). They consume $C_{u,t}$, supply labor $N^h_{u,t}$ to unions, save in domestic (foreign) nominal riskless bonds $B_{u,t}$ ($B^*_{u,t}$), and receive profits from firm ownership $DIV^F_{u,t}$. Their lifetime utility is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \Phi_t \left(\frac{C_{u,t}^{1-\sigma}}{1-\sigma} - \chi \frac{(N^h_{u,t})^{1+\varphi}}{1+\varphi} \right), \quad \text{where} \quad \Phi_t = \Phi_{t-1} \beta \exp\left\{ \epsilon_\beta \left(\frac{C_{u,t-1}}{C_u} - 1 \right) \right\}.$$

The endogenous discount factor Φ_t ensures that the net foreign asset position returns to a unique steady state following temporary shocks. Unconstrained households maximise their lifetime utility subject to their budget constraint

$$W^h_t N^h_{u,t} + DIV^L_{u,t} + B_{u,t} + \mathcal{E}_t B^*_{u,t} + DIV^F_{u,t} - T_{u,t} = P_t C_{u,t} + R_{t-1} B_{u,t-1} + \mathcal{E}_t R^*_{t-1} B^*_{u,t-1}, \tag{2.1}$$

where R_{t-1} (R^*_{t-1}) denotes the gross nominal rate of return on domestic (foreign) bonds, P_t is the price of the consumption good,[14] $\mathcal{E}_t$ is the nominal exchange rate, $DIV^F_{u,t}$ represents profits derived from firm ownership, $DIV^L_{u,t}$ are profits transferred to the household by labor unions and $T_{u,t}$ are government lump-sum transfers. The unconstrained household's consumption-savings Euler equation is given by

$$1 = \mathbb{E}_t \left[\Lambda_{u,t,t+1} \frac{R_t}{\Pi_{t+1}} \right], \tag{2.2}$$

where $\Pi_t \equiv P_t / P_{t-1}$, $\Lambda_{u,t,t+1} \equiv \Phi_{t+1}/\Phi_t \left(C_{u,t}/C_{u,t+1} \right)^{\sigma}$ and the UIP condition is given by

$$0 \quad = \quad \mathbb{E}_t \left[\Lambda_{u,t,t+1} \frac{1}{\Pi_{t+1}} \left(R_t - R^*_t \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right) \right]. \tag{2.3}$$

[12]Under Cobb-Douglas production, cost shares are constant regardless of the monopolistic competition distortion. This means that natural (distorted) output falls just as much as efficient output following an oil price shock, and perfectly stabIlizing prices is the optimal policy to follow.

[13]Several papers also study how monetary policy should respond depending on whether the energy price shock is demand or supply driven. Plante (2014) considers this question in a closed economy model, while Stevens (2015) considers this in an open economy model and finds that despite differences in the transmission of an energy demand and an energy supply shock, optimal monetary policy remains largely the same. However, Bodenstein et al. (2008) show that the source of an oil shock matters greatly for the optimal monetary policy response to fluctuations in energy prices.

[14]We assume that households' consumption basket only consists of the domestically produced good.

Constrained Households The remaining fraction λ of households are *financially constrained* (denoted by c). They only receive labor income, hence their consumption is given by

$$P_t C_{c,t} = W_t^h N_{c,t}^h + DIV_{c,t}^L - T_{c,t}. \tag{2.4}$$

The wage received by households W_t^h is determined as a function of a weighted average of unconstrained and constrained households marginal rate of substitution. Aggregate consumption is

$$C_t = (1-\lambda)C_{u,t} + \lambda C_{c,t}. \tag{2.5}$$

We define the consumption gap as the ratio between unconstrained and constrained consumption

$$\Gamma_t \equiv \frac{C_{u,t}}{C_{c,t}}. \tag{2.6}$$

2.2 Production Input Complementarity

Final good packers Final good packers operate in a competitive market. They produce the final good Z_t by combining a continuum of varieties $Z_t(i)$ with measure one so that $Z_t = \left(\int_0^1 (Z_t(i))^{\frac{\epsilon_Z - 1}{\epsilon_Z}} di\right)^{\frac{\epsilon_Z}{\epsilon_Z - 1}}$. Optimization implies the following demand function for variety i

$$Z_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_Z} Z_t,$$

where $P_t \equiv \left(\int_0^1 (P_t(i))^{1-\epsilon_Z} di\right)^{\frac{1}{1-\epsilon_Z}}$ is the price of the final good. It can be shown that $P_t Z_t = \int_0^1 P_t(i) Z_t(i) di$.

Final good producers A continuum of final output producing firms, indexed by $i \in [0,1]$, operate in a monopolistically competitive environment. Hence, each firm produces a single-differentiated good and operates as a monopoly in its own market. A key element of our model is the production structure. Firm i produces the final output variety $Z_t(i)$ using the following CES production technology with imported energy ($M_t(i)$) and labor ($N_t(i)$) as inputs

$$Z_t(i) = \varepsilon_t^{TFP} \left((1-\alpha)^{\frac{1}{\psi}} (N_t(i))^{\frac{\psi-1}{\psi}} + \alpha^{\frac{1}{\psi}} (M_t(i))^{\frac{\psi-1}{\psi}}\right)^{\frac{\psi}{\psi-1}}, \tag{2.7}$$

where ε_t^{TFP} represents productivity and $\psi < 1$ is the elasticity of substitution between energy and labor.

Cost Minimisation Cost minimization by final output goods producers yields the following demand functions for labor and energy, respectively[15]

$$\begin{aligned} W_t &= (1-\alpha)^{\frac{1}{\psi}} \frac{MC_t^Z}{\tau_t^Z} \left(\frac{Z_t(i)}{N_t(i)}\right)^{\frac{1}{\psi}} \left(\varepsilon_t^{TFP}\right)^{\frac{\psi-1}{\psi}} \\ P_t^M &= (\alpha)^{\frac{1}{\psi}} \frac{MC_t^Z}{\tau_t^Z} \left(\frac{Z_t(i)}{M_t(i)}\right)^{\frac{1}{\psi}} \left(\varepsilon_t^{TFP}\right)^{\frac{\psi-1}{\psi}} \end{aligned}$$

[15] See Appendix B.3.1 for details.

where the Lagrange multiplier $MC_t^Z(i)$ is the (nominal) shadow cost of producing one more unit of final output, i.e. the nominal marginal cost, and τ_t^Z is a shock to final output marginal costs that is isomorphic to a markup shock.

Price Setting Firms face price stickiness à la Calvo, resetting prices in every period with probability $(1-\phi_z)$. A firm that is able to reset prices in period t chooses the price $P_t^{\#}$ that maximizes the sum of discounted profits subject to the demand faced in $t+k$

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\phi_z)^k \{\Lambda_{u,t,t+k}(P_t^{\#} Z_{t+k|t} - MC_{t+k}^Z Z_{t+k|t})\} \quad s.t. \quad Z_{t+k|t} = \left(\frac{P_t^{\#}}{P_{t+k}}\right)^{-\epsilon_z} Z_{t+k}.$$

Profit maximization implies

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\phi_z)^k \{\Lambda_{u,t,t+k} Z_{t+k|t}(P_t^{\#} - \mathcal{M}_z MC_{t+k|t}^Z)\} = 0,$$

where $\mathcal{M}_z \equiv \frac{\epsilon_z}{\epsilon_z - 1}$ is the desired final output price markup and $MC_{t+k|t}^Z$ the nominal marginal cost.

2.3 Remaining Features

2.3.1 Wage Stickiness

We incorporate wage stickiness following the standard in the literature (refer to Appendix (B.2)).

2.3.2 GDP and the GDP Deflator

Nominal GDP is defined as

$$P_t^Y Y_t \equiv P_t Z_t - P_t^M M_t, \tag{2.8}$$

where $M_t \equiv \int_0^1 M_{j,t} dj$ and the GDP deflator P_t^Y is implicity defined by the following expression

$$P_t = \frac{Y}{Z} P_t^Y + \frac{M}{Z} P_t^M. \tag{2.9}$$

2.3.3 The External Sector

Foreign demand for the domestically produced non-energy export good is $X_t = \vartheta^* \left(\frac{P_t}{\mathcal{E}_t P_t^*}\right)^{-\varsigma^*} Y_t^*$. Assuming the law of one price holds, so that the terms of trade is given by $S_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t}$, we have

$$X_t = \vartheta^* S_t^{\varsigma^*} Y_t^*.$$

2.3.4 Monetary policy

The central bank follows a Taylor rule that responds to deviations of inflation and employment from their targets,

$$R_t = R^{1-\theta_R} R_{t-1}^{\theta_R} \left(\frac{\Pi_t^{annual}}{\bar{\Pi}^{annual}}\right)^{\frac{(1-\theta_R)\theta_\Pi}{4}} \left(\hat{N}_t\right)^{(1-\theta_R)\theta_N}.$$

2.3.5 Shock Processes

The model includes a shock to the price of energy, which follows the exogenous process

$$log\left(\frac{P_t^{M*}}{P_t^*}\right) = \rho_m log\left(\frac{P_{t-1}^{M*}}{P_{t-1}^*}\right) + \eta_{m,t},$$

and a shock to firms' productivity

$$log\left(\varepsilon_t^{TFP}\right) = \rho_{TFP}\log\left(\varepsilon_{t-1}^{TFP}\right) + \eta_{TFP,t}.$$

3 The demand side effects of an energy price shock

3.1 The IS equation

From (2.2), (2.5), (2.6) and (2.8) we can derive the following IS equation[16]

$$y_t = \mathbb{E}_t y_{t+1} - \frac{PC}{PZ}\frac{1}{\sigma}\mathbb{E}_t\left(r_t - \pi_{t+1} + \Delta\phi_{t+1}\right) + \frac{PC}{PZ}\frac{\lambda}{\lambda+\Gamma(1-\lambda)}\mathbb{E}_t\Delta\gamma_{t+1} - \frac{PX}{PZ}\mathbb{E}_t\Delta y_{t+1}^* - \mathbb{E}_t\left(\frac{P^M M}{P^Y Y}\psi\Delta p_{t+1}^M + \frac{PX}{PZ}\varsigma^*\Delta s_{t+1} + \frac{P^M M}{P^Y Y}(\Delta\epsilon_{t+1}^{TFP} + \psi\Delta\mu_{t+1})\right). \quad (3.1)$$

Solving forward the IS equation, we obtain

$$y_t = -\frac{PC}{PZ}\frac{1}{\sigma}\mathbb{E}_t\sum_{k=0}^{\infty}\left(r_{t+k} - \pi_{t+k+1}\right) - \frac{PC}{PZ}\frac{\lambda}{\lambda+\Gamma(1-\lambda)}\gamma_t + \frac{PX}{PZ}y_t^* + \frac{P^M M}{P^Y Y}\psi p_t^M + \frac{PX}{PZ}\varsigma^* s_t + \frac{P^M M}{P^Y Y}(\epsilon_t^{TFP} + \psi\mu_t) + \frac{PC}{PZ}\frac{1}{\sigma}\phi_t. \quad (3.2)$$

According to equation (3.2), GDP is dependent on the path of the real interest rate, the consumption gap (γ_t), and foreign demand for the domestic good (determined by foreign activity (y_t^*) and the terms of trade (s_t)). Finally, GDP also depends on the relative price of energy (p_t^M), as variations in this variable lead to substitution between imported energy and labor. An increase in the consumption gap reflects redistribution against the constrained workers, and hence a drop in aggregate demand that brings GDP down. The effect of the consumption gap on GDP is increasing in the share of constrained households.

The IS equation illustrates the channels through which an energy price shock affects economic activity. The term involving the path of the real interest rate captures the usual channel through which supply shocks depress economic activity in a RANK model. Given the inflationary pressures derived from such shocks, the central bank responds by tightening monetary policy. The ensuing increase in the real rate is the source of an economic recession. This means that in the RANK, the supply shock is not contractionary by itself, instead, the economic downturn is a result of the monetary policy response to inflation. A new channel for supply shocks is present in the TANK economy, as indicated by the term involving the consumption gap. This term captures a demand side effect of the energy shock. In this economy, an increase in the price of energy translates into a contraction in households' income, as more

[16] Capital letters without the time subscript represent steady state levels while lowercase letters denote variables in low deviation from steady state.

resources must be devoted to the purchase of the energy input. Given the financial constraints faced by a fraction of households, aggregate demand falls, leading to an economic recession.

Next, we discuss the aforementioned demand side effect of energy price variations by analyzing how such shocks affect the consumption gap.

3.2 The consumption gap

Letting $INC_{u,t}$ and $INC_{c,t}$ denote unconstrained and constrained households' current income, and using the budget constraints (2.4) and (2.1), we can express the consumption gap as follows[17]

$$\Gamma_t = \frac{INC_{u,t} + \mathcal{E}_t \Delta B^*_{u,t} - \mathcal{E}_t (R^*_{t-1} - 1) B^*_{u,t-1}}{INC_{c,t}}.$$

Defining the income gap between unconstrained and constrained households as $\Gamma^{inc}_t \equiv \frac{INC_{u,t}}{INC_{c,t}}$ we can rewrite the above equation as

$$\Gamma_t = \Gamma^{inc}_t + \frac{\mathcal{E}_t \Delta B^*_{u,t} - \mathcal{E}_t (R^*_{t-1} - 1) B^*_{u,t-1}}{INC_{c,t}}. \tag{3.3}$$

Equation (3.3) illustrates how an energy price shock affects the consumption gap through a differential impact on constrained and unconstrained households consumption. An unequal consumption response can have two sources. One source is through changes in the income gap, which reflects the different impact of the shock on current income (due to differences in income composition). The other source is access to borrowing, which allows unconstrained households to insure their consumption from the fall in income following an increase in energy prices.

Let's consider how these two components of the consumption gap are determined. From the economy's budget constraint, we know that the evolution of debt depends on the balance of trade. Therefore, the consumption gap can be rewritten as

$$\Gamma_t = \Gamma^{inc}_t - \frac{1}{1-\lambda} \frac{TB_t}{INC_{c,t}}, \tag{3.4}$$

where $TB_t = P_t X_t - P^M_t M_t$ is the trade balance. Using the definitions of unconstrained and constrained total income and imposing $N_{u,t} = N_{c,t} = N_t$, the above expression can be written as follows

$$\Gamma_t = \underbrace{1 + \frac{1}{1-\lambda} \frac{\mathcal{M}_t - 1}{\Xi_t}}_{\text{income gap}} + \underbrace{\frac{1}{1-\lambda} \left(\frac{1}{\Xi_t} - 1 - \frac{TB^{NM}_t}{INC_{c,t}} \right)}_{\text{borrowing}}, \tag{3.5}$$

where $\mathcal{M}_t \equiv \frac{P_t}{MC^Z_t}$ is firms' average markup, $\Xi_t \equiv \frac{W_t N_t}{W_t N_t + P^M_t M_t}$ is the labor share in firms' total expenditure and $TB^{NM}_t \equiv P_t X_t$ is the balance of trade net of the energy imports. Equation (3.5) tells us that the effect of a change in energy prices on the consumption gap (and hence, on aggregate demand) is determined by the impact of the shock on two key variables, firms' markups ($\mathcal{M}_t$) and the labor share (Ξ_t).

The income gap (and hence, the consumption gap) depends positively on firms' markup, since an increase in the markup redistributes resources towards the unconstrained firm owners. The income gap

[17] The expression for the consumption gap takes into account that domestic bonds must equal zero in equilibrium.

also increases in response to a reduction of the labor share in total factor expenditure, since a reduction in the labor share redistributes resources against the constrained workers and towards the import of energy.

A reduction in the labor share also increases the consumption gap due to unconstrained households' ability to insure their consumption by borrowing. The reduction in the labor share reflects an increase in the resources devoted to import energy (an increase in the energy share), which must be financed via an increase in debt. Borrowing from the foreign sector is used by unconstrained households to finance their consumption, hence increasing the consumption gap.

Finally, to understand how the energy price shock affects firms' average markup and the labor share, notice that these two objects are linked to the price of energy according to the following expressions[18]

$$\mathcal{M}_t = \frac{\varepsilon_t^{TFP} P_t}{\left((1-\alpha) W_t^{1-\psi} + \alpha \left(P_t^M\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}} \tag{3.6}$$

$$\Xi_t = \left(1 + \frac{\alpha}{1-\alpha}\left(\frac{P_t^M}{W_t}\right)^{1-\psi}\right)^{-1}. \tag{3.7}$$

Notice from (3.6) that, given price rigidities, an increase in energy prices (P_t^M) reduces firms' markups. This implies a redistribution of income in favor of workers, reflected in an reduction of the consumption gap (equation (3.5)). This boosts aggregate demand, and hence, activity (equation (3.2)).

Equation (3.7) shows that the impact of higher energy prices on the labor share crucially depends on the elasticity of substitution between energy and labor (ψ). In the case of a Cobb-Douglas production technology ($\psi = 1$) we have $\Xi_t = 1 - \alpha$, implying that the price of energy has no impact on the labor share. If the elasticity of substitution is larger than one ($\psi > 1$), higher energy prices increase the labor share. The reason is that costlier energy triggers a strong substitution from energy towards labor. The resulting redistribution of income in favor of workers is reflected in a reduction in the consumption gap, which boosts aggregate demand and activity. Alternatively, if energy cannot easily be substituted for by labor ($\psi < 1$), an increase in energy prices reduces the labor share. The shock therefore redistributes against the constrained workers, increasing the consumption gap. The consequent drop in aggregate demand depresses economic activity.

As we will see later, the empirical evidence points to a low substitutability between labor and energy. In this scenario, we should expect that an increase in energy prices will reduce both the labor share and firms markups (i.e., the profit share). The relative impact of the shock on this two objects will determine whether the constrained workers of firm owners are mostly affected, and hence, the size of the demand side effect of the energy price shock.

4 Results

4.1 Parameterization

To stay close to the literature, we calibrate our model using some common parameterizations. We assume a discount factor, β, of 0.9994. The elasticities of substitution across goods varieties (ϵ_z) and across worker types (ϵ_w) are both set to 11, which implies a markup of 10% in steady state. We assume goods

[18]The expression for the labor share is obtained using firms' demand functions for energy and labor.

prices and wages are adjusted with Calvo parameter ϕ_z and ϕ_w of 0.75, which implies an average duration of 4 quarters for each. We set the response to inflation (θ_π) and slack (θ_n) in the Taylor rule to 1.5 and 0.25, respectively. The interest rate smoothing parameter (θ_R) is set to 0.9. The productivity process parameters are set to $\rho_{TFP} = 0.9$ and $\sigma^2_{TFP} = 0.01$. The energy price shock has persistence $\rho_m = 0.8$ and $\sigma^2_m = 0.85$ so that prices increase by 50% on impact. The population share of constrained worker households (λ) is set to 0.3.

We limit our discussion to the following key parameters: the steady state share of energy in production ($\alpha = 0.05$) and the elasticity of substitution between energy and labor in production ($\psi = 0.1$). There are a wide range of estimates for the elasticity of substitution between production inputs in the literature. Higher estimates, such as those provided by Bodenstein et al. (2012) (0.42) are motivated by estimates of the short-run price elasticity of oil demand from structural econometric models. Natal (2012) sets this parameter to 0.3, while Plante (2014) suggests a calibration of 0.25 so that the own price elasticity of oil is approximately -0.25. Montoro (2012) sets the value of the elasticity of substitution between oil and labor at 0.2, equal to the average value reported by Hamilton (2009). On the low end of estimates is Adjemian and Darracq Paries (2008) and Backus and Crucini (2000), at 0.09. However, their production function is Cobb-Douglas in labor and a capital services-energy mix, where the latter is combined via CES. Finally, Stevens (2015) suggests an elasticity of substitution between oil and value-added of 0.03, where value-added is a Cobb-Douglas function with labor and capital inputs. This parameter is equivalent to the short-run oil demand elasticity and is chosen to be consistent with reduced-form evidence on the slope of the oil demand curve that lie between 0 and 0.11. Between the extreme cases of zero or infinite substitutability, the effects of an energy price shock on macroeconomic aggregates also depends on the share of energy in production. The share of energy in production ranges from 2% in Natal (2012) for the US, 4% in Bachmann et al. (2022) for Germany, and 5% in Stevens (2015).

4.2 Impulse response functions

We provide IRFs to illustrate how the strength of the channels discussed in the previous section depend on the severity of financial frictions and the degree of substitutability between energy and labor in production.

Figure 1 shows the baseline response to an increase in energy prices. In the RANK economy, an energy price shock places upward pressure on production costs, leading to a surge in inflation. The central bank responds by tightening monetary policy, which induces a contraction in activity. Relative to the RANK, the Gas-TANK economy experiences a deeper contraction. Moreover, while the recession in the RANK originates from the contractionary policy implemented by the central bank, in the Gas-TANK it is largely driven by the direct impact of higher energy prices on aggregate demand. Since production inputs are complementary in our Gas-TANK economy, higher energy prices reduce the labor share of total income, implying a drop in workers' earnings. Given borrowing constraints for worker households, this translates into a fall in aggregate demand. Due to the adverse effect of the energy price shock on demand, monetary policy in the Gas-TANK is much looser. Finally, note how the energy price shock has a different effect on constrained workers' and unconstrained capitalists' consumption. While workers' consumption largely falls due to the drop in their income, capitalists are able to insure their consumption by borrowing from the external sector. This is reflected in an increase in the consumption gap. Over time, as firms pass the costs of the shock to workers through an increase in prices, the income gap goes up as well, further raising the gap in consumption.

FIGURE 1: Dynamic Responses to a Global Energy Price Shock: Benchmark Case

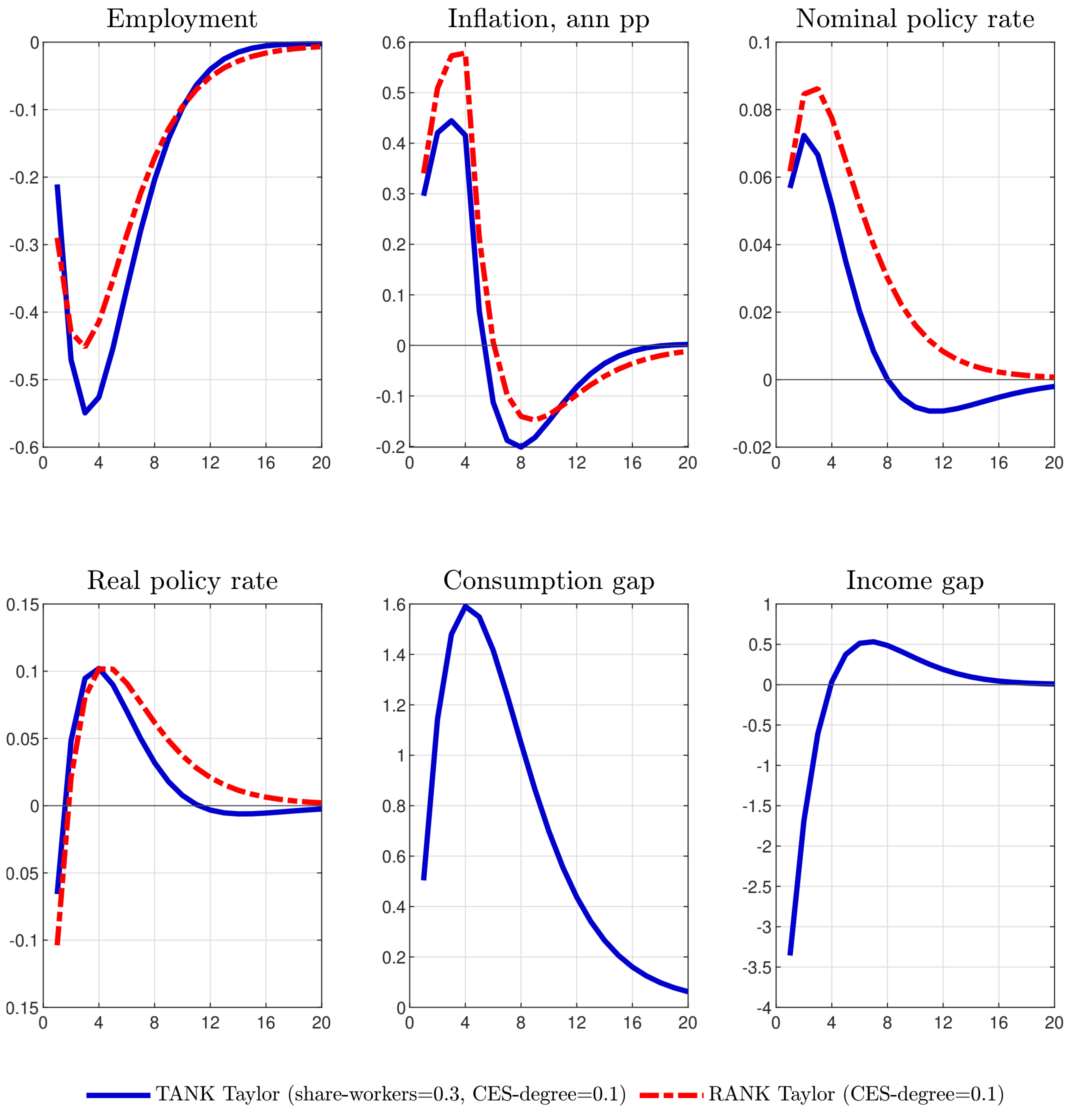


Figure 2 illustrates the effects of the energy shock when we assume a larger share of constrained households. Given more severe borrowing constraints, consumption becomes more responsive to the drop in households' income. It follows that the increase in energy prices induces a stronger fall in aggregate demand, leading to a deeper recession than in the baseline case (Figure 1).

FIGURE 2: Dynamic Responses to a Global Energy Price Shock: Higher Share of Constrained Households

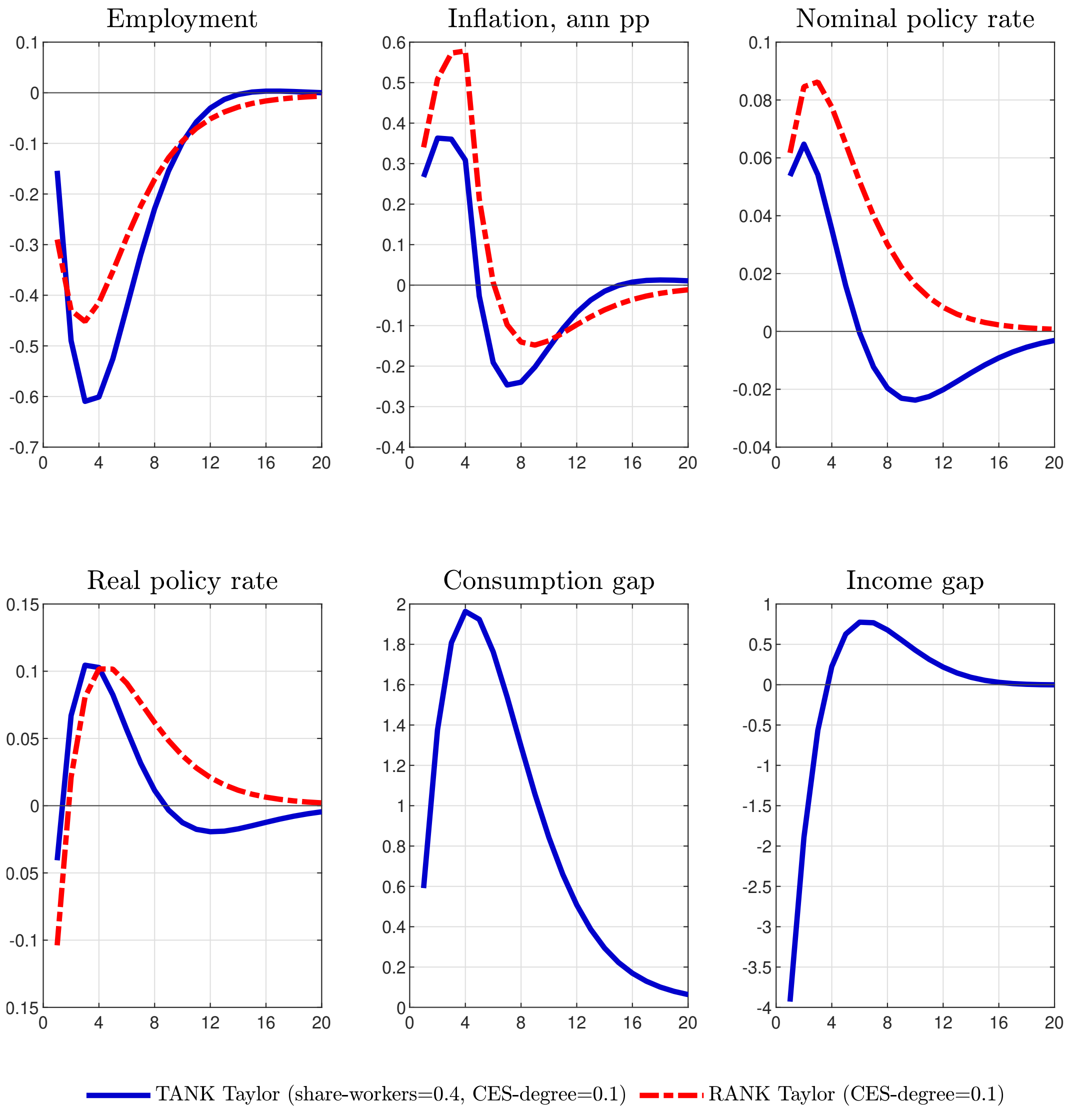
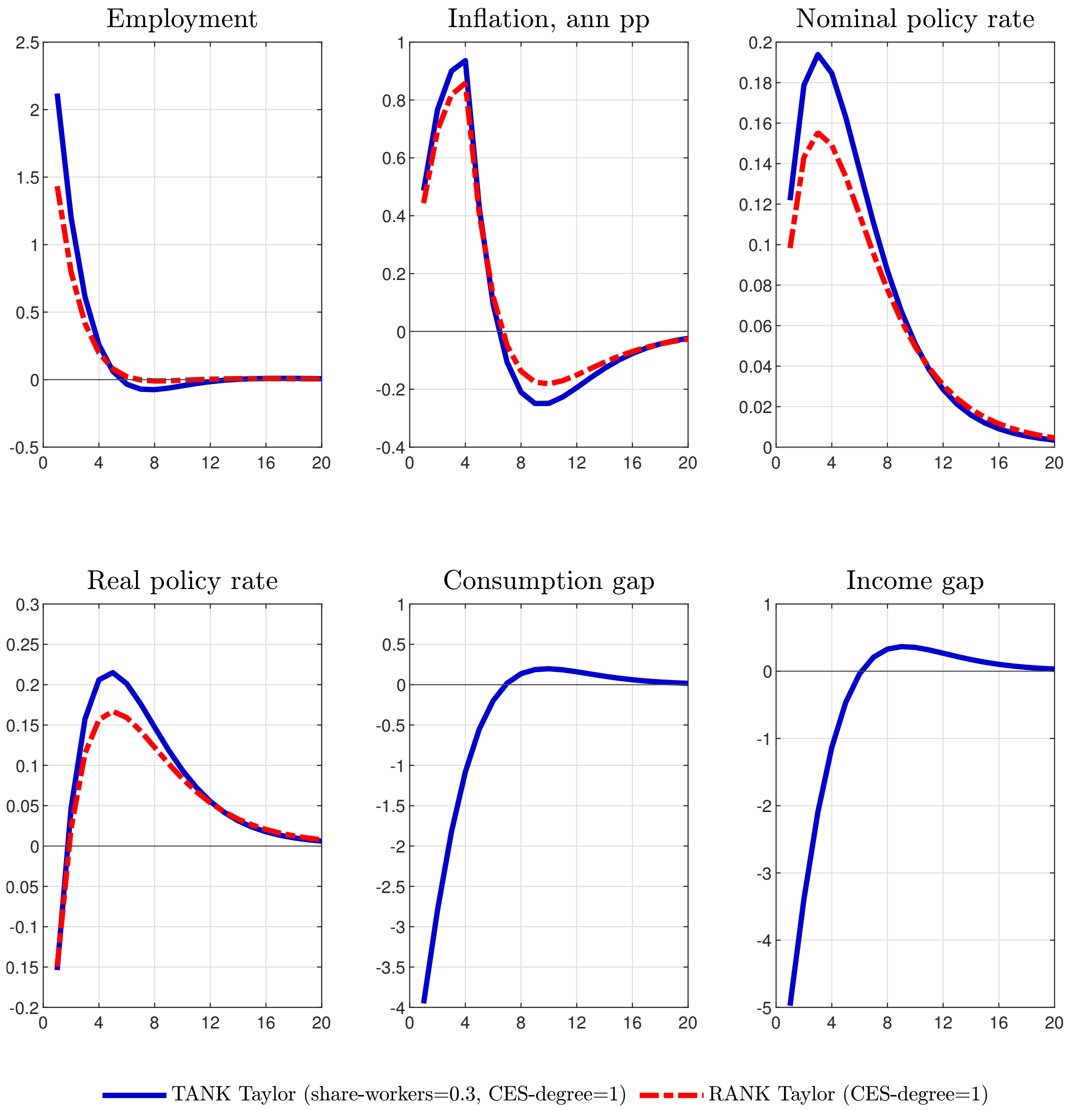


Figure 3 illustrates the effects of the energy price shock under a higher degree of substitutability between labor and energy in production (Cobb-Douglas, $\psi = 1$). The increase in the price of energy imports imply a fall in the relative price of labor, which now leads to greater degree of substitution towards labor. The labor demand schedule shifts upwards and employment increases. For $\psi = 1$, higher employment fully compensates for the lower relative wage, leaving the labor share constant. While the labor share remains constant, firms' markups experience a reduction due higher marginal costs. Redistribution in favor of constrained workers, reflected in a reduction of the consumption gap, boosts aggregate demand. Given the positive effect of the shock on demand, the TANK economy experiences a milder recession relative to its RANK counterpart.

FIGURE 3: Dynamic Responses to a Global Energy Price Shock: Cobb-Douglas instead of CES



5 The demand side effects of alternate supply shocks

Can the economic effects of an energy price shock be appropriately proxied with a TFP shock, since both shocks restrain supply? In this section, we explore whether the demand contraction that follows a rise in energy prices is a common feature of supply disturbances. Equations (3.5) to (3.7) are used to analyze the demand side effect of a disturbance to firms' TFP. For simplicity, assume a closed economy environment, where only labor is used in production. The consumption gap becomes

$$\Gamma_t = 1 + \frac{1}{1-\lambda}\left(\mathcal{M}_t - 1\right) \tag{5.1}$$

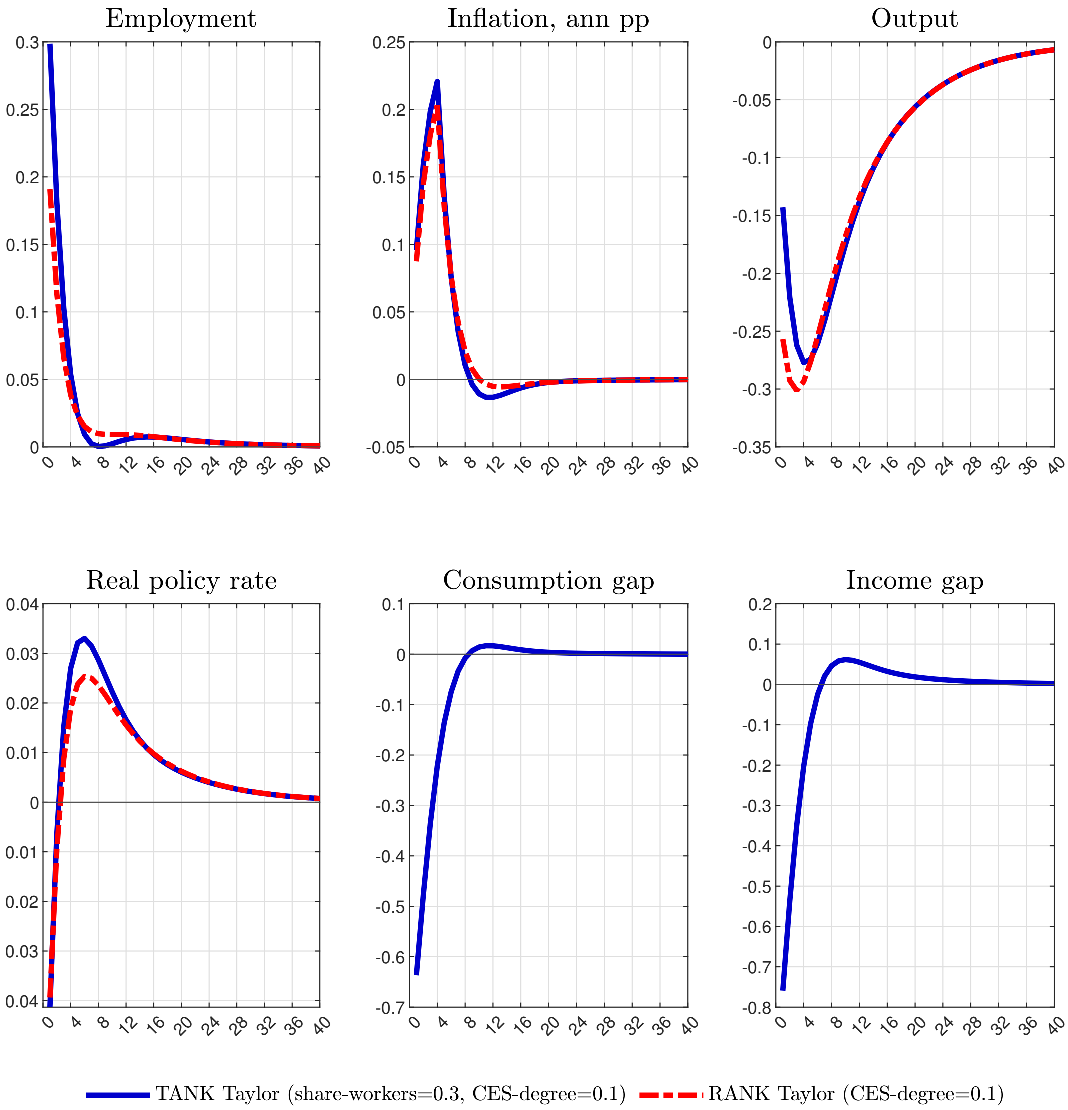
where

$$\mathcal{M}_t = \frac{\varepsilon^{TFP} P_t}{W_t}. \tag{5.2}$$

It is easy to check that an adverse TFP shock leads to a fall in markups. The reason is that with lower productivity firms must hire more labor to produce each unit of the good. This implies lower markups and an increase in workers' income. It follows that the consumption gap falls, leading to an increase in GDP (equation 3.2).

The IRFs to an adverse TFP shock in Figure 4 illustrate this intuition. Similar to the energy price shock, the TFP shock leads to higher marginal costs, which places upward pressure on inflation. The consequent response of the central bank to higher inflation leads to a drop in output. While both energy and TFP shocks generate similar supply side effects, this is not the case for the demand side effect. Lower TFP implies that more labor is required to produce each unit of the good, which explains the observed increase in employment. Workers' income thus increases, boosting aggregate demand. As a consequence, the TANK economy features a milder contraction in consumption and output relative to the RANK. Energy and TFP shocks therefore diverge in terms of their impact on demand. Whereas the former reduces workers' income, the latter increases it, leading to a different profile for aggregate demand. We conclude that no generalization can be made about the effects of supply shocks on aggregate demand, as the nature of the shock crucially affects the way resources are redistributed in the economy.

FIGURE 4: Dynamic Responses to a TFP Shock

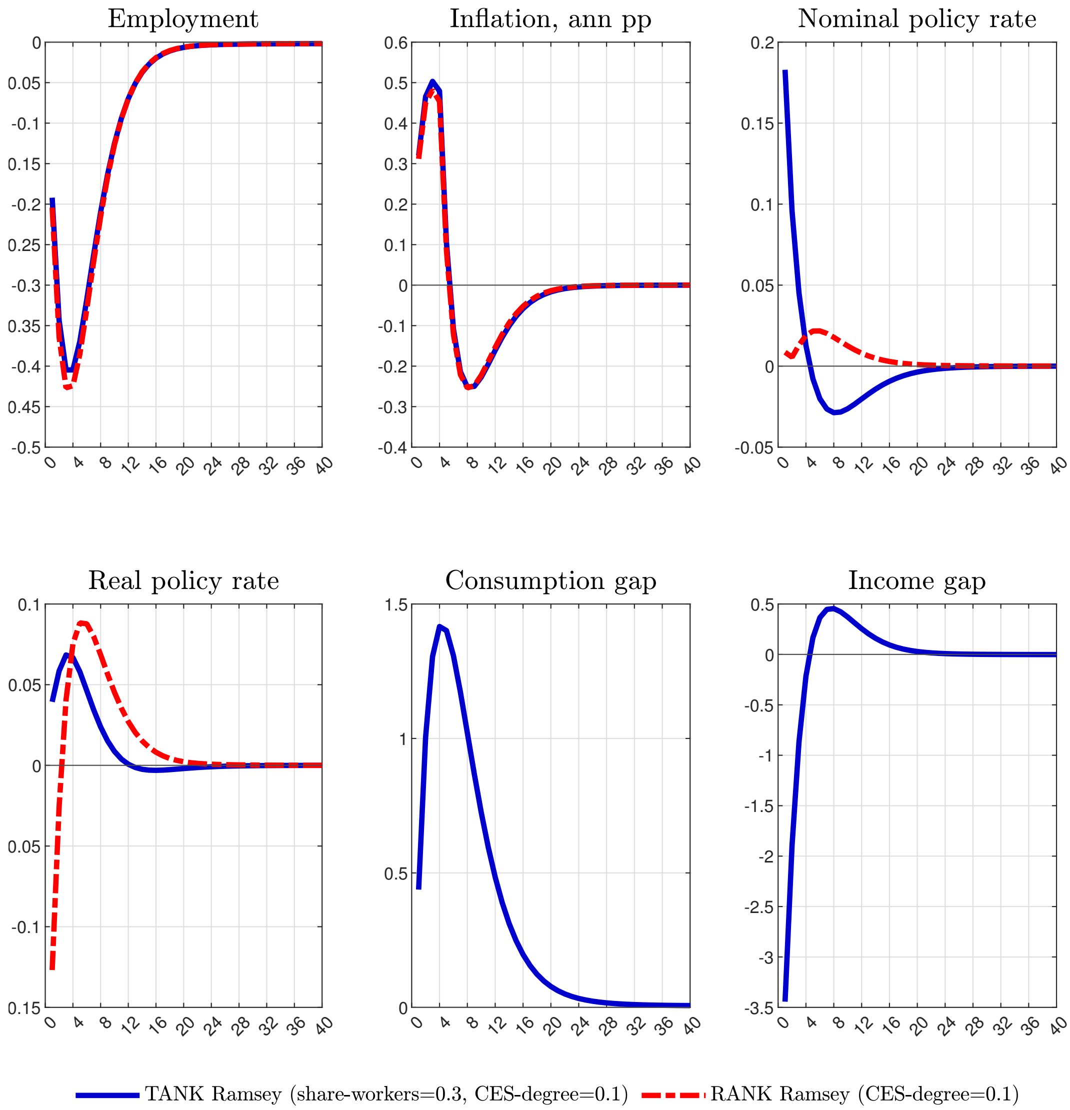


6 Optimal monetary policy

Finally, we study the optimal response of the central bank to the energy price shock.[19] Figure 5 presents the IRFs under the Ramsey policy. We compare the optimal policy in the TANK versus the RANK model. The figure shows that although optimal policy leads to very similar paths for inflation and employment in the two economies, the implementation is different. In both cases, the policymaker implements contractionary policy in order to counteract the inflationary effect of the shock. However, the required increase in the interest rate is milder in the TANK. This is explained by the direct contractionary effect of higher energy prices on households' income. In the TANK, the lower income translates into lower aggregate demand, which contains the inflationary pressures of the shock. Hence, a milder response of the central bank is needed.

[19]To compute the optimal Ramsey policy we maximize households' lifetime utility subject to the non-linear system of equations that describe private agents' optimality conditions

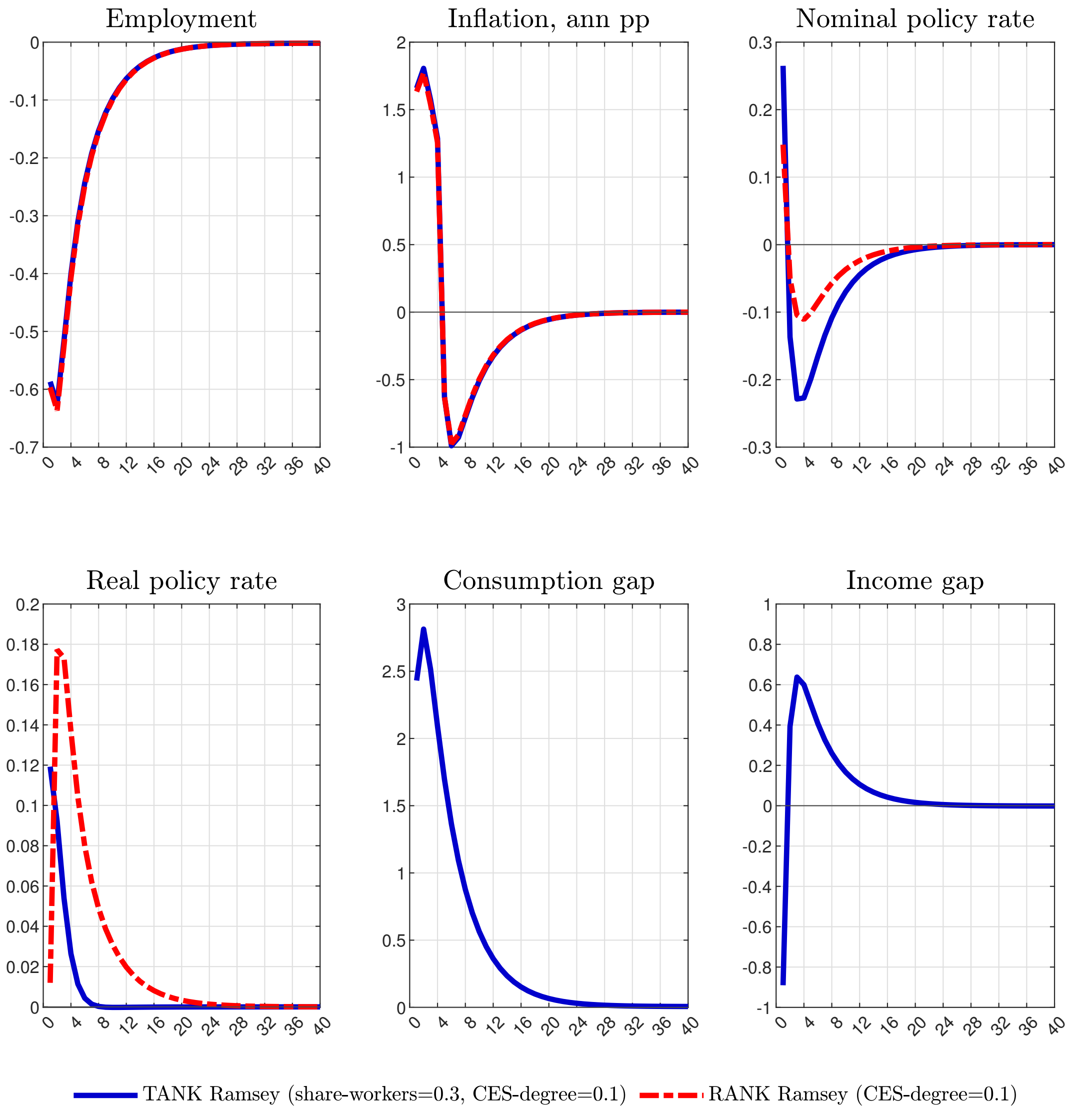
FIGURE 5: Dynamic Responses to a Global Energy Price Shock: Optimal Policy



As stressed earlier, the demand effect of higher energy prices depends on the evolution of firms' markups. If inflation remains contained in spite of the costlier energy input, firms largely absorb the costs of the shock. This would be reflected in a reduction in markups. Conversely, if prices go up strongly to preserve markups, firms can pass the costs of the shock to workers, who will experience a more severe reduction in their income. The degree to which prices react to the shock thus determines who takes the hit, and hence, its impact on aggregate demand. We then explore a scenario where firms raise prices more aggressively in response to the costlier energy in an attempt to preserve profits. To this end, we repeat the optimal policy exercise assuming a higher degree of price flexibility.[20] Results are presented in Figure 6. A comparison with Figure 5 illustrates that when firms react to an energy price shock by raising prices strongly, constrained households experience a more severe drop in their income relative to unconstrained households, as reflected by the income gap. Since the constrained households are more severely affected, there is a deeper contraction in aggregate demand. As a consequence, optimal monetary policy in the TANK is now much looser relative to its RANK counterpart.

[20]For this simulation we set the Calvo parameter to 0.3.

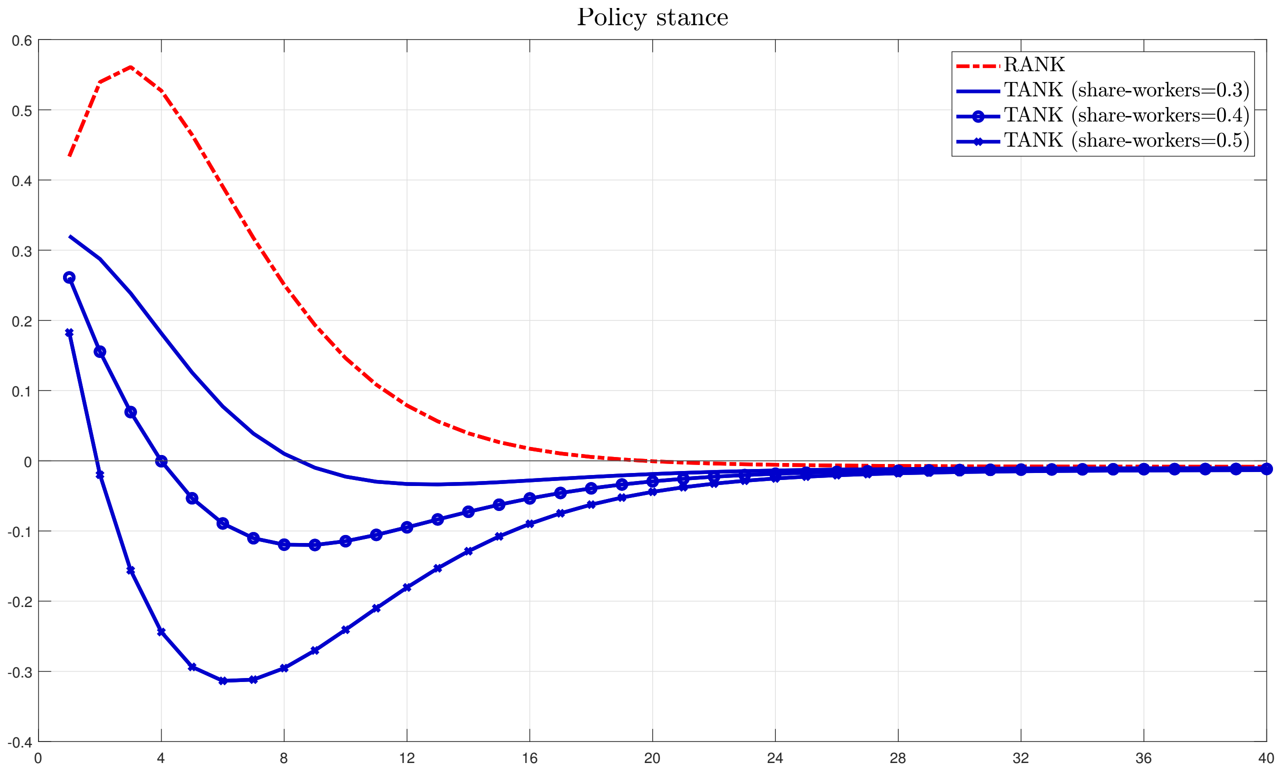
FIGURE 6: Dynamic Responses to a Global Energy Price Shock: Optimal Policy with Higher Price Flexibility



Next, we explore whether optimal policy may actually be expansionary in response to an adverse supply shock. We can expect that as the contractionary effect of the shock on demand strengthens, it should be optimal for the policymaker to loosen policy. For this exercise, we introduce a measure for the monetary policy stance, which indicates whether policy is contractionary or expansionary. From (2.2) we know that the demand of households whose consumption responds to interest rates is determined by the expected path of the real interest rate, rather than the current real rate. Therefore, we define the *policy stance* as $st_t \equiv \frac{1}{\sigma}\mathbb{E}_t \sum_{k=0}^{\infty} (r_t - \pi_{t+k+1})$.

Figure 7 presents the IRFs for the policy stance over an increasingly larger share of constrained agents, which allows the energy price shock to yield a correspondingly larger fall in household consumption. In the RANK, monetary policy remains contractionary throughout the period of higher energy prices in order to counteract inflation. Meanwhile, in the TANK, the policy stance quickly turns expansionary as financial constraints become more severe. Optimal policy can be expansionary when the energy price shock has a larger adverse effect on the demand side.

FIGURE 7: Dynamic Responses to a Global Energy Price Shock: Optimal Policy with Stronger Credit Constraints



7 Conclusion

We build an open economy model with household heterogeneity and low substitutability between energy and labor to highlight the demand side effects of an energy price shock. We show that an energy price shock has different effects on households, depending on their sources of income and borrowing constraints.

The transmission of an energy price shock to aggregate variables differs significantly from a RANK. An energy price shock reduces the labor share of total factor expenditures, thereby redistributing income against constrained worker households, which depresses aggregate demand. The increase in resources used for energy imports must be financed by an increase in debt. Borrowing from the foreign sector is used by unconstrained households to finance their consumption. This redistributes income in favor of unconstrained worker households, thereby depressing aggregate demand. The magnitude of these channels depend on the degree of price rigidity and the elasticity of substitution between energy and labor. An energy price shock therefore has a self-correcting effect, as the consequent contraction in economic activity dampens inflationary pressures.

In our model, an energy price shock has features of an adverse productivity shock, but there are important differences. Although the supply side effects of both shocks are the same in our model, the demand side effect is completely different. Both an adverse productivity shock and an energy price shock lead to an increase in inflation. However, while a negative productivity shock leads to an increase in aggregate demand, the opposite is true for an energy price shock.

The demand side effect of an energy price shock in our model implies that optimal monetary policy is less contractionary, relative to a RANK model. In some cases, it may even be expansionary (i.e., when credit constraints are severe).

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A Small-scale Gas-TANK

A.1 The IS equation

Nominal value added is defined as

$$P_t^Y Y_t \equiv P_t Z_t - P_t^M M_t,$$

where Z_t is the final good and $M_t = \int_0^1 M_{j,t} dj$ are imports. Assume domestic households only consume domestic goods. However, part of the domestic production is consumed by foreign households. Then we have that $Z_t = C_t + X_t$ where X_t represents exports. Rearrange to obtain

$$Y_t = \frac{P_t}{P_t^Y} C_t + \frac{P_t}{P_t^Y} X_t - \frac{P_t^M}{P_t^Y} M_t.$$

Log-linearizing this expression

$$y_t = \frac{PC}{P^Y Y} c_t + \frac{PX}{P^Y Y} x_t - \frac{P^M M}{P^Y Y} (p_t^M + m_t) - p_t^Y,$$

since $\frac{PC_c + PX - P^M M}{P^Y Y} = 1$. As $p_t^Y = -\frac{P^M M}{P^Y Y} p_t^M$, then

$$y_t = \frac{PC}{P^Y Y} c_t + \frac{PX}{P^Y Y} x_t - \frac{P^M M}{P^Y Y} m_t. \tag{A.1}$$

Next, decompose aggregate consumption

$$C_t = (1-\lambda) C_{u,t} + \lambda C_{c,t}.$$

Defining the consumption gap between unconstrained and constrained households as $\Gamma_t \equiv \frac{C_{u,t}}{C_{c,t}}$, the above equation can be rewritten as

$$\frac{C_t}{C_{u,t}} = (1-\lambda) + \lambda \frac{1}{\Gamma_t}.$$

Log-linearizing

$$\frac{C}{C_u}(c_t - c_{u,t}) = -\lambda \frac{1}{\Gamma} \gamma_t.$$

In steady state we have $C = (1-\lambda) C_u + \lambda C_c$, which implies $\frac{C}{C_u} = (1-\lambda) + \lambda \frac{C_c}{C_u}$, $\frac{C}{C_u} = (1-\lambda) + \lambda \frac{1}{\Gamma}$, $\frac{C}{C_u} = \frac{(1-\lambda)\Gamma + \lambda}{\Gamma}$. Then

$$c_t = c_{u,t} - \frac{\lambda}{\lambda + \Gamma(1-\lambda)} \gamma_t. \tag{A.2}$$

Substituting A.2 into A.1

$$y_t = \frac{PC}{P^Y Y}\left(c_{u,t} - \frac{\lambda}{\lambda + \Gamma(1-\lambda)} \gamma_t \right) + \frac{PX}{P^Y Y} x_t - \frac{P^M M}{P^Y Y} m_t.$$

Combining A.3 with unconstrained households' Euler equation and the demand functions for imports and exports we obtain the IS equation

$$y_t = \mathbb{E}_t y_{t+1} - \frac{PC}{PZ}\frac{1}{\sigma}\mathbb{E}_t\left(r_t - \pi_{t+1} + \Delta\phi_{t+1}\right) + \frac{PC}{PZ}\frac{\lambda}{\lambda + \Gamma(1-\lambda)}\mathbb{E}_t\Delta\gamma_{t+1} - \frac{PX}{PZ}\mathbb{E}_t\Delta y^*_{t+1} - \\ \mathbb{E}_t\left(\frac{P^M M}{P^Y Y}\psi\Delta p^M_{t+1} + \frac{PX}{PZ}\varsigma^*\Delta s_{t+1} + \frac{P^M M}{P^Y Y}(\Delta\epsilon^{TFP}_{t+1} + \psi\Delta\mu_{t+1})\right). \quad \text{(A.3)}$$

A.2 The consumption gap

Domestic households only consume domestic goods. However, part of the domestic production is consumed by foreign households. Then we have that $Z_t = C_t + X_t$, where X_t represents exports. Multiply by P_t to obtain $P_t Z_t = P_t C_t + P_t X_t$. Using the fact that the sum of households income must equal the sum of firms total income net of energy import expenditure, i.e., $INC_t = P_t Z_t - P^M_t M_t$, we have $INC_t + P^M_t M_t = P_t C_t + P_t X_t$. Decomposing aggregate income and aggregate consumption, we obtain

$$(1-\lambda)INC_{u,t} + \lambda INC_{c,t} + P^M_t M_t = P_t((1-\lambda)C_{u,t} + \lambda C_{c,t}) + P_t X_t.$$

But $INC_{c,t} = P_t C_{c,t}$, then

$$(1-\lambda)INC_{u,t} + P^M_t M_t = (1-\lambda)P_t C_{u,t} + P_t X_t.$$

Rearrange to obtain unconstrained households' consumption

$$P_t C_{u,t} = INC_{u,t} + \frac{1}{(1-\lambda)}(P^M_t M_t - P_t X_t). \quad \text{(A.4)}$$

Letting $\Gamma^{inc}_t \equiv \frac{INC_{u,t}}{INC_{c,t}}$ denote the income gap between unconstrained and constrained households we get

$$\Gamma_t = \Gamma^{inc}_t - \frac{1}{1-\lambda}\frac{TB_t}{INC_{c,t}}, \quad \text{(A.5)}$$

where $TB_t = P_t X_t - P^M_t M_t$ is the trade balance.

B Medium-scale Gas-TANK Derivation

B.1 Households

A share $0 < \lambda < 1$ of all households have access to domestic and international financial markets and are able to save and borrow in an unconstrained manner. The remaining share, $1-\lambda$, are 'constrained' households. Those households have only limited access to financial markets, their marginal propensity to consume out of their labor income is higher. Unconstrained (constrained) household quantities are denoted with subscript u (c).

B.1.1 Unconstrained Households

Members of unconstrained households consume, work, save, pay taxes and receive profits from firm ownership. In any period $t = s$ an unconstrained household maximises his/her lifetime utility $\mathcal{U}_{u,s}$

$$\mathcal{U}_{u,s} = E_s\left[\sum_{t=s}^{\infty}\Phi_t\left\{U_{u,t}\left(C_{u,t}, N^h_{u,t}\right)\right\}\right], \quad \text{where} \quad U_{u,t} = \left[\frac{\left(C_{u,t} - hC_{u,t-1}\right)^{1-\sigma} - 1}{1-\sigma} - \nu_L\frac{\left(N^h_{u,t}\right)^{1+\varphi}}{1+\varphi}\right]$$

where $N^h_{u,t}$ is the unconstrained household's labor supply and φ is the elasticity of labor supply, ν_L is the relative weight on the disutility of working. Utility is maximised subject to the budget constraint

$$W^h_t N^h_{u,t} + R_{t-1}B_{u,t-1} + R^*_{t-1}B^*_{u,t-1}\mathcal{E}_t + DIV^F_{u,t} + DIV^L_{u,t} \quad = \quad P^C_t C_{u,t} + B_{u,t} + B^*_{u,t}\mathcal{E}_t + T^F_{u,t} + T^L_{u,t}$$

where W^h_t denotes the nominal wage received by households, $B_{u,t}$ and $B^*_{u,t}$ denote domestic and foreign nominal risk-less bonds, which provide a nominal gross returns of R_t and R^*_t to the household. $\mathcal{E}_t$ denotes the nominal exchange rate (domestic currency relative to foreign currency), $DIV^j_{u,t}$ $j \in \{F, L\}$ are the profits made by monopolistic firms and unions that are re-distributed lump-sum to unconstrained households. Total firm profits consist of the profits of final output (Z), import (M) and export (X) firms

$$DIV^F_{u,t} \quad = \quad DIV^Z_{u,t} + DIV^M_{u,t} + DIV^X_{u,t}.$$

$T^F_{u,t}$ and $T^L_{u,t}$ are a lump-sum taxes imposed on unconstrained households (to subsidize firms costs in order to get a steady state in which the distortion from monopolistic competition is eliminated).

Lagrangian Each unconstrained household solves the following Lagrangian in any arbitrary period t

$$\begin{aligned}\mathcal{L}_{u,t} \quad = \quad & \sum_{\mathcal{S}^t}\pi_{\mathcal{S}^t}\sum_{t=0}^{\infty}\Phi_t\Bigg\{U_{u,t} + \Lambda_{u,t}\Bigg[W^h_t N^h_{u,t} + \\ & R_{t-1}B_{u,t-1} + R^*_{t-1}B^*_{u,t-1}\mathcal{E}_t + DIV^F_{u,t} + DIV^L_{u,t} - P^C_t C_{u,t} - B_{u,t} - B^*_{u,t}\mathcal{E}_t - T^F_{u,t} - T^L_{u,t}\Bigg]\Bigg\}\end{aligned}$$

where $\Lambda_{u,t}$ is the Lagrange multiplier associated with the unconstrained households' resource constraint. The endogenous discount factor

$$\Phi_t = \Phi_{t-1}\beta\exp\left\{\epsilon_\beta\left(\frac{C_{u,t-1}}{C_{u,ss}} - 1\right)\right\} \tag{B.1}$$

ensures that net foreign asset position returns to a unique steady state following temporary shocks.

Optimal Choice of C_u The first-order condition for unconstrained household consumption is

$$\Lambda_{u,t} \quad = \quad \frac{U^C_{u,t}}{P^C_t}, \quad \lambda_{u,t} \equiv U^C_{u,t} = \left(C_{u,t} - hC_{u,t-1}\right)^{-\sigma}. \tag{B.2}$$

where we define the marginal utility of unconstrained consumption as $\lambda_{u,t}$.

Optimal Choice of N_u^h The first order conditions for unconstrained household labor supply are

$$\Lambda_{u,t} = -\frac{U_{u,t}^N}{W_t^h} \quad \Leftrightarrow \quad W_t^h = MRS_{u,t} \equiv -\frac{U_{u,t}^N}{\Lambda_{u,t}}$$

We define $w_t^h \equiv W_t^h / P_t$ and denote the real wage and the marginal rate of substitution in real terms

$$w_t^h = mrs_{u,t} \tag{B.3}$$

$$mrs_{u,t} = -\frac{U_{u,t}^N}{\lambda_{u,t}} \tag{B.4}$$

$$U_{u,t}^N = -\nu_L \left(N_{u,t}^h\right)^\varphi \tag{B.5}$$

Optimal Choice of B_u and B_u^* The first order conditions for domestic and foreign bonds are

$$\Lambda_{u,t} = E_t\left[\frac{\Phi_{t+1}}{\Phi_t}\Lambda_{u,t+1}R_t\right], \quad \Lambda_{u,t} = E_t\left[\frac{\Phi_{t+1}}{\Phi_t}\Lambda_{u,t+1}R_t^*\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right].$$

The domestic saving-consumption Euler equation is then given by

$$\frac{1}{P_t^C}\lambda_{u,t} = E_t\left[\frac{\Phi_{t+1}}{\Phi_t}\frac{1}{P_{t+1}^C}\lambda_{u,t+1}\right]R_t, \quad 1 = E_t\left[\frac{\Phi_{t+1}}{\Phi_t}\frac{\lambda_{u,t+1}}{\lambda_{u,t}}\left(\Pi_{t+1}^C\right)^{-1}\right]R_t. \tag{B.6}$$

We use the definition of the real exchange rate and for consumer price inflation

$$\mathcal{Q}_t \equiv \mathcal{E}_t\frac{P_t^*}{P_t}, \quad \Pi_t^C \equiv \frac{P_t^C}{P_{t-1}^C}.$$

P_t (P_t^*) denotes the domestic (foreign) final output price level and P_t^C denotes the price level for consumption goods.[21] The foreign saving-consumption Euler equation

$$1 = E_t\left[\frac{\Phi_{t+1}}{\Phi_t}\frac{\lambda_{u,t+1}}{\lambda_{u,t}}\left(\Pi_{t+1}^C\right)^{-1}\frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t}\frac{\frac{P_{t+1}}{P_{t+1}^*}}{\frac{P_t}{P_t^*}}\right]R_t^* \quad \Leftrightarrow \quad 1 = E_t\left[\frac{\Phi_{t+1}}{\Phi_t}\frac{\lambda_{u,t+1}}{\lambda_{u,t}}\left(\Pi_{t+1}^C\right)^{-1}\frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t}\frac{\Pi_{t+1}}{\Pi_{t+1}^*}\right]R_t^*.$$

We can derive the uncovered interest rate parity (UIP) condition by combining the two Euler equations

$$E_t\left[\frac{\Phi_{t+1}}{\Phi_t}\frac{\lambda_{u,t+1}}{\lambda_{u,t}}\left(\Pi_{t+1}^C\right)^{-1}\left(R_t - R_t^*\frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t}\frac{\Pi_{t+1}}{\Pi_{t+1}^*}\right)\right] = 0 \tag{B.7}$$

We define the unconstrained household's stochastic discount factor as

$$\Lambda_{u,t,t+1} \equiv E_t\left[\frac{\Phi_{t+1}}{\Phi_t}\frac{\lambda_{u,t+1}}{\lambda_{u,t}}\right] \tag{B.8}$$

Unconstrained Household Budget in real terms

$$w_t^h N_{u,t}^h + R_{t-1}B_{u,t-1}\frac{1}{P_t} + R_{t-1}^*B_{u,t-1}^*\mathcal{E}_t\frac{1}{P_t} + div_{u,t}^F + div_{u,t}^L = \frac{P_t^C}{P_t}C_{u,t} + B_{u,t}\frac{1}{P_t} + B_{u,t}^*\mathcal{E}_t\frac{1}{P_t} + t_{u,t}^F + t_{u,t}^L$$

[21] We will show that these two are equivalent below when discussing the retailer.

$$w_t^h N_{u,t}^h + \frac{R_{t-1} b_{u,t-1}}{\Pi_t} + \frac{R_{t-1}^* b_{u,t-1}^* \mathcal{Q}_t}{\Pi_t^*} + div_{u,t}^F + div_{u,t}^L \quad = \quad p_t^C C_{u,t} + b_{u,t} + b_{u,t}^* \mathcal{Q}_t + t_{u,t}^F + t_{u,t}^L \quad \text{(B.9)}$$

Detrending Total Profits from Firm Ownership

$$\begin{aligned} DIV_{u,t}^F &= DIV_{u,t}^Z + DIV_{u,t}^M + DIV_{u,t}^X, \quad div_{u,t}^F \equiv \frac{1}{P_t} DIV_{u,t}^F \\ div_{u,t}^F &= div_{u,t}^Z + div_{u,t}^M + div_{u,t}^X. \end{aligned} \quad \text{(B.10)}$$

B.1.2 Constrained Households

Members of constrained households consume, save and work to maximise their lifetime utility $\mathcal{U}_{c,s}$

$$\mathcal{U}_{c,s} = E_s \left[\sum_{t=s}^{\infty} \beta^t \left\{ U_{c,t}\left(C_{c,t}, N_{c,t}^h \right) \right\} \right], \quad U_{c,t} = \left[\frac{(C_{c,t} - \psi_C C_{c,t-1})^{1-\sigma} - 1}{1-\sigma} - \nu_L \frac{\left(N_{c,t}^h \right)^{1+\varphi}}{1+\varphi} \right]$$

where $N_{c,t}^h$ is the constrained household's labor supply and φ is the elasticity of labor supply, ν_L is the relative weight on the disutility of working. Utility is maximised subject to the budget constraint

$$W_t^h N_{c,t}^h + R_{t-1} B_{c,t-1} + DIV_{c,t}^L \quad = \quad P_t^C C_{c,t} + B_{c,t} + P_t \Psi_{c,t} + T_{c,t}^L, \quad \Psi_{c,t} = \frac{\Psi}{2} \left(\frac{B_{c,t}}{P_t} - \bar{b}_c \right)^2$$

where $B_{c,t}$ denotes a domestic nominal risk-less bond, which provides a nominal gross returns of R_t to the constrained household and $DIV_{c,t}^L$ are the profits made by monopolistically competitive labor unions. $T_{c,t}^L$ is a transfer to the union in order to subsidize its cost.

Lagrangian Each constrained household solves the following Lagrangian in any arbitrary period t

$$\mathcal{L}_{c,t} = \sum_{\mathcal{S}^t} \pi_{\mathcal{S}^t} \sum_{t=0}^{\infty} \beta^t \left\{ U_{c,t} + \Lambda_{c,t} \left[W_t^h N_{c,t}^h + R_{t-1} B_{c,t-1} + DIV_{c,t}^L - P_t^C C_{c,t} - B_{c,t} - P_t \Psi_{c,t} - T_{c,t}^L \right] \right\}$$

where $\Lambda_{c,t}$ is the constrained household Lagrange multiplier associated with the resource constraint.

Optimal Choice of C_c The first-order condition for constrained consumption is

$$\Lambda_{c,t} \quad = \quad \frac{U_{c,t}^C}{P_t^C}, \quad \lambda_{c,t} \equiv U_{c,t}^C = (C_{c,t} - h C_{c,t-1})^{-\sigma}. \quad \text{(B.11)}$$

where we define the marginal utility of constrained household consumption as $\lambda_{c,t}$.

Optimal Choice of B_c The constrained household first order conditions for domestic bonds are

$$\begin{aligned} \frac{\partial \mathcal{L}_{c,t}}{\partial B_{c,t}} &= \pi_{\mathcal{S}^t} \beta^t \left\{ \Lambda_{c,t} \left[-\left(1 + P_t \frac{\partial \Psi_{c,t}}{\partial B_{c,t}} \right) \right] \right\} + \pi_{\mathcal{S}^{t+1|t}} \beta^{t+1} \left\{ \Lambda_{c,t+1} \left[R_t \right] \right\} = 0 \\ 0 &= \left\{ \Lambda_{c,t} \left[-\left(1 + P_t \frac{\partial \Psi_{c,t}}{\partial B_{c,t}} \right) \right] \right\} + \beta E_t \left\{ \Lambda_{c,t+1} \left[R_t \right] \right\} \end{aligned}$$

The constrained household saving-consumption Euler equation is then given by

$$1 + \Psi(b_{c,t} - \bar{b}_c) \quad = \quad E_t \left[\beta \frac{\lambda_{c,t+1}}{\lambda_{c,t}} \left(\Pi_{t+1}^C \right)^{-1} \right] R_t. \quad \text{(B.12)}$$

We define the constrained household's stochastic discount factor as

$$\Lambda_{c,t,t+1} \equiv E_t\left[\beta\frac{\lambda_{c,t+1}}{\lambda_{c,t}}\right] \tag{B.13}$$

Optimal Choice of N_c^s The first order conditions for labor supply is

$$\Lambda_{c,t} = -\frac{U_{c,t}^N}{W_t^h} \quad \Leftrightarrow \quad W_t^h = MRS_{c,t} \equiv -\frac{U_{c,t}^N}{\Lambda_{c,t}}$$

We define $w_t^h \equiv W_t^h/P_t$ and denote the wage and the marginal rate of substitution in real terms

$$w_t^h = mrs_{c,t} \tag{B.14}$$

$$mrs_{c,t} = -\frac{U_{c,t}^N}{\lambda_{c,t}} \tag{B.15}$$

$$U_{c,t}^N = -\nu_L\left(N_{c,t}^h\right)^{\varphi} \tag{B.16}$$

Real-term constrained household budget

$$w_t^h N_{c,t}^h + \frac{R_{t-1}b_{c,t-1}}{\Pi_t} + div_{c,t}^L = p_t^C C_{c,t} + b_{c,t} + \frac{\Psi}{2}\left(b_{c,t}-\bar{b}_c\right)^2 + t_{c,t}^L \tag{B.17}$$

Auxiliary Household Equations We define the consumption gap as the ratio between unconstrained and constrained consumption

$$\Gamma_t \equiv \frac{C_{u,t}}{C_{c,t}}. \tag{B.18}$$

Moreover, we use

$$t_t^F = t_t^Z + t_t^M + t_t^X \tag{B.19}$$

$$t_t^Z = (1-\tau_t^Z)(w_t N_t^h + p_t^M M_t) \tag{B.20}$$

$$t_t^M = (1-\tau_t^M)p_t^{X*}\mathcal{Q}_t M_t \tag{B.21}$$

$$t_t^X = (1-\tau_t^X)p_t^X X_t \tag{B.22}$$

$$t_t^L = (1-\tau_t^W)w_t^h N_t^h \tag{B.23}$$

B.1.3 Aggregation and Market Clearing

$$C_t = \lambda C_{c,t} + (1-\lambda)C_{u,t} \tag{B.24}$$

$$b_t = \lambda b_{c,t} + (1-\lambda)b_{u,t} \tag{B.25}$$

$$N_t^h = \lambda N_{c,t}^h + (1-\lambda)N_{u,t}^h \tag{B.26}$$

$$\Lambda_{t,t+1} = \lambda\Lambda_{c,t,t+1} + (1-\lambda)\Lambda_{u,t,t+1} \tag{B.27}$$

$$\begin{aligned}
div_t^F &= (1-\lambda)div_{u,t}^F \\
div_t^L &= \lambda div_{c,t}^L + (1-\lambda)div_{u,t}^L \\
b_t^* &= (1-\lambda)b_{u,t}^* \\
t_t^F &= (1-\lambda)t_{u,t}^F \\
t_t^L &= \lambda t_{c,t}^L + (1-\lambda)t_{u,t}^L
\end{aligned}$$

Domestic Bond Market Clearing We assume that domestic government bonds are in zero net supply

$$b_t = 0 \tag{B.28}$$

This implies that constrained and unconstrained households can lend to each other

$$0 = b_t = \lambda b_{c,t} + (1-\lambda) b_{u,t}, \qquad \Leftrightarrow \qquad \lambda b_{c,t} = -(1-\lambda) b_{u,t}$$

but recall the constrained household is subject to bond adjustment costs. Moreover, the unconstrained household can also hold foreign bonds.

Firm and Union Profits net of monopolistic competion correction subsidy

$$\begin{aligned}
div_t^L - t_t^L &= w_t N_t - \tau_t^W w_t^H N_t^h - w_t^H N_t^h + \tau_t^W w_t^H N_t^h = w_t N_t - w_t^H N_t^h, \quad t_t^L = (1-\tau_t^L) w_t^h N_t^h \\
div_t^Z - t_t^Z &= Z_t - \tau_t^Z \left(w_t N_t + p_t^M M_t \right) - t_t^Z = Z_t - \left(w_t N_t + p_t^M M_t \right) \\
div_t^M - t_t^M &= p_t^M M_t - \tau_t^M p_t^{X,*} \mathcal{Q}_t M_t - t_t^M = p_t^M M_t - p_t^{X,*} \mathcal{Q}_t M_t \\
div_t^X - t_t^X &= p_t^{EXP} \mathcal{Q}_t X_t - \tau_t^X p_t^X X_t - t_t^X = p_t^{EXP} \mathcal{Q}_t X_t - p_t^X X_t
\end{aligned}$$

Combine Firm Profits net of monopolistic competion correction subsidy

$$\begin{aligned}
div_t^F - t_t^F &= Z_t - \left(w_t N_t + p_t^M M_t \right) + p_t^M M_t - p_t^{X,*} \mathcal{Q}_t M_t + p_t^{EXP} \mathcal{Q}_t X_t - p_t^X X_t \\
div_t^F - t_t^F &= Z_t - w_t N_t - p_t^{X,*} \mathcal{Q}_t M_t + p_t^{EXP} \mathcal{Q}_t X_t - p_t^X X_t
\end{aligned}$$

Goods Market Clearing - Combine Household Budgets Recall the real-term household budgets

$$\begin{aligned}
w_t^h N_{u,t}^h + \frac{R_{t-1} b_{u,t-1}}{\Pi_t} + \frac{R_{t-1}^* b_{u,t-1}^* \mathcal{Q}_t}{\Pi_t^*} + div_{u,t}^F + div_{u,t}^L &= p_t^C C_{u,t} + b_{u,t} + b_{u,t}^* \mathcal{Q}_t + t_{u,t}^F + t_{u,t}^L \\
w_t^h N_t^h + \frac{R_{t-1} b_{c,t-1}}{\Pi_t} + div_{c,t}^L &= p_t^C C_{c,t} + b_{c,t} + \frac{\Psi}{2} \left(b_{c,t} - \bar{b} \right)^2 + t_{c,t}^L
\end{aligned}$$

and re-arrange for consumption, pre-multiplied with their household-type share λ to get

$$\begin{aligned}
p_t^C C_t = {} & (1-\lambda) \left(w_t^h N_{u,t}^h + \frac{R_{t-1} b_{u,t-1}}{\Pi_t} + \frac{R_{t-1}^* b_{u,t-1}^* \mathcal{Q}_t}{\Pi_t^*} + div_{u,t}^F - t_{u,t}^F + div_{u,t}^L - t_{u,t}^L - b_{u,t} - b_{u,t}^* \mathcal{Q}_t \right) \\
& + \lambda \left(w_t^h N_{c,t}^h + \frac{R_{t-1} b_{c,t-1}}{\Pi_t} + div_{c,t}^L - t_{c,t}^L - b_{c,t} - \frac{\Psi}{2} \left(b_{c,t} - \bar{b} \right)^2 \right)
\end{aligned}$$

which can be simplified to

$$\begin{aligned}
p_t^C C_t = {} & \underbrace{w_t^h N_t^h + div_t^L - t_t^L}_{= w_t N_t^h} + \frac{R_{t-1} b_{t-1}}{\Pi_t} + \frac{R_{t-1}^* b_{t-1}^* \mathcal{Q}_t}{\Pi_t^*} + \underbrace{div_t^F - t_t^F}_{Z_t - w_t N_t^h - p_t^{X,*} \mathcal{Q}_t M_t + p_t^{EXP} \mathcal{Q}_t X_t - p_t^X X_t} \\
& - b_t - b_t^* \mathcal{Q}_t - \frac{\lambda \Psi}{2} \left(b_{c,t} - \bar{b} \right)^2
\end{aligned}$$

to get

$$p_t^C C_t + p_t^X X_t = \underbrace{\frac{R_{t-1} b_{t-1}}{\Pi_t} - b_t}_{=0} - \underbrace{\left(b_t^* \mathcal{Q}_t - \frac{R_{t-1}^* b_{t-1}^* \mathcal{Q}_t}{\Pi_t^*} \right)}_{NFA_t} + Z_t + \underbrace{p_t^{EXP} \mathcal{Q}_t X_t - p_t^{X,*} \mathcal{Q}_t M_t}_{NFA_t} - \frac{\lambda \Psi}{2} \left(b_{c,t} - \bar{b}_c \right)^2$$

and finally

$$P_t^C C_t + P_t^X X_t \quad = \quad P_t \left(Z_t - \frac{\lambda \Psi}{2} \left(b_{c,t} - \bar{b} \right)^2 \right) \tag{B.29}$$

and

$$NFA_t \quad = \quad b_t^* \mathcal{Q}_t - \frac{R_{t-1}^* b_{t-1}^* \mathcal{Q}_t}{\Pi_t^*} = p_t^{EXP} \mathcal{Q}_t X_t - p_t^{X,*} \mathcal{Q}_t M_t \tag{B.30}$$

Introduce Definition for Value-added Output Recall

$$
\begin{aligned}
p_t^C C_t &= \underbrace{w_t^h N_t^h + div_t^L - t_t^L}_{=w_t N_t^h} + \frac{R_{t-1} b_{t-1}}{\Pi_t} + \frac{R_{t-1}^* b_{t-1}^* \mathcal{Q}_t}{\Pi_t^*} + \underbrace{div_t^F - t_t^F}_{Z_t - w_t N_t - p_t^{X,*} \mathcal{Q}_t M_t + p_t^{EXP} \mathcal{Q}_t X_t - p_t^X X_t} \\
&\quad - b_t - b_t^* \mathcal{Q}_t - \frac{\lambda \Psi}{2} \left(b_{c,t} - \bar{b}\right)^2
\end{aligned}
$$

$$
\frac{\lambda \Psi}{2} \left(b_{c,t} - \bar{b}\right)^2 + p_t^X X_t + p_t^C C_t = \frac{R_{t-1}^* b_{t-1}^* \mathcal{Q}_t}{\Pi_t^*} + \underbrace{Z_t - p_t^{X,*} \mathcal{Q}_t M_t}_{\equiv p_t^Y Y_t} + p_t^{EXP} \mathcal{Q}_t X_t - b_t^* \mathcal{Q}_t
$$

$$
\frac{\lambda \Psi}{2} \left(b_{c,t} - \bar{b}\right)^2 + \left(p_t^X - p_t^{EXP} \mathcal{Q}_t\right) X_t + p_t^C C_t + \underbrace{\left(b_t^* \mathcal{Q}_t - \frac{R_{t-1}^* b_{t-1}^* \mathcal{Q}_t}{\Pi_t^*}\right)}_{NFA_t} = \underbrace{Z_t - p_t^{X,*} \mathcal{Q}_t M_t}_{\equiv p_t^Y Y_t}
$$

Assume Infinite Bond Adjustment Cost

$$
\left(p_t^X - p_t^{EXP} \mathcal{Q}_t\right) X_t + p_t^C C_t + \underbrace{\left(b_t^* \mathcal{Q}_t - \frac{R_{t-1}^* b_{t-1}^* \mathcal{Q}_t}{\Pi_t^*}\right)}_{NFA_t} = \underbrace{Z_t - p_t^{X,*} \mathcal{Q}_t M_t}_{\equiv p_t^Y Y_t}
$$

Assume Fully Flexible Prices in the Export Sector and Unity Gross Markup

$$
\begin{aligned}
\frac{f_t^{EXP,1}}{f_t^{EXP,2}} &= 1 = \frac{p_t^X}{p_t^{EXP}} \mathcal{Q}_t mc_t^X = \frac{p_t^X}{p_t^{EXP}} \mathcal{Q}_t \\
f_t^{EXP,1} &= \frac{p_t^X}{p_t^{EXP}} \mathcal{Q}_t mc_t^X X_t \\
f_t^{EXP,2} &= X_t
\end{aligned}
$$

$$
\underbrace{\left(p_t^X - p_t^X\right) X_t}_{=0} + p_t^C C_t + \underbrace{\left(b_t^* \mathcal{Q}_t - \frac{R_{t-1}^* b_{t-1}^* \mathcal{Q}_t}{\Pi_t^*}\right)}_{NFA_t} = \underbrace{Z_t - p_t^{X,*} \mathcal{Q}_t M_t}_{\equiv p_t^Y Y_t}
$$

Assume Fully Flexible Prices in the Import Sector and Unity Gross Markup

$$
\begin{aligned}
p_t^C C_t + NFA_t &= \underbrace{Z_t - \overbrace{p_t^{X,*} \mathcal{Q}_t}^{p_t^M} M_t}_{\equiv p_t^Y Y_t} \\
p_t^C C_t &= p_t^Y Y_t - NFA_t = p_t^Y Y_t + p_t^M M_t - p_t^X X_t \quad \Leftrightarrow \quad \frac{P_t^C}{P_t} C_t + \frac{P_t^X}{P_t} X_t = Z_t
\end{aligned}
$$

Recall

$$
NFA_t = b_t^* \mathcal{Q}_t - \frac{R_{t-1}^* b_{t-1}^* \mathcal{Q}_t}{\Pi_t^*} = \underbrace{p_t^{EXP} \mathcal{Q}_t}_{p_t^X} X_t - \underbrace{p_t^{X,*} \mathcal{Q}_t}_{p_t^M} M_t
$$

B.1.4 Steady State

$$NFA_{ss} = \left[b^*_{ss}\mathcal{Q}_{ss} - \left(\frac{R^*_{ss}b^*_{ss}\mathcal{Q}_{ss}}{\Gamma^H\Pi^*_{ss}}\right)\right], \quad NFA_{ss}/\mathcal{Q}_{ss} = \left[b^*_{ss}\left(1-\frac{R^*_{ss}}{\Gamma^H\Pi^*_{ss}}\right)\right] \quad b^*_{ss} = \frac{NFA_{ss}}{\mathcal{Q}_{ss}\left(1-\frac{R^*_{ss}}{\Gamma^H\Pi^*_{ss}}\right)}$$

$$\begin{aligned}
C_{c,t} &= w^h_t N^h_{c,t} + div^L_{c,t} - t^L_{c,t} + \frac{R_{t-1}b_{c,t-1}}{\Pi_t} - b_{c,t} - \frac{\Psi}{2}\left(b_{c,t}-\bar{b}_c\right)^2 = w_t N^h_{c,t} + \frac{R_{t-1}b_{c,t-1}}{\Pi_t} - b_{c,t} - \frac{\Psi}{2}\left(b_{c,t}-\bar{b}_c\right)^2 \\
C_{c,ss} &= w_{ss}L^{w,s}_{ss} + b_{c,ss}\left(\frac{R_{ss}}{\Pi_{ss}}-1\right) \\
b_{c,ss} &= \bar{b}_c = 0 \quad \text{(this is an assumption)} \\
C_{c,ss} &= w_{ss}L^{w,s}_{ss}, \qquad \lambda_{c,ss} = ((1-\psi_C)C_{c,ss})^{-\sigma} \\
C_{u,t} &= \left(w^h_t N^h_{u,t} + div^L_{u,t} - t^L_{u,t} + \frac{R_{t-1}b_{u,t-1}}{\Pi_t} + \frac{R^*_{t-1}b^*_{u,t-1}\mathcal{Q}_t}{\Pi^*_t} + div^F_{u,t} - t^F_{u,t} - b_{u,t} - b^*_{u,t}\mathcal{Q}_t\right) \\
C_{u,ss} &= w_{ss}L^{c,s}_{ss} + b_{u,ss}\left(\frac{R_{ss}}{\Pi_{ss}}-1\right) + b^*_{u,ss}\left(\frac{R^*_{ss}\mathcal{Q}_{ss}}{\Pi^*_{ss}} - \mathcal{Q}_{ss}\right) + div^F_{u,ss} - t^F_{u,ss} \\
b_{c,ss} &= \bar{b}_c = 0 \quad \Rightarrow \quad \bar{b}_u = 0 \\
C_{u,ss} &= w_{ss}L^{c,s}_{ss} + \frac{b^*_{ss}}{1-\lambda}\left(\frac{R^*_{ss}\mathcal{Q}_{ss}}{\Pi^*_{ss}} - \mathcal{Q}_{ss}\right) + \frac{div^F_{ss} - t^F_{ss}}{1-\lambda}, \qquad \lambda_{u,ss} = ((1-\psi_C)C_{u,ss})^{-\sigma}
\end{aligned}$$

Recall

$$\begin{gathered}
L_{ss} = \lambda L_{c,ss} + (1-\lambda)L_{u,ss} \quad , \quad mrs_{c,ss} = -\frac{U^L_{c,ss}}{\lambda_{c,ss}} \\
U^L_{c,ss} = -\nu_L\left(L_{c,ss}\right)^{\varphi} \quad , \quad w^H_{ss} = mrs_{ss} \\
mrs_{ss} = -\frac{U^L_{u,ss}}{\lambda_{u,ss}} \quad , \quad U^L_{u,ss} = -\nu_L\left(L_{u,ss}\right)^{\varphi}
\end{gathered}$$

which implies

$$\begin{aligned}
\frac{\lambda_{u,ss}}{\lambda_{c,ss}} = \frac{U^L_{u,ss}}{U^L_{c,ss}}, \quad \frac{((1-\psi_C)C_{u,ss})^{-\sigma}}{((1-\psi_C)C_{c,ss})^{-\sigma}} &= \frac{U^L_{u,ss}}{U^L_{c,ss}} \\
\left(\frac{w_{ss}L_{u,ss} + \frac{b^*_{ss}}{1-\lambda}\left(\frac{R^*_{ss}\mathcal{Q}_{ss}}{\Pi^*_{ss}} - \mathcal{Q}_{ss}\right) + \frac{div^F_{ss}-t^F_{ss}}{1-\lambda}}{w_{ss}L_{c,ss}}\right)^{-\epsilon_C} &= \left(\frac{L_{u,ss}}{L_{c,ss}}\right)^{\varphi} \\
\left(L_{u,ss} + \frac{1}{w_{ss}}\frac{b^*_{ss}}{1-\lambda}\left(\frac{R^*_{ss}\mathcal{Q}_{ss}}{\Pi^*_{ss}} - \mathcal{Q}_{ss}\right) + \frac{1}{w_{ss}}\frac{div^F_{ss}-t^F_{ss}}{1-\lambda}\right)^{-\epsilon_C}\left(L_{c,ss}\right)^{\epsilon_C} &= \left(L_{u,ss}\right)^{\varphi}\left(L_{c,ss}\right)^{-\varphi}
\end{aligned}$$

Combine this with

$$\begin{aligned}
1 \underbrace{=}_{\text{assume}} L_{ss} &= \lambda L_{c,ss} + (1-\lambda)L_{u,ss} \\
\frac{1-(1-\lambda)L_{u,ss}}{\lambda} &= L_{c,ss} \\
\frac{1-\lambda L_{c,ss}}{(1-\lambda)} &= L_{u,ss}
\end{aligned}$$

and use a solver the get $L_{c,ss}$.

B.2 labor Packers and Unions

The introduction of wage stickiness into the model involves two types of agents: (i) perfectly competitive labor packers and (ii) monopolistically competitive unions. After households have chosen how much labor to supply in a given period, $N^h_{k,t}(j)$, $k \in \{u,c\}$, this labor is supplied to a union, in return for a nominal wage W^h_t. The union unpacks the homogenous labor supplied by households and differentiates it into different varieties $N_t(j)$, $j \in [0,1]$ and sells these units of labor varieties at wage $W_t(j)$. The union acts as monopolist since each labor variety is only imperfectly substitutable with each other. The profits of the monopolistically competitive union are rebated back to households.

Perfectly Competitive labor Packers Varieties $N_t(j)$ are assembled by labor packers according to

$$N_t = \left[\int_0^1 (N_t(j))^{\frac{\epsilon_w - 1}{\epsilon_w}} dj\right]^{\frac{\epsilon_w}{\epsilon_w - 1}}, \quad \mathcal{M}_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}.$$

where $N_t(j)$ denotes the demand for a specific labor variety j and N_t denotes aggregate labor demand. ϵ_w is the elasticity of substitution between labor varieties and thus $\mathcal{M}_w$ is the corresponding gross wage markup of monopolistically competitive unions. After the packers have assembled the labor bundle they sell it to firms at wage W_t who then use it in the production process. The cost minimisation of labor packers implies the following demand schedule for a labor variety j

$$N_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{\frac{\mathcal{M}_w}{1-\mathcal{M}_w}} N_t, \quad W_t \equiv \left(\int_0^1 (W_t(j))^{\frac{1}{1-\mathcal{M}_w}} dj\right)^{1-\mathcal{M}_w}$$

where W_t is the aggregate wage index. Optimal packer behaviour implies that $W_t N_t = \int_0^1 W_t(j) N_t(j) dj$.

Monopolistically Competitive labor Unions Each individual labor union who sells its imperfectly substitutable labor variety $N^h_t(j)$ to the packer is subject to *nominal wage rigidities*. The probability that the union cannot reset its wage is ϕ_w. It is convenient to split the problem of a monopolistically competitive labor union into two steps: (i) the intra-temporal cost minimisation problem and (ii) the inter-temporal wage setting problem.

Cost Minimisation Problem A union will choose to minimise its costs $\tau^W_t W^h_t N^h_t(j)$ subject to meeting the packer's labor demand

$$\mathcal{L} = -\tau^W_t \left(W^h_t N^h_t(j)\right) + MC^W_t(j) \left(N^h_t(j) - \left(\frac{W_t(j)}{W_t}\right)^{-\frac{\mathcal{M}_w}{\mathcal{M}_w - 1}} N_t\right).$$

The Lagrange multiplier $MC^W_t(j)$ is the union's (nominal) shadow cost of providing one more unit of labor, i.e. the nominal marginal cost and τ^W_t is a shock to marginal costs that is isomorphic to a wage markup shock. Note that the Lagrange multiplier of an individual union j does not depend on its own quantities of inputs demanded, so that all unions have the same marginal costs $MC^W_t(j) = MC^W_t$. The wage paid to the household[22], W^h_t corresponds to the marginal rate of substitution so that

$$\frac{\partial \mathcal{L}}{\partial N^h_t(j)} = 0 \Leftrightarrow \tau^W_t W^h_t = MC^W_t(j),\ MC^W_t = \tau^W_t W^h_t = \tau^W_t MRS_t$$

Recall that we use lower cases to denotes real (final output price level) terms $w^h_t \equiv W^h_t / P_t$ so that

$$mc^W_t = \tau^W_t w^h_t = \tau^W_t mrs_t \tag{B.31}$$

[22] We assume that both, unconstrained and constrained household receive the same wage.

Pricing Problem The objective of each union j is to maximise its nominal profits $DIV_t^L(j)$

$$DIV_t^L(j) = W_t(j)N_t^h(j) - \left\{\tau_t^W\left(W_t^h N_t^h(j)\right)\right\}, \quad div_t^N = \left(w_t - mc_t^W\right)N_t^h \tag{B.32}$$

With probability ϕ_w a union is stuck with its previous-period wage indexed to a composite

$$W_t(j) = \begin{cases} W_t^{\#}(j) & \textit{with probability: } 1-\phi_w \\ W_{t-1}(j)\left(\left(\Pi_{ss}^W\right)^{1-\xi_w}\left(\Pi_{t-1}^W\right)^{\xi_w}\right) & \textit{with probability: } \phi_w \end{cases}$$

where $\xi_w \in [0,1]$ is the weight attached to the previous period wage inflation. Consider a union who can reset its wage in the current period $W_t(j) = W_t^{\#}(j)$ and who is then stuck with its wage until future period $t+s$. The wage in this case would be

$$W_{t+s}(j) \;=\; W_t^{\#}(j)\left(\Pi_{ss}^W\right)^{s(1-\xi_w)}\left(\prod_{g=0}^{s-1}\left(\left(\Pi_{t+g}^W\right)^{\xi_w}\right)\right) = W_t^{\#}(j)\left[\left(\Pi_{ss}^W\right)^{s(1-\xi_w)}\left(\frac{W_{t+s-1}}{W_{t-1}}\right)^{\xi_w}\right]$$

Subject to the above derived demand constraint and assuming that a union j always meets the demand for its labor at the current wage labor unions solve the following optimisation problem

$$\max_{W_t^{\#}(j)} E_t \sum_{s=0}^{\infty}(\phi_W)^s \frac{\Phi_{t+s}}{\Phi_t}\frac{\lambda_{it+s}}{\lambda_{i,t}}\left[\left(W_t^{\#}(j)\right)^{1-\frac{\mathcal{M}_w}{\mathcal{M}_w-1}}\left(\left[\left(\Pi_{ss}^W\right)^{s(1-\xi_w)}\left(\frac{W_{t+s-1}}{W_{t-1}}\right)^{\xi_w}\right]\right)^{1-\frac{\mathcal{M}_w}{\mathcal{M}_w-1}}\right.$$
$$\left.\left[\left(\frac{1}{W_{t+s}}\right)^{-\frac{\mathcal{M}_w}{\mathcal{M}_w-1}} L_{t+s}^d\right] - MC_{t+s}^W\left[\left(W_t^{\#}(j)\right)^{-\frac{\mathcal{M}_w}{\mathcal{M}_w-1}}\left(\frac{\left[\left(\Pi_{ss}^W\right)^{s(1-\xi_w)}\left(\frac{W_{t+s-1}}{W_{t-1}}\right)^{\xi_w}\right]}{W_{t+s}}\right)^{-\frac{\mathcal{M}_w}{\mathcal{M}_w-1}} N_{t+s}\right]\right].$$

$N_{t+s|t}(j)$ denotes the labor supplied in period $t+s$ by a union j that last reset its wage in period t. Taking the derivative with respect to $W_t^{\#}(j)$ delivers the familiar wage inflation schedule (B.33)

$$\frac{f_t^{W,1}}{f_t^{W,2}}\mathcal{M}_w \;=\; w_t^* = \left(\frac{1-\phi_W(\zeta_t^W)^{\frac{1}{\mathcal{M}_w-1}}}{1-\phi_W}\right)^{1-\mathcal{M}_w} \tag{B.33}$$

$$f_t^{W,1} \;=\; \frac{1}{w_t}mc_t^W N_t + \phi_W E_t\left[\Lambda_{t,t+1}\frac{\Pi_{t+1}^W}{\Pi_{t+1}}(\zeta_{t+1}^W)^{\frac{\mathcal{M}_w}{\mathcal{M}_w-1}} f_{t+1}^{W,1}\right] \tag{B.34}$$

$$f_t^{W,2} \;=\; N_t + \phi_W E_t\left[\Lambda_{t,t+1}\frac{\Pi_{t+1}^W}{\Pi_{t+1}}(\zeta_{t+1}^W)^{\frac{1}{\mathcal{M}_w-1}} f_{t+1}^{W,2}\right] \tag{B.35}$$

$$\zeta_t^W \;=\; \frac{\Pi_t^W}{(\Pi_{ss}^W)^{1-\xi_w}(\Pi_{t-1}^W)^{\xi_w}} \tag{B.36}$$

$$w_t \;=\; \frac{\Pi_t^W}{\Pi_t}w_{t-1} \tag{B.37}$$

Recursively written wage dispersion is given by

$$\mathcal{D}_t^W \;=\; (1-\phi_W)\left(\frac{1-\phi_W\left(\zeta_t^W\right)^{\frac{1}{\mathcal{M}_w-1}}}{1-\phi_W}\right)^{\mathcal{M}_w} + \phi_W\left(\zeta_t^W\right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w-1}}\mathcal{D}_{t-1}^W. \tag{B.38}$$

The aggregate supply of hours worked in the economy is given by $N_t^h = N_t\mathcal{D}_t^W$.

B.3 Firms

There are three domestic firm sectors in our model: (i) final output good producers, (ii) import good producers and (iii) export good producers. All three sectors are characterised by monopolistic competition and nominal rigidities so that each sector will be associated with a Phillips Curve.

B.3.1 Final Output Goods Sector

Final output goods production involves two types of agents: (i) perfectly competitive final output packers and (ii) monopolistically competitive final output producers.

Final Output Good packers Final output packers demand and aggregate infinitely many varieties of final output goods $Z_t(i)$, $i \in [0,1]$ into a final output good Z_t according to

$$Z_t = \left[\int_0^1 (Z_t(i))^{1-\frac{1}{\epsilon_Z}} di\right]^{\frac{\epsilon_Z}{\epsilon_Z - 1}}, \quad \mathcal{M}_z \equiv \frac{\epsilon_Z}{\epsilon_Z - 1}$$

$Z_t(i)$ denotes the demand for a specific variety i of the final output good and Z_t denotes the aggregate demand of the final output good. ϵ_Z is the elasticity of substitution and $\mathcal{M}_z$ is the corresponding gross markup of monopolistically competitive final output good producers. Final output packers purchase a single variety at given prices $P_t(i)$ and sell the final output good Z_t at price P_t to a sectoral retailer[23] who transforms the final output good into consumption and export goods. Optimal behaviour implies the standard demand schedule of the final output good packers and price index P_t

$$Z_t(i) \quad = \quad \left(\frac{P_t(i)}{P_t}\right)^{\frac{\mathcal{M}_z}{1-\mathcal{M}_z}} Z_t, \qquad P_t \equiv \left(\int_0^1 (P_t(i))^{\frac{1}{1-\mathcal{M}_z}} di\right)^{1-\mathcal{M}_z}, \quad P_t Z_t = \int_0^1 P_t(i) Z_t(i) di.$$

Final Output Good Producers Each variety $Z_t(i)$ that the final output good packer demands and assembles is produced and supplied by a single *monopolistically competitive* final output producer $i \in [0,1]$ according to the final output CES production function

$$Z_t(i) \quad = \quad \varepsilon^{TFP} \left((1-\alpha)^{\frac{1}{\psi}} (N_t(i))^{\frac{\psi-1}{\psi}} + (\alpha)^{\frac{1}{\psi}} (M_t(i))^{\frac{\psi-1}{\psi}} \right)^{\frac{\psi}{\psi-1}} \tag{B.39}$$

The production inputs demanded by a specific firm i are labor $N_t(i)$ and imported energy goods $M_t(i)$. α denotes the share of energy in production and ψ denotes the elasiticity of substitution between labor and the import good. Both, labor and import goods, are provided by monopolistically competitive unions and importers.[24] Firm i uses an index of imports $M_t(i) = \left[\int_0^1 (M_t(i,j))^{1/\mathcal{M}^m} dj\right]^{\mathcal{M}^m}$. Optimal firm behaviour implies $P_t^M M_t(i) = \int_0^1 P_t^M(j) M_t(i,j) dj$ where the price indices are defined as $P_t^M \equiv \left(\int_0^1 (P_t^M(j))^{\frac{1}{1-\mathcal{M}^m}} dj\right)^{1-\mathcal{M}^m}$ and the demand of final output firm i for a single variety j of the import good is given by

$$M_t(i,j) = \left(\frac{P_t^M(j)}{P_t^M}\right)^{-\frac{\mathcal{M}^m}{\mathcal{M}^m - 1}} M_t(i).$$

Each individual final output producer is subject to *nominal rigidities*. The probability that they cannot reset their price is ϕ_z. We split the firms problem into two steps: (i) the intra-temporal cost minimisation problem and (ii) the inter-temporal price setting problem.

[23]The problem of the sectoral retailers will be described below.
[24]The production and price setting process of these importers will be described in detail below.

Cost Minimisation Problem A final output firm chooses its inputs to minimise its costs

$$\min_{N_t(i),M_t(i)} \left\{ \tau_t^Z \left(W_t N_t(i) + P_t^M M_t(i) \right) \right\}, \qquad s.t. \quad Z_t(i) \geq \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\mathcal{M}_Z}{\mathcal{M}_Z - 1}} Z_t.$$

The Lagrangian is given by

$$\mathcal{L}_t^Z = -\tau_t^Z \left(W_t N_t(i) + P_t^M M_t(i) \right) + MC_t^Z(i) \left(Z_t(i) - \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\mathcal{M}_Z}{\mathcal{M}_Z - 1}} Z_t \right)$$

and the Lagrange multiplier $MC_t^Z(i)$ is the (nominal) shadow cost of producing one more unit of final output, e.g. the nominal marginal cost and τ_t^Z is a shock to final output marginal costs that is isomorphic to a markup shock. The optimality conditions are given by

$$w_t = (1-\alpha)^{\frac{1}{\psi}} \frac{mc_t^Z}{\tau_t^Z} \left(\frac{Z_t(i)}{N_t(i)} \right)^{\frac{1}{\psi}} \left(\varepsilon^{TFP} \right)^{\frac{\psi-1}{\psi}} \tag{B.40}$$

$$p_t^M = (\alpha)^{\frac{1}{\psi}} \frac{mc_t^Z}{\tau_t^Z} \left(\frac{Z_t(i)}{M_t(i)} \right)^{\frac{1}{\psi}} \left(\varepsilon^{TFP} \right)^{\frac{\psi-1}{\psi}} \tag{B.41}$$

Combine the first order conditions to obtain

$$\frac{W_t}{P_t^M} = \left(\frac{1-\alpha}{\alpha} \right)^{\frac{1}{\psi}} \left(\frac{N_t(i)}{M_t(i)} \right)^{-\frac{1}{\psi}}$$

Rearrange to obtain the optimal trade-off between production factors as a function of their relative price,

$$\frac{N_t(i)}{M_t(i)} = \frac{1-\alpha}{\alpha} \left(\frac{W_t}{P_t^M} \right)^{-\psi}. \tag{B.42}$$

Factor Demand Schedules Combine the optimality condition (B.42) with the production function

$$Z_t(i) = \left((1-\alpha)^{\frac{1}{\psi}} \left(\frac{1-\alpha}{\alpha} \left(\frac{W_t}{P_t^M} \right)^{-\psi} M_t(i) \right)^{\frac{\psi-1}{\psi}} + \alpha^{\frac{1}{\psi}} (M_t(i))^{\frac{\psi-1}{\psi}} \right)^{\frac{\psi}{\psi-1}}$$

$$Z_t(i) = \left((1-\alpha)^{\frac{1}{\psi}} \left(\frac{1-\alpha}{\alpha} \right)^{\frac{\psi-1}{\psi}} \left(\frac{W_t}{P_t^M} \right)^{1-\psi} + \alpha^{\frac{1}{\psi}} \right)^{\frac{\psi}{\psi-1}} M_t(i)$$

$$Z_t(i) = \left(\frac{(1-\alpha)\alpha^{-\frac{\psi-1}{\psi}} W_t^{1-\psi} + \alpha^{\frac{1}{\psi}} (P_t^M)^{1-\psi}}{(P_t^M)^{1-\psi}} \right)^{\frac{\psi}{\psi-1}} M_t(i)$$

$$Z_t(i) = \alpha^{-1} \left(\frac{(1-\alpha) W_t^{1-\psi} + \alpha (P_t^M)^{1-\psi}}{(P_t^M)^{1-\psi}} \right)^{\frac{\psi}{\psi-1}} M_t(i)$$

Rearrange to obtain the demand function for $M_t(i)$,

$$M_t(i) = \alpha \left(\frac{P_t^M}{\left((1-\alpha) W_t^{1-\psi} + \alpha (P_t^M)^{1-\psi} \right)^{\frac{1}{1-\psi}}} \right)^{-\psi} Z_t(i).$$

Equivalently, for $N_t(i)$ we obtain the following demand function

$$N_t(i) \quad = \quad (1-\alpha)\left(\frac{W_t}{\left((1-\alpha)W_t^{1-\psi}+\alpha(P_t^M)^{1-\psi}\right)^{\frac{1}{1-\psi}}}\right)^{-\psi} Z_t(i).$$

Final Output Marginal Cost To obtain the marginal cost, raise the first order condition with respect to $N_t(i)$ to the power $1-\psi$ and multiply by $1-\alpha$,

$$\begin{aligned}
(1-\alpha)W_t^{1-\psi} &= (1-\alpha)^{\frac{1-\psi}{\psi}+1}(MC_t^Z(i))^{1-\psi}(Z_t(i))^{\frac{1-\psi}{\psi}}N_t(i)^{-\frac{1-\psi}{\psi}} \\
\alpha(P_t^M)^{1-\psi} &= \alpha^{\frac{1-\psi}{\psi}+1}(MC_t^Z(i))^{1-\psi}(Z_t(i))^{\frac{1-\psi}{\psi}}(M_t(i))^{-\frac{1-\psi}{\psi}} \\
\left((1-\alpha)W_t^{1-\psi}+\alpha(P_t^M)^{1-\psi}\right)^{\frac{1}{1-\psi}} &= \left((1-\alpha)^{\frac{1-\psi}{\psi}+1}(MC_t^Z(i))^{1-\psi}Z_t(i)^{\frac{1-\psi}{\psi}}(N_t(i))^{-\frac{1-\psi}{\psi}}\right. \\
&\quad \left. +\alpha^{\frac{1-\psi}{\psi}+1}(MC_t^Z(i))^{1-\psi}(Z_t(i))^{\frac{1-\psi}{\psi}}(M_t(i))^{-\frac{1-\psi}{\psi}}\right)^{\frac{1}{1-\psi}}.
\end{aligned}$$

Rearrange to obtain the marginal cost,

$$\begin{aligned}
\left((1-\alpha)W_t^{1-\psi}+\alpha(P_t^M)^{1-\psi}\right)^{\frac{1}{1-\psi}} &= \left((1-\alpha)^{\frac{1}{\psi}}N_t(j)^{\frac{\psi-1}{\psi}}+\alpha^{\frac{1}{\psi}}M_{j,t}^{\frac{\psi-1}{\psi}}\right)^{\frac{1}{1-\psi}} MC_t^Z(i)(Z_t(i))^{\frac{1}{\psi}} \\
\left((1-\alpha)W_t^{1-\psi}+\alpha(P_t^M)^{1-\psi}\right)^{\frac{1}{1-\psi}} &= \left(\left((1-\alpha)^{\frac{1}{\psi}}N_t(j)^{\frac{\psi-1}{\psi}}+\alpha^{\frac{1}{\psi}}M_{j,t}^{\frac{\psi-1}{\psi}}\right)^{\frac{\psi}{\psi-1}}\right)^{-\frac{1}{\psi}} MC_t^Z(i)(Z_t(i))^{\frac{1}{\psi}} \\
\left((1-\alpha)W_t^{1-\psi}+\alpha(P_t^M)^{1-\psi}\right)^{\frac{1}{1-\psi}} &= (Z_t(i))^{-\frac{1}{\psi}}MC_t^Z(i)(Z_t(i))^{\frac{1}{\psi}} \\
MC_t^Z(i) &= MC_t^Z = \left((1-\alpha)W_t^{1-\psi}+\alpha(P_t^M)^{1-\psi}\right)^{\frac{1}{1-\psi}}
\end{aligned}$$

Note that the Lagrange multiplier of an individual final output producing firm i does not depend on its own quantities of labor demanded, so that all final output firms have the same multiplier $MC_t^Z(i) = MC_t^Z$.

Pricing Problem The objective of each final output producing firm is to maximise its nominal profits

$$DIV_t^Z(i) \quad = \quad P_t(i)Z_t(i) - \left\{\tau_t^Z\left(W_tN_t(i)+P_t^MM_t(i)\right)\right\} \quad \Leftrightarrow \quad div_t^Z = \left(1-mc_t^Z\right)Z_t. \tag{B.43}$$

With probability ϕ_z a firm is stuck with its previous-period price indexed to a composite of previous-period inflation and steady state inflation so that

$$P_t(i) = \begin{cases} P_t^{\#}(i) & \textit{with probability: } 1-\phi_z \\ P_{t-1}(i)\left((\Pi_{ss})^{1-\xi_z}(\Pi_{t-1})^{\xi_z}\right) & \textit{with probability: } \phi_z \end{cases}$$

where $\xi_z \in [0,1]$ is the weight attached to previous period inflation. Consider a firm who can reset its price in the current period $P_t(i) = P_t^{\#}(i)$ and who is then stuck with its price until future period $t+s$. The price in this case would be

$$P_{t+s}(i) \quad = \quad P_t^{\#}(i)\left[(\Pi_{ss})^{s(1-\xi_z)}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\xi_z}\right].$$

Final output good producing firms solve the following optimisation problem

$$\max_{P_t^{\#}(i)} E_t \sum_{s=0}^{\infty} (\phi_Z)^s \frac{\Phi_{t+s}}{\Phi_t} \frac{\lambda_{u,t+s}}{\lambda_{u,t}} \left[P_{t+s}(i) Z_{t+s|t}(i) - MC_{t+s}^{Z} Z_{t+s|t}(i) \right]$$

subject to the above derived demand constraint and assuming that a firm z always meets the demand for its good at the current price. $Z_{t+s|t}(i)$ denotes the final output supplied in period $t+s$ by a firm i that last reset its price in period t. If one substitutes the demand schedule and $P_{t+s}(i)$ into the objective function one obtains

$$\max_{P_t^{\#}(i)} E_t \sum_{s=0}^{\infty} (\phi_Z)^s \frac{\Phi_{t+s}}{\Phi_t} \frac{\lambda_{u,t+s}}{\lambda_{u,t}}$$
$$\left[\left(P_t^{\#}(i)\right)^{1-\frac{\mathcal{M}_z}{\mathcal{M}_z-1}} \left(\left[(\Pi_{ss})^{s(1-\xi_z)} \left(\frac{P_{t+s-1}}{P_{t-1}} \right)^{\xi_z} \right] \right)^{1-\frac{\mathcal{M}_z}{\mathcal{M}_z-1}} \left[\left(\frac{1}{P_{t+s}} \right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z-1}} Z_{t+s} \right] \right.$$
$$\left. -MC_{t+s}^{Z} \left[\left(P_t^{\#}(i)\right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z-1}} \left(\frac{\left[(\Pi_{ss})^{s(1-\xi_z)} \left(\frac{P_{t+s-1}}{P_{t-1}} \right)^{\xi_z} \right]}{P_{t+s}} \right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z-1}} Z_{t+s} \right] \right].$$

Taking the derivative with respect to $P_t^{\#}(i)$ delivers the familiar price inflation schedule (B.44)

$$\frac{f_t^{Z,1}}{f_t^{Z,2}} \mathcal{M}_z = \left[\frac{1-(\phi_Z)\left(\zeta_t^Z\right)^{\frac{-1}{1-\mathcal{M}_z}}}{1-\phi_Z} \right]^{1-\mathcal{M}_z} \tag{B.44}$$

$$f_t^{Z,1} = mc_t^Z Z_t + \phi_Z E_t \left[\Lambda_{u,t,t+1} (\zeta_{t+1}^Z)^{\frac{\mathcal{M}_z}{\mathcal{M}_z-1}} f_{t+1}^{Z,1} \right] \tag{B.45}$$

$$f_t^{Z,2} = Z_t + \phi_Z E_t \left[\Lambda_{u,t,t+1} (\zeta_{t+1}^Z)^{\frac{1}{\mathcal{M}_z-1}} f_{t+1}^{Z,2} \right] \tag{B.46}$$

$$\zeta_t^Z = \frac{\Pi_t}{(\Pi_{ss})^{1-\xi_z} (\Pi_{t-1})^{\xi_z}} \tag{B.47}$$

Aggregation implies

$$\int_0^1 Z_t(i) di = \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z-1}} Z_t di = Z_t \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z-1}} di$$

where we define price dispersion as $\mathcal{D}_t^Z \equiv \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z-1}} di$ which can be written recursively

$$\mathcal{D}_t^Z = (1-\phi_Z) \left(\frac{1-\phi_Z \left(\zeta_t^Z\right)^{\frac{1}{\mathcal{M}_z-1}}}{1-\phi_Z} \right)^{\mathcal{M}_z} + \phi_Z \left(\zeta_t^Z\right)^{\frac{\mathcal{M}_z}{\mathcal{M}_z-1}} \mathcal{D}_{t-1}^Z. \tag{B.48}$$

B.3.2 Energy Import Goods Sector

Each energy import variety $M_t(j)$ that the final output good producer demands is supplied by a *monopolistically competitive* import good firm $j \in [0,1]$. They buy a homogenous tradeable good on the world market from foreign exporters at foreign currency price P_t^{X*}. One can transform this into domestic currency by multiplying by the *nominal* exchange rate so that $P_t^X \equiv P_t^{X*} \mathcal{E}_t$. If for example (from the UK's perspective as the domestic economy) the nominal exchange was $\mathcal{E}_t = 0.5$ £/\$ and the importer purchases oil on the world market for $P_t^{X*} = 100\$$ this would correspond to $P_t^X = (100\$) * (0.5£/\$) = 50£$. The importers then transform and differentiate the homogenous good they purchased $M_t(j) = X_t^*(j)$.

After the monopolistically competitive importers have transformed and differentiated the import good they sell it to domestic final output producers s.t. the above derived demand schedule. Each individual import good firm is subject to nominal rigidities to the extent that it may not be able to readjust their price in a given period t with probability ϕ_m. We split the problem of import good producers into two steps: (i) the intra-temporal cost minimisation problem and (ii) the inter-temporal price setting problem.

Cost Minimisation Problem The cost minimisation problem of importer j takes the simple form

$$\min_{X_t^*(j)} \left\{ \tau_t^M \left(P_t^{X*} X_t^*(j) \right) \right\} \quad s.t. \quad X_t^*(j) \geq \left(\frac{P_t^M(j)}{P_t^M} \right)^{-\frac{\mathcal{M}^m}{\mathcal{M}^m - 1}} M_t.$$
$$\mathcal{L} = -\tau_t^M \left(P_t^{X*} \mathcal{E}_t X_t^*(j) \right) + MC_t^M(j) \left(X_t^*(j) \right)$$

The Lagrange multiplier $MC_t^M(j)$ is the (nominal) shadow cost of purchasing one more unit of the generic tradeable good from foreign exporters, e.g. the nominal marginal cost and τ_t^M is a shock to marginal costs that is isomorphic to a markup shock. The optimality conditions are given by

$$\frac{\partial \mathcal{L}}{\partial X_t^*(j)} = 0 \Leftrightarrow \tau_t^M P_t^{X,*} \mathcal{E}_t = MC_t^M(j) \Leftrightarrow \tau_t^M P_t^{X,*} \mathcal{E}_t \frac{1}{P_t} \frac{P_t^*}{P_t^*} = \frac{MC_t^M}{P_t} \Leftrightarrow \tau_t^M p_t^{X,*} \mathcal{Q}_t = mc_t^M \tag{B.49}$$

We assume that the global export price level follows the exogenous process

$$p_t^{X,*} = \left(p_{ss}^{X,*} \right)^{1-\rho_{pX*}} \left(p_{t-1}^{X,*} \right)^{\rho_{pX*}} \varepsilon_t^{pX*} \tag{B.50}$$

where the price level shock process ε_t^{pX*} follows

$$\log \varepsilon_t^{pX*} = (1 - \rho_{pX*})^2 \sigma_{pX*} \eta_t^{pX*}, \qquad \eta_t^{pX*} \sim N(0,1).$$

Also, note that

$$p_t^{X,*} = \frac{P_t^{X,*}}{P_t^*} \quad \Leftrightarrow \quad p_t^{X,F} = \frac{\Pi_t^{X,*}}{\Pi_t^*} p_{t-1}^{X,*} \tag{B.51}$$

and

$$p_t^M = \frac{P_t^M}{P_t} \quad \Leftrightarrow \quad p_t^M = \frac{\Pi_t^M}{\Pi_t} p_{t-1}^M. \tag{B.52}$$

Pricing Problem Each energy import good firm maximises its nominal profits $DIV_t^M(j)$

$$DIV_t^M(j) = P_t^M(j) M_t(j) - \tau_t^X P_t^{X,*} \mathcal{E}_t X_t^*(j) \quad \Leftrightarrow \quad div_t^M = \left(1 - mc_t^M \right) p_t^M M_t \tag{B.53}$$

Some firms may not be able to set their desired price $P_t^{M\#}(j)$. With probability ϕ_m a firm cannot reset its price in period t. In this case the firm is stuck with its previous-period price indexed to a composite of previous-period inflation and steady state inflation so that

$$P_t^M(j) = \begin{cases} P_t^{M\#}(j) & \textit{with probability: } 1 - \phi_m \\ P_{t-1}^M(j) \left(\left(\Pi_{ss}^M \right)^{1-\xi_m} \left(\Pi_{t-1}^M \right)^{\xi_m} \right) & \textit{with probability: } \phi_m \end{cases}$$

where $\xi_m \in [0,1]$ is the weight attached to previous-period inflation. Consider a firm who can reset its price in the current period $P_t^M(j) = P_t^{M\#}(j)$ and who is then stuck with its price until future period $t+s$

$$P_{t+s}^M(j) \quad = \quad P_t^{M\#}(j)\left[\left(\Pi_{ss}^M\right)^{s(1-\xi_m)}\left(\frac{P_{t+s-1}^M}{P_{t-1}^M}\right)^{\xi_m}\right].$$

The import firms solve the following optimisation problem

$$\max_{P_t^{M\#}(j)} E_t \sum_{s=0}^{\infty}(\phi_m)^s \frac{\Phi_{t+s}}{\Phi_t}\frac{\Lambda_{u,t+s}}{\Lambda_{u,t}}\left[P_{t+s}^M(j)M_{t+s|t}(j) - MC_{t+s}^M M_{t+s|t}(j)\right]$$

subject to the above derived demand constraint and assuming that a firm m always meets the demand for its good at the current price $M_{t+s|t}(j) = M_{t+s}(j)$ where $M_{t+s|t}(j)$ denotes the imports supplied in period $t+s$ by a firm j that last reset its price in period t.

$$\max_{P_t^{M\#}(j)} E_t \sum_{s=0}^{\infty}(\phi_m)^s \frac{\Phi_{t+s}}{\Phi_t}\frac{\Lambda_{it+s}^C}{\Lambda_{i,t}^C}\left[\left(P_t^{M\#}(j)\right)^{1-\frac{\mathcal{M}^m}{\mathcal{M}^m-1}}\left(\left[\left(\Pi_{ss}^M\right)^{s(1-\xi_m)}\left(\frac{P_{t+s-1}^M}{P_{t-1}^M}\right)^{\xi_m}\right]\right)^{1-\frac{\mathcal{M}^m}{\mathcal{M}^m-1}}\right.$$
$$\left.\left[\left(\frac{1}{P_{t+s}^M}\right)^{\frac{\mathcal{M}^m}{1-\mathcal{M}^m}} M_{t+s}\right] - MC_{t+s}^M\left[\left(P_t^{M\#}(j)\right)^{\frac{\mathcal{M}^m}{1-\mathcal{M}^m}}\left(\frac{\left[\left(\Pi_{ss}^M\right)^{s(1-\xi_m)}\left(\frac{P_{t+s-1}^M}{P_{t-1}^M}\right)^{\xi_m}\right]}{P_{t+s}^M}\right)^{\frac{\mathcal{M}^m}{1-\mathcal{M}^m}} M_{t+s}\right]\right].$$

Taking the derivative with respect to $P_{m,t}^{M\#}$ delivers the import price inflation schedule (B.54)

$$\frac{f_t^{M,1}}{f_t^{M,2}}\mathcal{M}^m \quad = \quad \left[\frac{1-(\phi_m)\left(\Pi_t^M\right)^{\frac{-1}{1-\mathcal{M}^m}}}{1-\phi_m}\right]^{1-\mathcal{M}^m} \tag{B.54}$$

$$f_t^{M,1} \quad = \quad mc_t^M M_t + \phi_m E_t\left[\Lambda_{u,t,t+1}\frac{\Pi_{t+1}^M}{\Pi_{t+1}}\left(\zeta_{t+1}^M\right)^{\frac{\mathcal{M}^m}{\mathcal{M}^m-1}} f_{t+1}^{M,1}\right] \tag{B.55}$$

$$f_t^{M,2} \quad = \quad M_t + \phi_m E_t\left[\Lambda_{u,t,t+1}\frac{\Pi_{t+1}^M}{\Pi_{t+1}}\left(\zeta_{t+1}^M\right)^{\frac{1}{\mathcal{M}^m-1}} f_{t+1}^{M,2}\right] \tag{B.56}$$

$$\zeta_t^M \quad = \quad \frac{\Pi_t^M}{(\Pi_{ss}^M)^{1-\xi_m}(\Pi_{t-1}^M)^{\xi_m}} \tag{B.57}$$

Import price dispersion can be written recursively

$$\mathcal{D}_t^M \quad = \quad (1-\phi_m)\left(\frac{1-\phi_m\left(\zeta_t^M\right)^{\frac{1}{\mathcal{M}^m-1}}}{1-\phi_m}\right)^{\mathcal{M}^m} + \phi_m\left(\zeta_t^M\right)^{\frac{\mathcal{M}^m}{\mathcal{M}^m-1}}\mathcal{D}_{t-1}^M. \tag{B.58}$$

We denote the amount of aggregate imports $M_t \equiv M_t$ which implies that $M_t^s = M_t\mathcal{D}_t^M$.

B.3.3 Non-Energy Export Goods Sector

The non-energy export goods sector involves two types of agents: (i) perfectly competitive export good packers and (ii) monopolistically competitive export good firms.

Export Good Packers Export good packers assemble infinitely many varieties of export goods $X_t(j)$, $j \in [0,1]$, according to the CES production function

$$X_t = \left[\int_0^1 (X_t(j))^{1-\frac{1}{\epsilon_x}} dj\right]^{\frac{\epsilon_x}{\epsilon_x - 1}}, \quad \mathcal{M}_x \equiv \frac{\epsilon_x}{\epsilon_x - 1}$$

where ϵ_x is the elasticity of substitution and $\mathcal{M}_x$ is the corresponding gross markup. Non-energy export good packers operate in a perfectly competitive market, they domestically purchase a single export variety at domestic-currency prices $P_t^X(j)$ and sell the assembled and homogenised export good X_t at foreign-currency price P_t^{EXP} on the world market. Optimal behaviour by the packers implies $P_t^{EXP} X_t = \int_0^1 P_t^{EXP}(j) X_t(j) dj$. The standard demand schedule for a specific variety j and the export price index are given by

$$X_t(j) = \left(\frac{P_t^{EXP}(j)}{P_t^{EXP}}\right)^{-\frac{\mathcal{M}_x}{\mathcal{M}_x - 1}} X_t, \qquad P_t^{EXP} \equiv \left(\int_0^1 \left(P_t^{EXP}(j)\right)^{\frac{1}{1-\mathcal{M}_x}} dj\right)^{1-\mathcal{M}_x}$$

Export Good Producers Each variety $X_t(j)$ that the packer demands and assembles is supplied by a *monopolistically competitive* export good firm $j \in [0,1]$. They buy a homogenous non-energy export good on the domestic market from final output retailers at domestic-currency price P_t^X. They re-brand this good and turn it into an imperfectly substitutable variety j. The 'production' of non-energy export goods works via a simple transformation of final output goods into the expenditure components C, X, so that the supply of a specific export variety is given by $X_t(j) = Z_t^X(j)$. The non-energy exporter producers sell the transformed export good to the packers facing the above-derived demand schedule. Each exporter is subject to nominal rigidities. We split the problem of monopolistically competitive export good firms into two steps: (i) the intra-temporal cost minimisation problem and (ii) the inter-temporal price setting problem. Also, note that

$$p_t^{EXP} = \frac{P_t^{EXP}}{P_t^*} \quad \Leftrightarrow \quad p_t^{EXP} = \frac{\Pi_t^{EXP}}{\Pi_t^*} p_{t-1}^{EXP} \tag{B.59}$$

Cost Minimisation Problem The cost minimisation problem of exporter j takes the simple form

$$\min_{Z_t^X(j)} \left\{\tau_t^X \left(P_t^X Z_t^X(j)\right)\right\} \quad s.t. \quad Z_t^X(j) \geq \left(\frac{P_t^{EXP}(j)}{P_t^{EXP}}\right)^{-\frac{\mathcal{M}_x}{\mathcal{M}_x - 1}} X_t,\ \mathcal{L}_t = -\tau_t^X \left(P_t^X Z_t^X(j)\right) + MC_t^X(j) \left(Z_t^X(j)\right).$$

The Lagrange multiplier $MC_t^X(j)$ is the (nominal) shadow cost of purchasing one more unit of the generic good that can be turned into exports from final output retailers, e.g. the nominal marginal cost and τ_t^X is a shock to marginal costs that is isomorphic to a markup shock. The optimality conditions are

$$\frac{\partial \mathcal{L}}{\partial Z_t^X(j)} = 0 \ \Leftrightarrow \ \tau_t^X P_t^X = MC_t^X(j) \Leftrightarrow \ \tau_t^X p_t^X = mc_t^X \tag{B.60}$$

Pricing Problem The objective of each export good firm is to maximise its nominal profits $DIV_t^X(j)$

$$DIV_t^X(j) = P_t^{EXP}(j) \mathcal{E}_t X_t(j) - \tau_t^X P_t^X Z_t^X(j) \ \Leftrightarrow \ div_t^X = \left(p_t^{EXP} \mathcal{Q}_t - mc_t^X\right) X_t \tag{B.61}$$

With probability ϕ_x a firm is stuck with its previous-period price indexed to a composite of previous-period inflation and steady state inflation so that

$$P_t^{EXP}(j) = \begin{cases} P_t^{EXP\#}(j) & \textit{with probability: } 1 - \phi_x \\ P_{t-1}^{EXP}(j) \left(\left(\Pi_{ss}^{EXP}\right)^{1-\xi_x} \left(\Pi_{t-1}^{EXP}\right)^{\xi_x}\right) & \textit{with probability: } \phi_x \end{cases}$$

where $\xi_x \in [0,1]$ is the weight attached to previous-period export price inflation. Consider a firm who can reset its price in the current period $P_t^{EXP}(j) = P_t^{EXP\#}(j)$ and who is then stuck with its price until future period $t+s$. The price in this case would be

$$P_{t+s}^{EXP}(j) \quad = \quad P_t^{EXP\#}(j)\left[\left(\Pi_{ss}^{EXP}\right)^{s(1-\xi_x)}\left(\frac{P_{t+s-1}^{EXP}}{P_{t-1}^{EXP}}\right)^{\xi_x}\right].$$

Subject to the above derived demand constraint and assuming that a firm j always meets the demand for its good at the current price $X_{t+s|t}(j) = X_{t+s}(j)$ the export firms solve the following optimisation problem

$$\max_{P_t^{EXP\#}(j)} E_t \sum_{s=0}^{\infty} (\phi_x)^s \frac{\Phi_{t+s}}{\Phi_t}\frac{\Lambda_{u,t+s}}{\Lambda_{u,t}}\left[\mathcal{E}_{t+s}\left(P_t^{EXP\#}(j)\right)^{\frac{1}{1-\mathcal{M}_x}}\left(\left(\Pi_{ss}^{EXP}\right)^{s(1-\xi_x)}\left(\frac{P_{t+s-1}^{EXP}}{P_{t-1}^{EXP}}\right)^{\xi_x}\right)^{\frac{1}{1-\mathcal{M}_x}}\right.$$

$$\left.\left(\frac{1}{P_{t+s}^{EXP}}\right)^{\frac{\mathcal{M}_x}{1-\mathcal{M}_x}} X_{t+s} - MC_{t+s}^X\left(P_t^{EXP\#}(j)\right)^{\frac{\mathcal{M}_x}{1-\mathcal{M}_x}}\left(\frac{\left[\left(\Pi_{ss}^{EXP}\right)^{s(1-\xi_x)}\left(\frac{P_{t+s-1}^{EXP}}{P_{t-1}^{EXP}}\right)^{\xi_x}\right]}{P_{t+s}^{EXP}}\right)^{\frac{\mathcal{M}_x}{1-\mathcal{M}_x}} X_{t+s}\right].$$

where $X_{xt+s|t}^s$ denotes the exports supplied in period $t+s$ by a firm x that last reset its price in period t. Taking the derivative with respect to $P_{x,t}^{EXP\#}$ delivers the export price inflation schedule (B.62)

$$\frac{f_t^{EXP,1}}{f_t^{EXP,2}}\mathcal{M}^X = \left[\frac{1-(\phi_x)\left(\Pi_t^{EXP}\right)^{\frac{-1}{1-\mathcal{M}_x}}}{1-\phi_x}\right]^{1-\mathcal{M}_x} \tag{B.62}$$

$$f_t^{EXP,1} = \frac{p_t^X}{p_t^{EXP}}\mathcal{Q}_t mc_t^X X_t + \phi_x E_t\left[\Lambda_{u,t,t+1}\frac{\Pi_{t+1}^{EXP}}{\Pi_{t+1}}\left(\zeta_{t+1}^{EXP}\right)^{\frac{\mathcal{M}_x}{\mathcal{M}_x-1}} f_{t+1}^{EXP,1}\right] \tag{B.63}$$

$$f_t^{EXP,2} = X_t + \phi_x E_t\left[\Lambda_{u,t,t+1}\frac{\Pi_{t+1}^{EXP}}{\Pi_{t+1}}\left(\zeta_{t+1}^{EXP}\right)^{\frac{1}{\mathcal{M}_x-1}} f_{t+1}^{EXP,2}\right] \tag{B.64}$$

$$\zeta_t^{EXP} = \frac{\Pi_t^{EXP}}{(\Pi_{ss}^{EXP})^{1-\xi_x}(\Pi_{t-1}^{EXP})^{\xi_x}} \tag{B.65}$$

Export price dispersion can be written recursively

$$\mathcal{D}_t^{EXP} \quad = \quad (1-\phi_x)\left(\frac{1-\phi_x\left(\zeta_t^{EXP}\right)^{\frac{1}{\mathcal{M}_x-1}}}{1-\phi_x}\right)^{\mathcal{M}_x} + \phi_x\left(\zeta_t^{EXP}\right)^{\frac{\mathcal{M}_x}{\mathcal{M}_x-1}}\mathcal{D}_{t-1}^{EXP}. \tag{B.66}$$

B.3.4 Retailers

There is a continuum of perfectly competitive retailers defined on the unit interval, who buy final output goods from the final output good packers at price P_t and convert them into differentiated goods representing each expenditure component: consumption and export goods. Retailer r in sector N converts goods using the following linear technology:

$$N_t(r) = Z_t^N(r), \text{ for } N \in \{C, X\}$$

where the input $Z_t^N(r)$ is the amount of the final output good bundle Z_t demanded by retail firm r in expenditure sector N and where the final good bundle, Z_t, is defined by its above stated CES aggregator. Each retailer r in sector N chooses its input $Z_t^N(r)$ to maximise profits, taking the price of its output,

$P_t^N, N \in \{C, X\}$ and the price of the final output good, P_t as given. They solve

$$\max_{Z_t^N(r)} P_t^N Z_t^N(r) - P_t Z_t^N(r)$$

with first-order condition given by

$$P_t^N = P_t, \quad N \in \{C, X\} \quad \Leftrightarrow p_t^X = P_t^X / P_t = 1, \quad p_t^C = P_t^C / P_t = 1 \tag{B.67}$$

$$\Pi_t = \Pi_t^C \tag{B.68}$$

B.4 Monetary Policy

The monetary policy maker follows a simple rule for the nominal interest rate in which it responds to persistent deviations of annual CPI inflation, $\Pi_t^{C,annual}$, from its target, $\bar{\Pi}^{annual}$, and a measure of the output gap, $\hat{Y}_t$. This gives the following rule:

$$R_t = R^{1-\theta_R} R_{t-1}^{\theta_R} \left(\frac{\Pi_t^{C,annual}}{\bar{\Pi}^{annual}} \right)^{\frac{(1-\theta_R)\theta_\Pi}{4}} \left(\hat{Y}_t\right)^{(1-\theta_R)\theta_Y} \varepsilon_t^R \tag{B.69}$$

with

$$\Pi_t^{C,annual} = \frac{P_t}{P_{t-4}^C} = \frac{P_t}{P_{t-1}^C} \frac{P_{t-1}^C}{P_{t-2}^C} \frac{P_{t-2}^C}{P_{t-3}^C} \frac{P_{t-3}^C}{P_{t-4}^C} = \Pi_t^C \Pi_t^{C,lag1} \Pi_t^{C,lag2} \Pi_{t-1}^{C,lag2} \tag{B.70}$$

$$\Pi_t^{C,lag1} = \Pi_{t-1}^C \tag{B.71}$$

$$\Pi_t^{C,lag2} = \Pi_{t-1}^{C,lag1} \tag{B.72}$$

and where $\bar{\Pi}^{annual} = (\bar{\Pi})^4$ and

$$\hat{Y}_t \equiv \frac{L_t}{L_t^{flex}} \tag{B.73}$$

where L_t^{flex} is the level of employment that would be observed if all prices and wages were flexible (to be defined below), R is the steady state nominal interest rate consistent with steady-state inflation being at target, and ε_t^R is a monetary policy shock which follows

$$\log \varepsilon_t^R = \sigma_R \eta_t^R, \qquad \eta_t^R \sim N(0,1)$$

Given the interest rate rule, the central bank will supply any quantity of money demanded at that rate. Money supply therefore equals money demand.

B.5 The World Block

B.5.1 Foreign Households

Foreign households consume, work, save and pay taxes. They own global firms and receive firm profits. We do not distinguish between unconstrained and constrained households on the world level. Each household derives utility from the sum of the utilities of the individual household members. We can express the utility function for the representative foreign household in per capita terms. In any arbitrary period $t = s$ a household maximises his/her lifetime utility $\mathcal{U}_s^*$

$$\mathcal{U}_s^* = E_s \left[\sum_{t=s}^{\infty} \beta^t \left\{ U_t^* \left(C_t^*, L_t^* \right) \right\} \right], \quad U_t^* = \left[\frac{\left(C_t^* - hC_{t-1}^* \right)^{1-\sigma} - 1}{1-\sigma} - \nu_L \frac{(L_t^*)^{1+\varphi}}{1+\varphi} \right]$$

where L_t^* is the foreign household's labor supply and φ is the elasticity of labor supply, ν_L is the relative weight on the disutility of working. Utility is maximised with respect to the budget constraint

$$W_t^* L_t^* + R_{t-1}^* B_{t-1}^* + DIV_t^{V,*} \quad = \quad P_t^{C*} C_t^* + B_t^* + T_t^{V,*}$$

where W_t^* denotes the nominal wage received by households, B_t^* denote foreign nominal risk-less bonds, which provide a nominal gross return R_t^* to the household. $DIV_t^{V,*}$ refers to the profits made by monopolistic firms that are re-distributed lump-sum to households. $T_t^{V,*}$ are lump-sum taxes imposed on households and it is transferred to value-added good producers to subsidize their costs in order to get a steady state in which the distortion from monopolistic competition is eliminated.

Lagrangian Each optimising household, i, solves the following Lagrangian in any arbitrary period t

$$\mathcal{L}_t^* \quad = \quad \sum_{\mathcal{S}^t} \pi_{\mathcal{S}^t} \sum_{t=0}^{\infty} \beta^t \left\{ U_t^* + \Lambda_t^* \left[W_t^* L_t^* + R_{t-1}^* B_{t-1}^* + DIV_t^{V,*} - P_t^{C*} C_t^* - B_t^* - T_t^{V,*} \right] \right\}$$

where Λ_t^* is the Lagrange multiplier associated with the households' resource constraint.

Optimal Choice of C^* The first-order condition for consumption is

$$\Lambda_t^* \quad = \quad \frac{U_t^{C^*}}{P_t^{C*}}, \quad \lambda_t^* \equiv U_t^{C^*} = \left(C_t^* - hC_{t-1}^* \right)^{-\sigma} \tag{B.74}$$

where we define the marginal utility of consumption as λ_t^*.

Optimal Choice of $L^{*,s}$ The first order conditions for labor supply are given by

$$\Lambda_t^* = -\frac{U_t^{L*}}{W_t^*} \quad \Leftrightarrow \quad W_t^* = MRS_t^* \equiv -\frac{U_t^{L*}}{\Lambda_t^*}$$

We define $w_t^* \equiv W_t^* / P_t^*$ and denote the real wage and the marginal rate of substitution in real terms

$$w_t^* \quad = \quad -\frac{U_t^{L,*}}{\lambda_t^*} \tag{B.75}$$

$$U_t^{L,*} \quad = \quad -\nu_L \left(L_t^* \right)^{\varphi} \tag{B.76}$$

Optimal Choice of B^* The first order conditions for bonds are

$$\Lambda_t^* \quad = \quad E_t \left[\beta \Lambda_{t+1}^* R_t^* \right] \quad \Leftrightarrow \quad 1 = E_t \left[\beta \frac{\lambda_{t+1}^*}{\lambda_t^*} \left(\Pi_{t+1}^{C*} \right)^{-1} \right] R_t^* \tag{B.77}$$

B.5.2 Household Aggregation and Market Clearing

$$\frac{1}{P_t^*}\left(W_t^* L_t^* + R_{t-1}^* B_{t-1}^* + DIV_t^{V,*}\right) \quad = \quad \frac{1}{P_t^*}\left(P_t^{C*} C_t^* + B_t^* + T_t^{V,*}\right)$$

$$w_t^* L_t^* + \frac{R_{t-1}^* b_{t-1}^*}{\Pi_t^*} + div_t^{V,*} - t_t^{V,*} \quad = \quad p_t^{C*} C_t^* + b_t^*, \quad t_t^{V,*} = (1-\tau^{\mathcal{M}^*}) w_t^* L_t^*$$

Global bonds in aggregate are in zero net supply $b_t^{*,agg} = \int_0^1 b_t^* di = 0$. Profits net of taxes are

$$div_t^{V,*} - t_t^{V,*} \quad = \quad V_t^* - \tau_t^{\mathcal{M}^*}(w_t^* L_t^*) - t_t^{V,*} = V_t^* - (w_t^* L_t^*)$$

such that

$$\begin{aligned} p_t^{C*} C_t^* \quad &= \quad w_t^* L_t^s + \underbrace{\frac{R_{t-1}^* b_{t-1}^{*,g}}{\Pi_t^*}}_{=0} + \underbrace{div_t^{V,*} - t_t^{V,*}}_{V_t^* - w_t^* L_t^*} - \underbrace{b_t^{*,g}}_{=0} \\ P_t^{C*} C_t^* \quad &= \quad P_t^* V_t^* \end{aligned} \tag{B.78}$$

B.5.3 World Trade

We assume that world trade $\mathcal{T}_t^*$ is a simple mapping of global value-added output (GDP) Y_t^* such that

$$\mathcal{T}_t^* \quad = \quad \left(\varepsilon_t^{\mathcal{T}*}\right) Y_t^*. \tag{B.79}$$

The shock $\varepsilon_t^{\mathcal{T}*}$ disturbs the relationship between world trade and world GDP. It can be interpreted as a 'world trade shock' and it follows an AR(1) process

$$\log \varepsilon_t^{\mathcal{T}*} \quad = \quad (1-\rho_{\mathcal{T}*}) \log \varepsilon^{\mathcal{T}*} + \rho_{\mathcal{T}*} \log \varepsilon_{t-1}^{\mathcal{T}*} + \left(1-\rho_{\mathcal{T}*}^2\right)^{1/2} \sigma_{\mathcal{T}*} \eta_t^{\mathcal{T}*} \qquad \eta_t^{\mathcal{T}*} \sim N(0,1).$$

Foreign Demand for Domestic Non-Energy Export Goods

The global demand schedule for the bundle of domestic non-energy exports X_t depends on the foreign currency price of domestic non-energy exports, P_t^{EXP}, relative to the world non-energy export price, P_t^{X*}, and on the world trade volume Z_t^*:

$$X_t \quad = \quad \vartheta_t^* \left(\frac{P_t^{EXP}}{P_t^{X*}}\right)^{-\varsigma^*} \mathcal{T}_t^* \quad \Leftrightarrow \quad X_t = \vartheta_t^* \left(\frac{p_t^{EXP}}{p_t^{X*}}\right)^{-\varsigma^*} \mathcal{T}_t^* \tag{B.80}$$

$$\vartheta_t^* \quad = \quad \vartheta_{ss}^* \varepsilon_t^{\vartheta*} \tag{B.81}$$

where the parameter ς_* is the elasticity of substitution between differentiated export goods in the rest of the world. ϑ_t^* can be interpreted as a shifter of the world's preference for domestic exports

$$\log \varepsilon_t^{\vartheta*} = (1-\rho_{\kappa^F}) \log \varepsilon^{\vartheta*} + \rho_{\vartheta*} \log \varepsilon_{t-1}^{\vartheta*} + \left(1-\rho_{\vartheta*}^2\right)^{1/2} \sigma_{\vartheta*} \eta_t^{\vartheta*}, \qquad \eta_t^{\vartheta*} \sim N(0,1)$$

B.5.4 Foreign Firms

Foreign output production involves two types of agents: (i) perfectly competitive foreign output packers and (ii) monopolistically competitive foreign output producers.

Perfectly Competitive Foreign Value-added Output Packers Foreign value-added output packers demand and *aggregate* infinitely many varieties of foreign value-added output goods $Y_t^*(i)$, $i \in [0,1]$ into an aggregated value-added output good Y_t^* according to the CES production function

$$Y_t^* = \left[\int_0^1 (Y_t^*(i))^{1-\frac{1}{\epsilon^*}} di\right]^{\frac{\epsilon^*}{\epsilon^*-1}}, \quad \mathcal{M}^* \equiv \frac{\epsilon^*}{\epsilon^*-1}$$

where $Y_t^*(i)$ denotes the demand for a specific foreign value-added output variety i of the value-added per-capita output good and Y_t^* denotes foreign *aggregate demand* of the value-added output good per-capita. ϵ^* is the elasticity of substitution and $\mathcal{M}^*$ is the corresponding gross markup. Optimal packer behaviour implies the standard demand schedule and price index

$$Y_t^*(i) = \left(\frac{P_t^*(i)}{P_t^*}\right)^{\frac{\mathcal{M}^*}{1-\mathcal{M}^*}} Y_t^*, \qquad P_t^* \equiv \left(\int_0^1 (P_t^*(i))^{\frac{1}{1-\mathcal{M}^*}} di\right)^{1-\mathcal{M}^*}.$$

Monopolistically Competitive Foreign Value-added Output Good Producers Each variety $Y_t^*(i)$ that the foreign value-added output good producer demands and assembles is produced and supplied according to the production function

$$Y_t^*(i) = \left(K_{t-1}^*(i)\right)^{1-\alpha_{L*}} (L_t^*(i))^{\alpha_{L*}} \tag{B.82}$$

The foreign value-added output producer will always choose to minimise its costs

$$\min_{K_{t-1}^*(i), L_t^*(i)} \left\{\tau_t^{\mathcal{M}^*}\left(W_t^* L_t^*(i) + \mathcal{R}_t^{K,*} K_{t-1}^*(i)\right)\right\} \quad s.t. \quad \left(K_{t-1}^*(i)\right)^{1-\alpha_*} (L_t^*(i))^{\alpha_*} \geq \left(\frac{P_t^*(i)}{P_t^*}\right)^{-\frac{\mathcal{M}^*}{\mathcal{M}^*-1}} Y_t^*$$

The Lagrange multiplier $MC_t^*(i)$ is the (nominal) shadow cost of producing one more unit of foreign value-added output, e.g. the nominal marginal cost and $\tau_t^{\mathcal{M}^*}$ is a shock to marginal costs that is isomorphic to a markup shock. The optimality conditions are given by

$$\frac{\partial \mathcal{L}_t}{\partial L_t^*(i)} = 0 \Leftrightarrow \tau_t^{\mathcal{M}^*} W_t^* = (\alpha_{L*}) MC_t^*(i) \left(\frac{K_{t-1}^*(i)}{L_t^*(i)}\right)^{1-\alpha_{L*}} \quad \Leftrightarrow w_t^* = \frac{mc_t^*}{\tau_t^{\mathcal{M}^*}} \frac{Y_t^*(i)}{L_t^*(i)} \tag{B.83}$$
$$\frac{\partial \mathcal{L}_t}{\partial K_{t-1}^*(i)} = 0 \Leftrightarrow \tau_t^{\mathcal{M}^*} \mathcal{R}_t^{K,*} = (1-\alpha_{L*}) MC_t^*(i) \left(\frac{K_{t-1}^*(i)}{L_t^*(i)}\right)^{\alpha_{L*}}$$

Note that the marginal cost of an individual foreign value-added output producing firm does not depend on its own quantities of capital demanded $K_{t-1}^*(i)$ or labor demanded $L_t^*(i)$, so that all firms have the same marginal cost $MC_t^*(i) = MC_t^*$. We make the simplifying assumption that on its balanced growth path world capital will be in fixed supply $\int_0^1 K_t^*(i) di = 1$ which allows us to abstract from capital and investment.

Inter-temporal Pricing Problem of foreign value-added output Producers

The objective of each foreign value-added output producing firm is to maximise its nominal profits $DIV_t^*(i)$

$$DIV_t^*(i) = P_t^*(i) Y_t^*(i) - \tau_t^{\mathcal{M}^*} (W_t^* L_t^*(i)) \quad \Leftrightarrow \quad div_t^* = Y_t^* - \tau_t^{\mathcal{M}^*} (w_t^* L_t^*)$$

Some firms may not be able to set their desired price $P_t^{*\#}(i)$ with probability ϕ_*. In this case the firm is stuck with its previous-period price indexed to a composite of $t-1$ inflation and steady state inflation

$$P_t^*(i) = \begin{cases} P_t^{*\#}(i) & \textit{with probability: } 1-\phi_* \\ P_{t-1}^*(i)\left((\Pi_{ss}^*)^{1-\xi_*}\,(\Pi_{t-1}^*)^{\xi_*}\right) & \textit{with probability: } \phi_* \end{cases}$$

where $\xi_* \in [0,1]$ is the weight attached to previous period inflation. Consider a firm who can reset its price in the current period $P_t^*(i) = P_t^{*\#}(i)$ and who is then stuck with its price until future period $t+s$. The price in this case would be

$$P_{t+s}^*(i) \quad = \quad P_t^{*\#}(i)\left[(\Pi_{ss}^*)^{s(1-\xi_*)}\left(\frac{P_{t+s-1}^*}{P_{t-1}^*}\right)^{\xi_*}\right].$$

Subject to the above derived demand constraint and assuming that a firm i always meets the demand for its good at the current price $Y_{t+s|t}^*(i) = Y_{t+s}^*(i)$ foreign value-added output good producing firms solve the following optimisation problem

$$\max_{P_t^{*\#}(i)} E_t \sum_{s=0}^{\infty}(\phi_*\beta^*)^s \frac{\Lambda_{t+s}^*}{\Lambda_t^*}\left[\left(P_t^{*\#}(i)\right)^{1-\frac{\mathcal{M}^*}{\mathcal{M}^*-1}}\left((\Pi_{ss}^*)^{s(1-\xi_*)}\left(\frac{P_{t+s-1}^*}{P_{t-1}^*}\right)^{\xi_*}\right)^{1-\frac{\mathcal{M}^*}{\mathcal{M}^*-1}}\right.$$

$$\left.\left(\frac{1}{P_{t+s}^*}\right)^{-\frac{\mathcal{M}^*}{\mathcal{M}^*-1}} Y_{t+s}^* - MC_{t+s}^*\left[\left(P_t^{*\#}(i)\right)^{-\frac{\mathcal{M}^*}{\mathcal{M}^*-1}}\left(\frac{\left[(\Pi_{ss}^*)^{s(1-\xi_*)}\left(\frac{P_{t+s-1}^*}{P_{t-1}^*}\right)^{\xi_*}\right]}{P_{t+s}^*}\right)^{-\frac{\mathcal{M}^*}{\mathcal{M}^*-1}} Y_{t+s}^*\right]\right].$$

where $Y_{t+s|t}^*(i)$ denotes the foreign value-added output supplied in $t+s$ by a firm i that last reset its price in period t. Taking the derivative with respect to $P_t^{*\#}(i)$ delivers the foreign inflation schedule (B.84)

$$\frac{f_t^{*,1}}{f_t^{*,2}}\mathcal{M}^* \quad = \quad \left[\frac{1-(\phi_*)\,(\Pi_t^*)^{\frac{-1}{1-\mathcal{M}^*}}}{1-\phi_*}\right]^{1-\mathcal{M}^*} \tag{B.84}$$

$$f_t^{*,1} \quad = \quad mc_t^* Y_t^* + \phi_* E_t\left[\beta\frac{\lambda_{t+1}^*}{\lambda_t^*}\left(\zeta_{t+1}^*\right)^{\frac{\mathcal{M}^*}{\mathcal{M}^*-1}} f_{t+1}^{*,1}\right] \tag{B.85}$$

$$f_t^{*,2} \quad = \quad Y_t^* + \phi_x E_t\left[\beta\frac{\lambda_{t+1}^*}{\lambda_t^*}\left(\zeta_{t+1}^*\right)^{\frac{1}{\mathcal{M}^*-1}} f_{t+1}^{*,2}\right] \tag{B.86}$$

$$\zeta_t^* \quad = \quad \frac{\Pi_t^*}{(\Pi_{ss}^*)^{1-\xi_*}(\Pi_{t-1}^*)^{\xi_*}} \tag{B.87}$$

Foreign price dispersion can be written recursively

$$\mathcal{D}_t^* \quad = \quad (1-\phi_*)\left(\frac{1-\phi_*\,(\zeta_t^*)^{\frac{1}{\mathcal{M}^*-1}}}{1-\phi_*}\right)^{\mathcal{M}^*} + \phi_*\,(\zeta_t^*)^{\frac{\mathcal{M}^*}{\mathcal{M}^*-1}}\,\mathcal{D}_{t-1}^*. \tag{B.88}$$

B.5.5 Foreign Retailers

There is a continuum of perfectly competitive foreign retailers defined on the unit interval, $r^* \in [0,1]$, who buy global value-added output goods from the packers at price P_t^* and convert them into consumption goods $C_t(r^*) = V_t(r*)$. Each retailer $r*$ chooses its input to maximise profits, taking the price of its output, P_t^{C*} and the price its input, P_t^* as given. The resulting first-order condition implies

$$P_t^{C*} = P_t^*, \quad \Leftrightarrow \quad \Pi_t^* \quad = \quad \Pi_t^{C*} \tag{B.89}$$

Foreign Monetary Policy

The foreign monetary policy maker follows a simple rule for the nominal interest rate in which it responds to persistent deviations of annual inflation, $\Pi_t^{C*,annual}$, from its target, $\bar{\Pi}^{C*,annual}$, and a measure of the foreign output gap, $\hat{Y}_t^*$. This gives the following rule

$$R_t^* = (R_{ss}^*)^{1-\theta_{R^*}} \left(R_{t-1}^*\right)^{\theta_{R^*}} \left(\frac{\Pi_t^{C*,annual}}{\bar{\Pi}^{C*,annual}}\right)^{\frac{(1-\theta_{R^*})\theta_{\Pi^*}}{4}} \left(\hat{Y}_t^*\right)^{(1-\theta_{R^*})\theta_{Y*}} \tag{B.90}$$

with

$$\Pi_t^{C*,annual} \quad = \quad \Pi_t^{C*}\Pi_t^{C*,lag1}\Pi_t^{C*,lag2}\Pi_{t-1}^{C*,lag2} \tag{B.91}$$

$$\Pi_t^{C*,lag1} \quad = \quad \Pi_{t-1}^{C*} \tag{B.92}$$

$$\Pi_t^{C*,lag2} \quad = \quad \Pi_{t-1}^{C*,lag1} \tag{B.93}$$

and where $\bar{\Pi}^{C*,annual} = \left(\bar{\Pi}^{C*}\right)^4$,

$$\hat{Y}_t^* \equiv Y_t^* / Y_t^{*flex} \tag{B.94}$$